Strong Mixing between Low and High Mass Scalar Mesons and Decay Processes of Scalar Mesons

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Fig. 1

- We assume the $f_0(1500)$ is the glueball candidate and this glueball mixes with $I = 0$ $L = 1$ $q\bar{q}$ scalar mesons. We analyze the strength of inter mixing between

  $\{I = 0$ $qq\bar{q}\bar{q}$ scalar mesons$\}$ and $\{I = 0$ $L = 1$ $q\bar{q}$ scalar mesons and glueball$\}$

  and the strengths of intra mixing among

  $\{I = 0$ law mass $qq\bar{q}\bar{q}$ scalar mesons$\}$

  and the strengths of intra mixing among

  $\{I = 0$ $L = 1$ $q\bar{q}$ scalar mesons and glueball$\}$.

- We analyze the decay processes of these low and high mass scalar mesons.

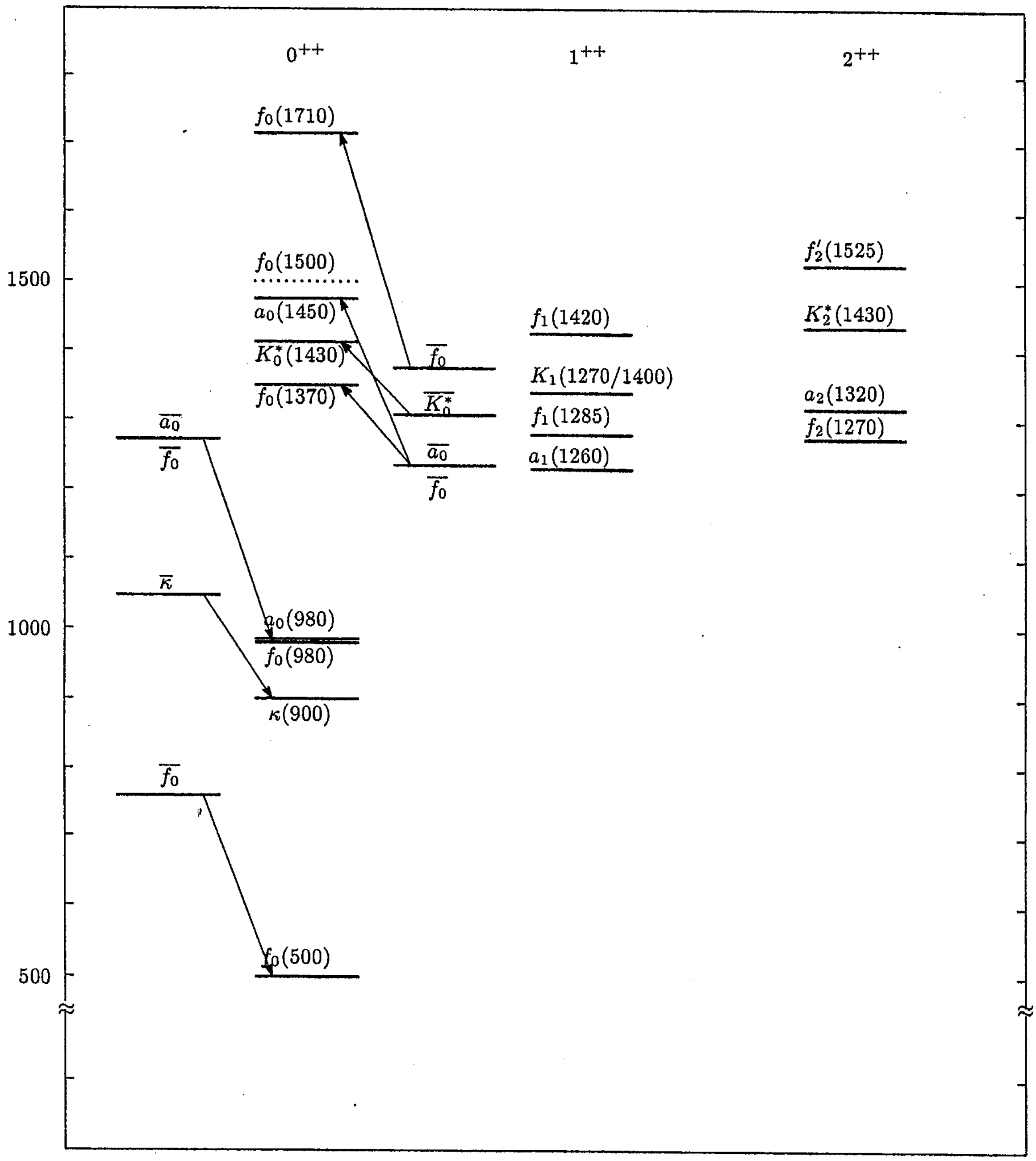


FIG. 1: The 2^{++}, 1^{++} and 0^{++} meson mass spectra. The particles with overline are particles before-mixing with masses estimated using the masses of 0^{++}, 1^{++} and 2^{++} mesons. (See the text).

2. Mixing between Low and High Mass Scalar Mesons

(2-1) Structure of low mass scalar mesons

- For the structures of the low mass scalar mesons, there are two possibilities. One is the chiral partner of the pseudoscalar nonet and the other is the $qq\bar{q}\bar{q}$ or $M\bar{M}$ molecule. We assume the $qq\bar{q}\bar{q}$ structure.
(Jaffe(1977), Black et al(1999))

- $qq\bar{q}\bar{q}$ nonet is represented by the 'dual 'quark $T_i = \epsilon_{ijk}\bar{q}^j\bar{q}^k$ and 'dual 'unti-quark filed $\bar{T}^i = \epsilon^{ijk}q_jq_k$ as

$$\bar{T}^i = \epsilon^{ijk}q_jq_k,\ T_i = \epsilon_{ijk}\bar{q}^j\bar{q}^k,\ N_i^j = T_i\bar{T}^j$$

$$\begin{array}{ccccc}
N_1^2 & \Longleftrightarrow & \bar{s}\bar{d}us & \Longleftrightarrow & a_0^+ \\
\frac{1}{2}(N_1^1 - N_2^2) & \Longleftrightarrow & \frac{1}{2}(\bar{s}\bar{d}ds - \bar{s}\bar{u}us) & \Longleftrightarrow & a_0^0 \\
N_2^1 & \Longleftrightarrow & \bar{s}\bar{u}ds & \Longleftrightarrow & a_0^- \\
N_1^3 & \Longleftrightarrow & \bar{s}\bar{d}ud & \Longleftrightarrow & \kappa^+ \\
N_2^3 & \Longleftrightarrow & \bar{s}\bar{u}ud & \Longleftrightarrow & \kappa^0 \\
N_3^2 & \Longleftrightarrow & \bar{u}\bar{d}us & \Longleftrightarrow & \overline{\kappa^0} \\
N_3^1 & \Longleftrightarrow & \bar{u}\bar{d}ds & \Longleftrightarrow & \kappa^- \\
\frac{1}{2}(N_1^1 + N_2^2) & \Longleftrightarrow & \frac{1}{2}(\bar{s}\bar{d}ds + \bar{s}\bar{u}us) & \Longleftrightarrow & f_0 \\
N_3^3 & \Longleftrightarrow & \bar{u}\bar{d}ud & \Longleftrightarrow & \sigma
\end{array}$$

- **Spectrum of low mass scalar mesons:** **Masses of** $f_0(980)$ **and** $f_0(500)$ **are represented by the masses** $a_0(980)$, $\kappa(900)$ **and mixing mass parameter** λ_0, **which describe the interaction strength of OZI rule suppression graph shown in Fig. 2.**

$$m_{f_0}^2 = m_{a_0}^2 + 2\lambda_0, \; m_{\sigma_0}^2 = 2m_\kappa^2 - m_{a_0}^2 + \lambda_0,$$

$$m_{f\sigma}^2 = \sqrt{2}\lambda_0$$

From the relation $m_s > m_{u,d}$, **we can get the desired spectrum,**

$$m_{f_0(980)}^2 \approx m_{a_0(980)}^2 > m_\kappa^2 > m_{f_0(500)}^2$$

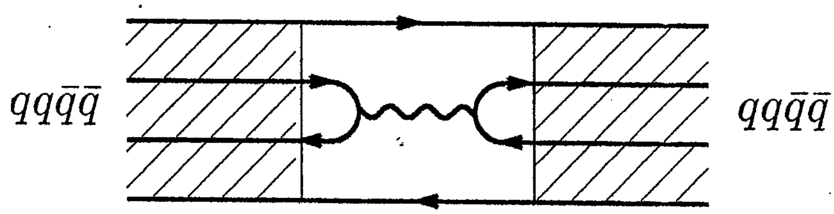


Fig.2

(2-2) **Inter-mixing between low mass $qq\bar{q}\bar{q}$ scalar mesons and high mass $L=1, q\bar{q}$ scalar mesons**

- **The inter-mixing interaction is caused by the graph shown in Fig.3 , which represents the OZI rule allowed interaction.**

$$L_{\rm int} = -\lambda_{01}\epsilon^{abc}\epsilon_{def}N_a^d N_b^{\prime e}\delta_c^f$$

$$= \lambda_{01}[a_0^+ a'^-_0 + a_0^- a'^+_0 + a_0^0 a'^0_0$$

$$+\kappa^+ K_0^{*-} + \kappa^- K_0^{*+} + \kappa^0 K_0^{*0} + \bar{\kappa}^+ \bar{K}_0^{*-}$$

$$-\sqrt{2} f_N f'_N - f_S f'_N - \sqrt{2} f_N f'_S]$$

where $N_b^{\prime a} = q_b \bar{q}^a$

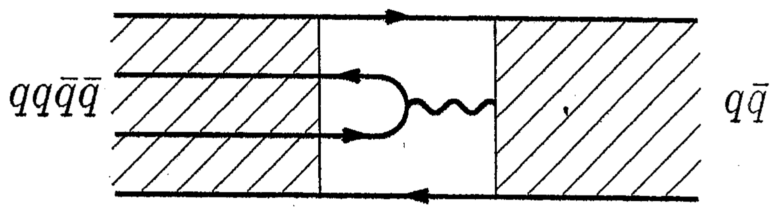


Fig.3

- **Strength of the inter-mixing**

(a) $a_0(1450)$ **and** $a_0(980)$ **mixing**

We estimate the masses before mixing

$$m_{\overline{a_0(980)}} = 1271 \pm 31 \text{MeV}, \quad m_{\overline{a_0(1450)}} = 1236 \pm 20 \text{MeV}$$

from the relation resulted from the $L \cdot S$ force.

$$m^2(2^{++}) - m^2(1^{++}) = 2(m^2(1^{++}) - m^2(0^{++}))$$

Diagonalising the mass matrix

$$\begin{pmatrix} m_{\overline{a_0(980)}} & \lambda_{01}^a \\ \lambda_{01}^a & m_{\overline{a_0(1450)}} \end{pmatrix}$$

and taking the eigenvalues of masses

$$m_{a_0(980)} = 984.8 \pm 1.4\text{MeV}, \quad m_{a_0(1450)} = 1474 \pm 19\text{MeV}$$

we can get the result

inter-mixing strength $\lambda_{01}^a = 0.600 \pm 0.028\text{GeV}^2$,

mixing angle $\theta_a = 47.1 \pm 3.5°$

(b) $\kappa(900)$ and $K_0^*(1430)$ mixing

The masses before mixing

$$m_{\overline{\kappa(900)}} = 1047 \pm 62\text{MeV}, \quad m_{\overline{K_0^*(1430)}} = 1307 \pm 11\text{MeV}$$

The eigenvalues of masses

$$m_{\kappa(900)} = 900 \pm 70\text{MeV}, \quad m_{K_0^*(1430)} = 1412 \pm 6\text{MeV}$$

From this, we get the results

Inter nixing strength $\lambda_{01}^K = 0.507 \pm 84\text{GeV}^2$,

mixing angle $\theta_K = 29.5 \pm 15.5°$

(2-3) Intra-mixing between $I = 0$ low and $I = 0$ high mass scalar mesons

- **Mixing between $I = 0, L = 1, q\bar{q}$ scalar mesons and glueball**

$$m_{f_N}^2 = m_{a_0}^2 + 2\lambda_1, \ m_{f_S}^2 = 2m_K^2 - m_{a_0}^2 + \lambda_1,$$

$$m_{NS}^2 = \sqrt{2}\lambda_1$$

$$m_{NG}^2 = \sqrt{2}\lambda_G, \ m_{SG}^2 = \lambda_G, \quad m_G^2 = \lambda_{GG}$$

λ_1 is the term of the OZI-rule suppression graph for $q\bar{q}$ shown in Fig. 4

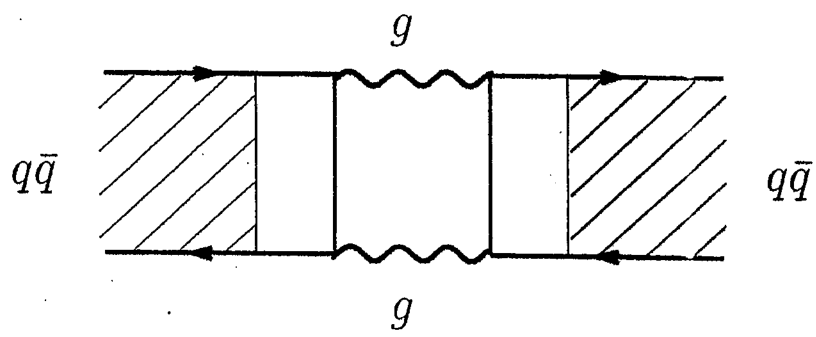


Fig.4

λ_{NG}, λ_{SG} are the transition between $q\bar{q}$ and glueball gg showed in Fig.5 (a) and λ_{GG} is the pure glueball mass shown in Fig.5 (b)

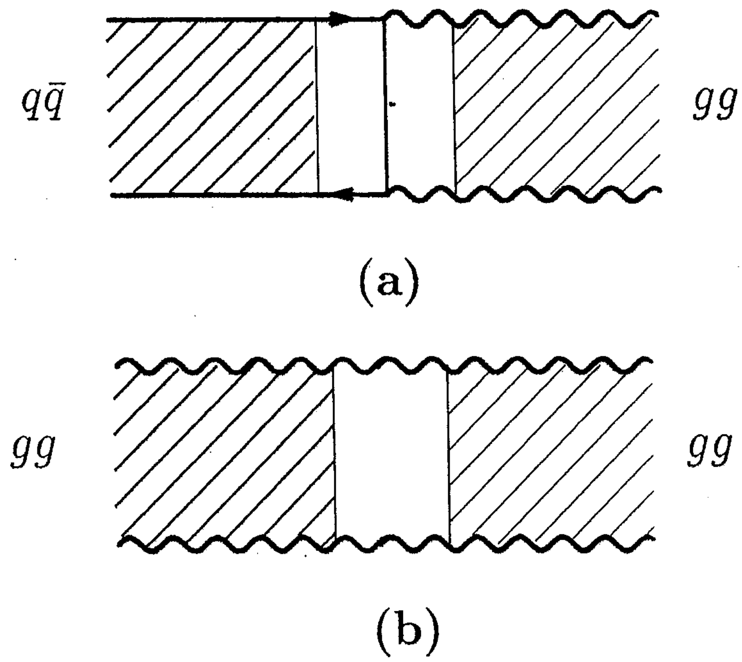


Fig. 5

- **Inter- and intra- mixing among $I = 0$ low mass and high mass scalar mesons mixing mass matrix**

$$
\begin{pmatrix}
m_N^2 + 2\lambda_0 & \sqrt{2}\lambda_0 & \lambda_{01} & \sqrt{2}\lambda_{01} & 0 \\
\sqrt{2}\lambda_0 & m_S^2 + \lambda_0 & \sqrt{2}\lambda_{01} & 0 & 0 \\
\lambda_{01} & \sqrt{2}\lambda_{01} & m_{N'}^2 + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\
\sqrt{2}\lambda_{01} & 0 & \sqrt{2}\lambda_1 & m_{S'}^2 + \lambda_1 & \lambda_G \\
0 & 0 & \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG}
\end{pmatrix}
$$

Input (unit:GeV)

$$m_N = 1.271 \pm 0.031, \;\; m_S = 0.760 \pm 0.179$$

$$m_{N'} = 1.236 \pm 0.02,\ m_{S'} = 1.374 \pm 0.003$$

$$m_{f_0(980)} = 0.980 \pm 0.010,\ m_{f_0(500)} = 0.500 \pm 0.100$$

$$m_{f_0(1370)} = 1.350 \pm 0.150,\ m_{f_0(1710)} = 1.715 \pm 0.007$$

$$m_{f_0(1500)} = 1.500 \pm 0.010.$$

We get the result:

(a) $f_0(1500)$ is assumed as glueball

$$\lambda_{01} = 0.53 \pm 0.04\mathrm{GeV}^2,\ \lambda_0 = 0.03 \pm 0.04\mathrm{GeV}^2$$

$$\lambda_1 = 0.07 \pm 0.05\mathrm{GeV}^2,\ \lambda_G = 0.23 \pm 0.06\mathrm{GeV}^2$$

$$\lambda_{GG} = (1.53 \pm 0.03)^2\mathrm{GeV}^2$$

$$\begin{pmatrix} f_0(980) \\ f_0(500) \\ f_0(1370) \\ f_0(1710) \\ f_0(1500) \end{pmatrix} = \begin{pmatrix} 0.719 \pm 0.061 & -0.389 \pm 0.096 & -0.095 \pm 0.059 & -0.558 \pm 0.041 & 0.111 \pm 0.026 \\ 0.264 \pm 0.065 & 0.789 \pm 0.080 & -0.531 \pm 0.086 & -0.104 \pm 0.060 & 0.108 \pm 0.035 \\ 0.048 \pm 0.047 & 0.433 \pm 0.062 & 0.681 \pm 0.042 & -0.480 \pm 0.042 & -0.311 \pm 0.053 \\ 0.533 \pm 0.100 & 0.174 \pm 0.029 & 0.464 \pm 0.044 & 0.527 \pm 0.044 & 0.445 \pm 0.164 \\ -0.403 \pm 0.107 & 0.050 \pm 0.047 & 0.089 \pm 0.088 & -0.361 \pm 0.122 & 0.813 \pm 0.102 \end{pmatrix}$$

$$
\times \begin{pmatrix} f_N \\ f_S \\ f'_N \\ f'_S \\ f_G \end{pmatrix}
$$

(b) $f_0(1710)$ is assumed as glueball

$$
\lambda_{01} = 0.42 \pm 0.05 \mathrm{GeV}^2, \;\; \lambda_0 = 0.04 \pm 0.02 \mathrm{GeV}^2
$$

$$
\lambda_1 = -0.08 \pm 0.05 \mathrm{GeV}^2, \;\; \lambda_G = 0.28 \pm 0.03 \mathrm{GeV}^2
$$

$$
\lambda_{GG} = (1.64 \pm 0.03)^2 \mathrm{GeV}^2
$$

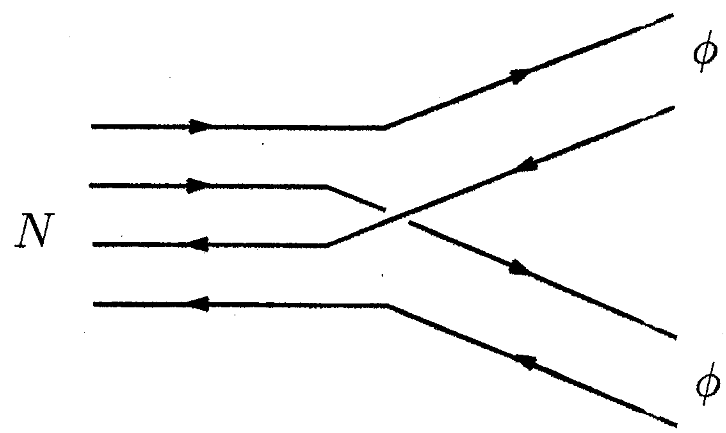
$$
\begin{pmatrix} f_0(980) \\ f_0(500) \\ f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.582 \pm 0.046 & -0.547 \pm 0.090 & -0.258 \pm 0.149 & -0.522 \pm 0.019 & 0.131 \pm 0.050 \\ 0.205 \pm 0.075 & 0.702 \pm 0.110 & -0.623 \pm 0.076 & -0.194 \pm 0.071 & 0.099 \pm 0.016 \\ 0.271 \pm 0.035 & 0.414 \pm 0.065 & 0.690 \pm 0.026 & -0.539 \pm 0.042 & -0.124 \pm 0.035 \\ -0.708 \pm 0.038 & -0.045 \pm 0.052 & -0.075 \pm 0.035 & -0.581 \pm 0.041 & 0.355 \pm 0.064 \\ 0.202 \pm 0.037 & 0.064 \pm 0.010 & 0.239 \pm 0.043 & 0.262 \pm 0.051 & 0.907 \pm 0.034 \end{pmatrix}
$$

$$
\times \begin{pmatrix} f_N \\ f_S \\ f'_N \\ f'_S \\ f_G \end{pmatrix}
$$

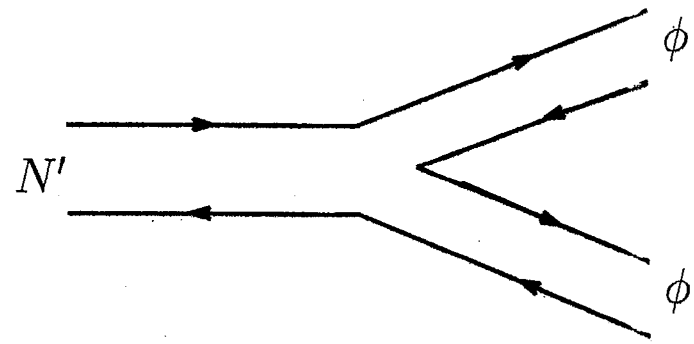
3. Decay processes of scalar mesons and glueball

- **$N\phi\phi$, $N'\phi\phi$ and $G\phi\phi$ Coupling**

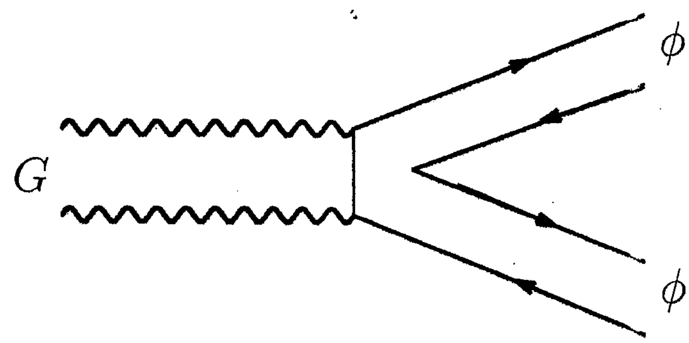
$$A\varepsilon^{abc}\varepsilon_{def}N_a^d\partial^\mu\phi_b^e\partial_\mu\phi_c^f \quad \text{for } N\phi\phi \text{ coupling,}$$



$$A'N_a'^b\{\partial^\mu\phi_b^c,\ \partial_\mu\phi_c^a\} \quad \text{for } N'\phi\phi \text{ coupling,}$$



$$A''G\{\partial^\mu\phi_a^b,\ \partial_\mu\phi_b^a\} \quad \text{for } G\phi\phi \text{ coupling,}$$



(2-1) $a_0(980)$, $a_0(1450)$ and $K_0^*(1450)$ decays

- **coupling constants**

$$\gamma_{a_0(980)\to K\bar{K}} = 2(A\cos\theta_a - A'\sin\theta_a)$$
$$\gamma_{a_0(980)\to\pi\eta} = 2(A\cos\theta_a\sin\theta_P - \sqrt{2}A'\sin\theta_a\cos\theta_P)$$
$$\gamma_{a_0(1450)\to K\bar{K}} = 2(A\sin\theta_a + A'\cos\theta_a)$$
$$\gamma_{a_0(1450)\to\pi\eta} = 2(A\sin\theta_a\sin\theta_P + \sqrt{2}A'\cos\theta_a\cos\theta_P)$$
$$\gamma_{a_0(1450)\to\pi\eta'} = 2(-A\sin\theta_a\cos\theta_P + \sqrt{2}A'\cos\theta_a\sin\theta_P)$$
$$\gamma_{K_0^*(1430)\to\pi K} = 2(A\sin\theta_K + A'\cos\theta_K)$$

θ_P **is** η-η' **mixing and** $\theta_P = \theta_{0-8} + 54.7^\circ$

- **decay widths**

$$\Gamma_{a_0(980)\to K\bar{K}} = \frac{\gamma^2_{a_0(980)\to K\bar{K}}}{32\pi}\frac{q_{a_0(980)\to K\bar{K}}}{m^2_{a_0(980)}}m^4_{a_0(980)\to K\bar{K}}$$
$$\Gamma_{a_0(980)\to\pi\eta} = \frac{\gamma^2_{a_0(980)\to\pi\eta}}{32\pi}\frac{q_{a_0(980)\to\pi\eta}}{m^2_{a_0(980)}}m^4_{a_0(980)\to\pi\eta}$$
$$\Gamma_{a_0(1450)\to K\bar{K}} = \frac{\gamma^2_{a_0(1450)\to K\bar{K}}}{32\pi}\frac{q_{a_0(1450)\to K\bar{K}}}{m^2_{a_0(1450)}}m^4_{a_0(1450)\to K\bar{K}}$$
$$\Gamma_{a_0(1450)\to\pi\eta} = \frac{\gamma^2_{a_0(1450)\to\pi\eta}}{32\pi}\frac{q_{a_0(1450)\to\pi\eta}}{m^2_{a_0(1450)}}m^4_{a_0(1450)\to\pi\eta}$$
$$\Gamma_{a_0(1450)\to\pi\eta'} = \frac{\gamma^2_{a_0(1450)\to\pi\eta'}}{32\pi}\frac{q_{a_0(1450)\to\pi\eta'}}{m^2_{a_0(1450)}}m^4_{a_0(1450)\to\pi\eta'}$$
$$\Gamma_{K_0^*(1430)\to\pi K} = \frac{3}{2}\frac{\gamma^2_{K_0^*(1430)\to\pi K}}{32\pi}\frac{q_{K_0^*(1430)\to\pi K}}{m^2_{K_0^*(1430)}}m^4_{K_0^*(1430)\to\pi K}$$

where

$$q_{M\to m_1m_2} = \sqrt{(\frac{M^2+m_2^2-m_1^2}{2M})^2 - m_2^2}$$

for $M \approx m_1 + m_2$

$$q_{M\to m_1 m_2} = \mathrm{Re}\frac{1}{\sqrt{2\pi}\Delta_M}\int_{M-10\Delta_M}^{M+10\Delta_M} e^{-\frac{(m-M)^2}{2\Delta_M^2}}$$

$$\times\sqrt{(\frac{m^2+m_2^2-m_1^2}{2m})^2 - m_2^2}\, dm$$

$$m_{M\to m_1 m_2} = \sqrt{M^2 - m_1^2 - m_2^2}$$

- **Experimental data**

$$\Gamma_{a_0(980)\to \mathrm{all}} = 75 \pm 25\mathrm{MeV}$$
$$\Gamma_{a_0(980)\to K\bar{K}}/\Gamma_{a_0(980)\to\pi\eta} = 0.177 \pm 0.024$$
$$\Gamma_{a_0(1450)\to \mathrm{all}} = 265 \pm 13\mathrm{MeV}$$
$$\Gamma_{a_0(1450)\to K\bar{K}}/\Gamma_{a_0(1450)\to\pi\eta} = 0.88 \pm 0.23$$
$$\Gamma_{a_0(1450)\to\pi\eta'}/\Gamma_{a_0(1450)\to\eta\pi} = 0.35 \pm 0.16$$
$$\Gamma_{K_0^*(1430)\to\pi K} = 273 \pm 44\mathrm{MeV}$$
$$\Gamma_{a_0(1450)\to \mathrm{all}}/\Gamma_{K_0^*(1430)\to \mathrm{all}} = 0.97 \pm 0.16$$
$$\theta_a = (47.1 + 3.5)^\circ, \quad \theta_K = (29.5 + 15.5)^\circ,$$

- **We get the allowed values for** A **and** A' **in the** $\chi^2 \leqq 8.383$ **corresponding to the 30% C.L. on**

degree of freedom 7.

$$A = -0.025 \pm 0.525, \quad A' = -2.7 \pm 0.3,$$
$$\theta_P = 49.7^\circ \pm 5.0^\circ$$
$$\Gamma_{a_0(980)\to \mathrm{all}} = 24 \pm 14\mathrm{MeV}$$
$$\Gamma_{a_0(980)\to K\bar{K}}/\Gamma_{a_0(980)\to \pi\eta} = 0.14$$
$$\Gamma_{a_0(1450)\to \mathrm{all}} = 267\mathrm{MeV}$$
$$\Gamma_{a_0(1450)\to K\bar{K}}/\Gamma_{a_0(1450)\to \pi\eta} = 0.62$$
$$\Gamma_{a_0(1450)\to \pi\eta'}/\Gamma_{a_0(1450)\to \eta\pi} = 0.46$$
$$\Gamma_{K_0^*(1430)\to \pi K} = 282\mathrm{MeV}$$
$$\Gamma_{a_0(1450)\to \mathrm{all}}/\Gamma_{K_0^*(1430)\to \mathrm{all}} = 0.94$$

(2-2) $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ **decays**

- **coupling constants for** $f_0(M)$, $(M = 980, 1370,$ $1500,\ 1710)$

$$\begin{aligned}
\gamma_{f_0(M)\to\pi\pi} &= 2(-AR_{f_0(M)S} + \sqrt{2}A'R_{f_0(M)N'} \\
&\quad +2A''R_{f_0(M)G}) \\
\gamma_{f_0(M)\to K\bar{K}} &= \sqrt{2}(-AR_{f_0(M)N} + A'R_{f_0(M)N'} \\
&\quad +\sqrt{2}A'R_{f_0(M)S'} + 2\sqrt{2}A''R_{f_0(M)G}) \\
\gamma_{f_0(M)\to\eta\eta} &= 2(-AR_{f_0(M)N}\cos\theta_P\sin\theta_P \\
&\quad +\tfrac{1}{2}AR_{f_0(M)S}\cos^2\theta_P \\
&\quad +\tfrac{1}{\sqrt{2}}A'R_{f_0(M)N'}\cos^2\theta_P \\
&\quad +A'R_{f_0(M)N'}\sin^2\theta_P + A''R_{f_0(M)G}) \\
\gamma_{f_0(M)\to\eta\eta'} &= 2(AR_{f_0(M)N}\cos 2\theta_P \\
&\quad +\tfrac{1}{2}AR_{f_0(M)S}\sin 2\theta_P \\
&\quad +\tfrac{1}{\sqrt{2}}A'R_{f_0(M)N'}\sin 2\theta_P \\
&\quad -A'R_{f_0(M)N'}\sin 2\theta_P)
\end{aligned}$$

where $R_{f_0(M)}$ **are matrix elements**

$$\begin{pmatrix} f_0(980) \\ f_0(500) \\ f_0(1370) \\ f_0(1710) \\ f_0(1500) \end{pmatrix} = [R_{f_0(M)I}] \begin{pmatrix} f_N \\ f_S \\ f'_N \\ f'_S \\ f_G \end{pmatrix}$$

$$I = N, S, N', S', G$$

- **decay widths for** $f_0(M)$, $(M = 980, 1370, 1500, 1710)$

$$\Gamma_{f_0(M)\to\pi\pi} = \frac{5}{4}\frac{\gamma^2_{f_0(M)\to\pi\pi}}{32\pi}\frac{q_{f_0(M)\to\pi\pi}}{m^2_{f_0(M)}} m^4_{f_0(M)\to\pi\pi}$$

$$\Gamma_{f_0(M)\to K\bar{K}} = 2\frac{\gamma^2_{f_0(M)\to K\bar{K}}}{32\pi}\frac{q_{f_0(M)\to K\bar{K}}}{m^2_{f_0(M)}} m^4_{f_0(M)\to K\bar{K}}$$

$$\Gamma_{f_0(M)\to\eta\eta} = \frac{\gamma^2_{f_0(M)\to\eta\eta}}{32\pi}\frac{q_{f_0(M)\to\eta\eta}}{m^2_{f_0(M)}} m^4_{f_0(M)\to\eta\eta}$$

$$\Gamma_{f_0(M)\to\eta\eta'} = \frac{\gamma^2_{f_0(M)\to\eta\eta'}}{32\pi}\frac{q_{f_0(M)\to\eta\eta'}}{m^2_{f_0(M)}} m^4_{f_0(M)\to\eta\eta'}$$

- **experimental data**

$$\Gamma_{f_0(980)\to all} = 70 \pm 30\text{MeV}$$
$$\Gamma_{f_0(980)\to\pi\pi}/\Gamma_{f_0(980)\to\pi\pi+K\bar{K}} = 0.74 \pm 0.07(\text{PD undecided})$$
$$\Gamma_{f_0(1370)\to \text{all}} = 350 \pm 150\text{MeV}$$
$$\Gamma_{f_0(1370)\to\pi\pi}/\Gamma_{f_0(1370)\to \text{all}} = 0.26 \pm 0.09(\text{PD undecided})$$
$$\Gamma_{f_0(1370)\to K\bar{K}}/\Gamma_{f_0(1370)\to \text{all}} = 0.35 \pm 0.13(\text{PD undecided})$$
$$\Gamma_{f_0(1500)\to \text{all}} = 109 \pm 7\text{MeV}$$
$$\Gamma_{f_0(1500)\to\pi\pi}/\Gamma_{f_0(1500)\to \text{all}} = 0.454 \pm 0.104(\text{PD undecided})$$
$$\Gamma_{f_0(1500)\to K\bar{K}}/\Gamma_{f_0(1500)\to \text{all}} = 0.044 \pm 0.021(\text{PD undecided})$$
$$\Gamma_{f_0(1500)\to\eta\eta}/\Gamma_{f_0(1500)\to\pi\pi} = 0.18 \pm 0.03$$
$$\Gamma_{f_0(1500)\to\eta\eta'}/\Gamma_{f_0(1500)\to\pi\pi} = 0.095 \pm 0.026$$
$$\Gamma_{f_0(1500)\to\eta\eta'}/\Gamma_{f_0(1500)\to\eta\eta} = 0.29 \pm 0.16$$
$$\Gamma_{f_0(1710)\to \text{all}} = 125 \pm 10\text{MeV}$$
$$\Gamma_{f_0(1710)\to\pi\pi}/\Gamma_{f_0(1710)\to K\bar{K}} = 0.39 \pm 0.14$$
$$\Gamma_{f_0(1710)\to K\bar{K}}/\Gamma_{f_0(1710)\to \text{all}} = 0.38 \pm 0.14(\text{PD undecided})$$
$$\Gamma_{f_0(1710)\to\eta\eta}/\Gamma_{f_0(1710)\to \text{all}} = 0.18 \pm 0.08(\text{PD undecided})$$
$$\Gamma_{f_0(1710)\to\eta\eta}/\Gamma_{f_0(1710)\to K\bar{K}} = 0.48 \pm 0.15$$

- **We cannot get the allowed values for A, A' and A'' in the $\chi^2 \leqq 16.222$ corresponding to the 30% C.L. on degree of freedom 14 for the case in which $f_0(1500)$ is glueball.**

- **The allowed values for A, A' and A'' in the $\chi^2 = 16.222$ corresponding to the 30% C.L. on degree of freedom 14 for the $f_0(1710)$ glue-**

ball case.

$$A = -2.9 \pm 0.15, \quad A' = -1.97 \pm 0.08,$$
$$A'' = 0.603 \pm 0.015, \quad \theta_P = (30.5 \pm 1.5)^\circ$$
$$\Gamma_{f_0(980)\to\pi\pi+\mathrm{K}\bar{\mathrm{K}}} = 32\mathrm{MeV}$$
$$\Gamma_{f_0(980)\to\pi\pi}/\Gamma_{f_0(980)\to\mathrm{all}} = 0.75$$
$$\Gamma_{f_0(1370)\to\pi\pi+\mathrm{K}\bar{\mathrm{K}}+\eta\eta} = 137\mathrm{MeV}$$
$$\Gamma_{f_0(1370)\to\pi\pi}/\Gamma_{f_0(1370)\to\mathrm{all}} = 0.322$$
$$\Gamma_{f_0(1370)\to K\bar{K}}/\Gamma_{f_0(1370)\to\mathrm{all}} = 0.004$$
$$\Gamma_{f_0(1500)\to\pi\pi+\mathrm{K}\bar{\mathrm{K}}+\eta\eta+\eta\eta'} = 52.5$$
$$\Gamma_{f_0(1500)\to\pi\pi}/\Gamma_{f_0(1500)\to\mathrm{all}} = 0.331$$
$$\Gamma_{f_0(1500)\to K\bar{K}}/\Gamma_{f_0(1500)\to\mathrm{all}} = 0.069$$
$$\Gamma_{f_0(1500)\to\eta\eta}/\Gamma_{f_0(1500)\to\pi\pi} = 0.17$$
$$\Gamma_{f_0(1500)\to\eta\eta'}/\Gamma_{f_0(1500)\to\pi\pi} = 0.075$$
$$\Gamma_{f_0(1500)\to\eta\eta'}/\Gamma_{f_0(1500)\to\eta\eta} = 0.439$$
$$\Gamma_{f_0(1710)\to\pi\pi+\mathrm{K}\bar{\mathrm{K}}+\eta\eta+\eta\eta'} = 122\mathrm{MeV}$$
$$\Gamma_{f_0(1710)\to\pi\pi}/\Gamma_{f_0(1710)\to K\bar{K}} = 0.397$$
$$\Gamma_{f_0(1710)\to K\bar{K}}/\Gamma_{f_0(1710)\to\mathrm{all}} = 0.628$$
$$\Gamma_{f_0(1710)\to\eta\eta}/\Gamma_{f_0(1710)\to\mathrm{all}} = 0.086$$
$$\Gamma_{f_0(1710)\to\eta\eta}/\Gamma_{f_0(1710)\to K\bar{K}} = 0.137$$

3. Conclusion

- Coupling constant A' for $N' \to \phi\phi$ is about -2.8 for $I = 1$, $I = 1/2$ and $I = 0$ scalar meson decays.

- Coupling constant A for $N \to \phi\phi$ is $-0.5 \sim 0.5$ for $I = 1$ and $I = 1/2$ scalar meson decays and about -2.9 for $I = 0$ scalar meson decays. This discrepancy should be explained in next work.

- Coupling constant A'' for $G \to \phi\phi$ is about 0.6.

- $f_0(1710)$ is preferred to be the glueball candidate rather than $f_0(1500)$.

- θ_P is $30^\circ \sim 50^\circ$.