2003. 2.

From Yukawa to M-Theory (S. Tanaka, 2003 2/24)

Let me first refer to the historical background of my talk. I moved to Kyoto university at the end of **1957**. Professor Yukawa(1907~81) was just 50 aged. He was in the middle of research of nonlocal field theory. I remember that he was at these days highly sensitive to the new proposal of **Heisenberg's Nonliner Spinor Theory**(1953), which was regarded as a work aiming at "a unified theory of universe". Surely, at these days, every things seemed exciting to us. Let me cite several important works at these days:

Heisenberg: S-Matrix(fundamental length l_0) (1943)
Yukawa: Nonlocal Field Theory (1947) … $[U, x_\mu] \neq 0$
*) **Snyder, Yang: Quantized Space-Time (1947)** … $[x_\mu, x_\nu] \neq 0$
Tomonaga, Schwinger, et al: Renormalization theory (1948)
Fermi-Yang: Composite Model of π-meson (1949)
Pauli-Villars: Regularization (1950)
Heisenberg: Nonlinear Spinor Theory (1953)
Yang-Mills: Nonabelian Gauge Theory (1954)
Sakata: Composite model of Hadrons (1956)

Here I remember also Philosophical and methodological disputes between Sakata and Yukawa about their different approaches in Composite Theory and Nonlocal Theory, as was seen later.

$ Yukawa's Challenge on Conventional "Spacetime and Local Field Theory" (1934) (マルク理論)

Yukawa's full-scale investigation of **Nonlocal theory** begins in 1940's, it should be noted, however, that its original idea goes back to April, in 1934, to our surprise, just in midst of proposal of his Meson Theory. In the annual meeting of P-M Society, he talked

1 *) [Jabier] the late 1930's
Heisenberg → Peierls → Pauli → Oppenheimer → Snyder

"On Provability Amplitude in Relativistic Quantum Mechanics" (Yukawa, 1934))

in a strong sympathy with Dirac's similar idea of

"Generalized Transformation Function" (Dirac, 1933)

Here, Dirac proposed Generalization of Concept of Probability in Quantum Mechanics, (by introducing Integral of lagrangian density over "Arbitrary form of Space-time region."

Yukawa tried to find in this Dirac's idea a possibility of fundamental breakthrough to overcome the so-called

"Divergence Difficulty in Quantum Field Theory,"

which was explicitly pointed out by the famous paper of Heisenberg-Pauli (1929) and gave serious influences on Yukawa over all his life to solve the problem. It ultimately led him to the firm belief:

"Fundamental Particles must never be point-like, but have their own proper space-time extension."

In fact, he tried to apply the above Dirac's action integral over arbitrary space-time region to the minimal space-time region of the order of Heisenberg's fundamental length l_0, by assuming that in such a minimal region, Conventional Causal Relation between definite initial and final times, t_1, t_2 no longer holds, and

{Inseparability between Cause and Effect becomes essential in such a small region relevant to the fundamental length of elementary particles.}

Conventional Cause-Effect

Effect

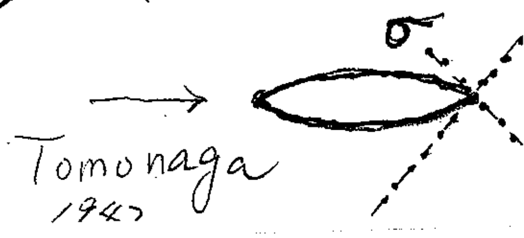
t2

$$A=\int_{t_1}^{t_2} dt\, \mathcal{L}$$

t1

Cause

Inseparable Cause-Effect



We find that this idea revives nearly **30 years later** in his Theory of Elementary Domain (1966).

$ Bilocal Field Theory

On the challenge to **nonlocal field theory**, however, which begins in a full-scale in 1947, Yukawa seems still careful to directly accept the above idea of minimal region, but **leaving space-time concept untouched**, and attempted to introduce **nonlocal field** U which is **non-commutative** with space-time coordinates

$$[U, X\mu] \neq 0$$

in accord with Markov(1940). Under the space-time coordinate representation basis,

$$| x_\mu >, \qquad \leftarrow \quad [x_\mu, x_\nu] = 0$$

the above relation immediately leads us to the **Bilocal field:**

$$U(x_\mu, x_\mu') \equiv < x_\mu | U | x_\mu' > = U(X_\mu, r_\mu)$$

that is, bilocal field or two-point field $U(x_\mu, x_\mu')$ is rewritten in terms of **center of coordinates** "external" $X_\mu = (x_\mu + x_\mu')/2$ and "**internal**" **coordinates:**

$$r_\mu = x_\mu - x_\mu'.$$

As seen in the so-called Fierz's Expansion (1950),

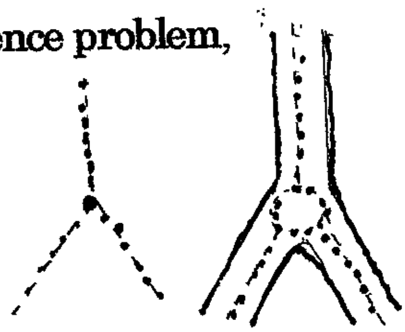
$$U(X_\mu, r_\mu) = \Sigma_{k} \int d\alpha \; \Phi_{\mu 1, \mu 2, \mu 3, \ldots \mu k}(X + \alpha\, r) \; F_{\mu 1, \mu 2, \mu 3, \ldots \mu k}(r)$$

Bilocal Field turns out to be **the infinite ensemble of local component fields** $\Phi_{\mu 1, \mu 2, \mu 3, \ldots \mu k}(X)$ **with various spin** k

This fact suggests that **nonlocal field theory** gives a possibility of **"Unified description of elementary Particles"**.

On the other hand, however, it leads us again to **Unwanted Local Component Fields** $\Phi_{\mu 1, \mu 2, \mu 3, \ldots \mu k}(X)$, possibly connected with UV divergence difficulty. (Fierz, Hara-Shimazu(1950)).

Yukawa expected that this dilemma might be overcome by virtue of the nonlocal interactions of local component fields, whose nonlocal form-factor could suitably reflect the whole effects of local component fields and the proper spatial extension of original bilocal fields. However, there remained unsolved many important questions concerning causality or unitarity, other than the divergence problem, all of which, I believe, are left unsolved even in the present Super-string theory, as will be remarked finally.



$ From Bilocal to Multi-local field

Although Yukawa's bilocal field theory was formulated precisely and rigorously (1949) under the Lorentz-covariance, the reciprocity principle (Born, 1938), the correspondence principle to the local field theory and so on, the bilocal field itself seemed to be too limited to describe hadrons in general, so Yukawa and his collaborators tried to extend the bilocal field as two-point function to multi-point fields:

$$U(x_\mu, x_\mu') \quad \rightarrow \quad U(x_\mu^{(1)}, x_\mu^{(2)}, x_\mu^{(3)} \ldots).$$

One sees, however, that this extension lacks a definite principle to formulate itself in contrast to bilocal field theory, but seems rather to imitate the wave-functions of many-body system in the composite model of hadrons. ?

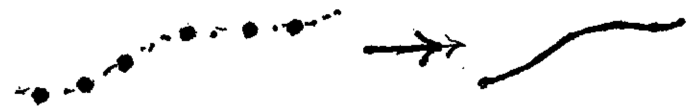
$ Urciton scheme (Ishida, 1971)

In this connection, I would like here to notice Professor Ishida's Urciton scheme (1971) as an attempt to connect both ideas of Yukawa's nonlocal theory and Sakata's composite theory, by introducing the exciton-like idea into hadronic level, which seems somewhat related to Yukawa's Elementary Domain or the present-day D(irechlet)-branes in Superstring theories stated later.

$ String model in the first stage or Pre-String theory (1968~)

The situation drastically changed, when the extreme limit of multi-point field was related to **string field or string model of one-dimensionally extended object**, $x_\mu(\sigma)$;

$U(x_\mu^{(1)}, x_\mu^{(2)}, x_\mu^{(3)} \ldots) \rightarrow U(x_\mu(\sigma))$,



which appeared immediately after **Veneciano model (1968)** devised for the explanation of the dual behavior of **Regge-poles and resonances** in high energy **hadron reactions**, and developed into the present-day Super-String Theories.

$ Elementary Domain (1966)

Before going into the problem, let us mention briefly on the Yukawa's idea of Elementary Domain. This was his first and final challenge to **space-time structure itself** in stead of simple **modification of point model of elementary particles as in Nonlocal field.** As seen in the title of the first paper (1966),

"Atomistics and the Divisibility of Space and Time",

it is clear that it originates in the idea of **"Minimal space-time region" proposed nearly 30 years ago**, explained before in detail.

Yukawa tried to describe the minimal region, that is, the so-called Elementary domain **D, in terms of various parameters:**

i) Center of Domain

$$X\mu = \int_D d^4x \; x_\mu \,/\, V_D$$

ii) Moments of Extension

$$I_{\mu 1, \mu 2, \ldots, \mu n} = \int_D d^4x \,(x_{\mu 1} - X_{\mu 1}) \ldots (x_{\mu n} - X_{\mu n}) / V_D$$

They are to be **observables describing various modes of deformation or excitation of an elementary domain, each of which are considered to correspond to the different kinds of elementary particles.**

"Space-time Description of Elementary Ptl"

Yukawa and his collaborators tried to introduce **Difference Equation** in place of usual differential equations, which gives the connection between the deformations of adjacent elementary domains. **The theory, however, remained unaccomplished.**

Yukawa left the following statement (1978): " **If one proceeds along this way, it might ultimately lead to the problem of quantization of space-time itself. The concept of Elementary Domain may be insufficient, because of the fact that it still presumes behind it Minkowski-space or the four-dimentional continuum. The solution, however, leaves entirely in the future.**" 合川 Yukawa's Revelation 「時空変革」

At this point, it is quite interesting to notice several important topics in the recent development of superstring theory, which seem deeply related to this Yukawa's concern or his long-sought goal. They are exemplified by the ideas of **D(irichlet)-branes or Holographic Hypothesis**, stated below.

$ M-Theory

As was already remarked, **the original string theory** appeared as **hadron models at the end 1960's.** The situation drastically changed **in the middle of 1970's**, when the **closed string theory seems to have a possibility of presenting naturally theoretical frameworks of quantum gravity as well as gauge theory.** In fact, after the middle of 1980's, especially Superstring Theory turned to be expected as a **Unified Theory of fundamental forces and matters in Nature including gravity.**

During the past decade, especially after **1995**, which is called the **second stage of Superstring theory**, it occurred that the familiar five superstring theories are unified (**Witten, 1995**), that is,

TYPE-I, TYPE-IIA, IIB, and two Hetrotic String theories all

formulated in D=10 are unified into a single fundamental theory, the so-called M-theory, F-theory, or S-theory in a hidden higher dimensional space-time , (D=11, 12, or 13).

$ Analogy with Quantum Mechanics in 1920's

The situation is sometimes expressed on the analogy of the well-known historical experience of the unification of Schrödinger's Wave-Mechanics and Heisenberg's Matrix-Dynamics at the birth of quantum mechanics in 1920's where it became clear that both approaches were merely due to **the different choice of representation bases in Hilbert space.**

As a matter of fact, the concept of string itself, as a linearly extended object, is **not necessarily so drastic**, but rather naturally conceivable in the continuous limit of multi-point particle system.

On the contrary, the so-called **Dirichlet-branes seem more radical:**

D-p branes: p-dimensionally Extended Objects arbitrary p

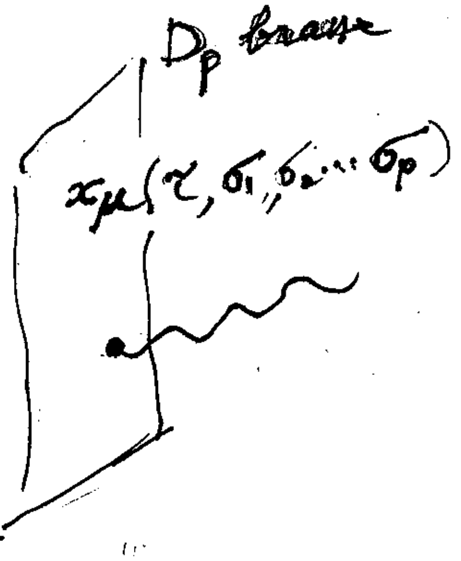
were introduced (Polchinski,1995) in the course of research of Superstrings, and played an important role in unification of various kinds of Superstring models mentioned above. In what follows, I would like to point out their important properties.

$p=-1$
0
1
2
⋮

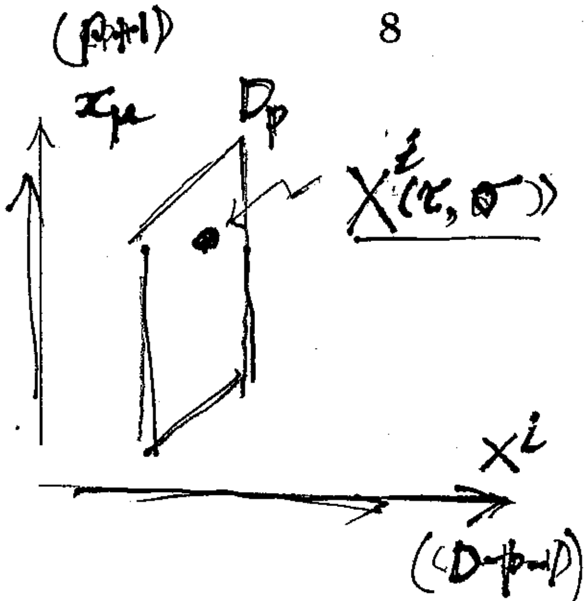
(i) Dp brane as Constraints on the End of Open String

D(ichlet) p brane was originally introduced through the Direchlet boundary condition imposed on the open string, where the end points of open string are **partially constrained on some p-dimensional hypersurface** described with p **continuous variables,** $\sigma_1, \sigma_2, \ldots \sigma_p$

together with time parameter τ . This p-dimensional Hypersurface is nothing but D-p brane.



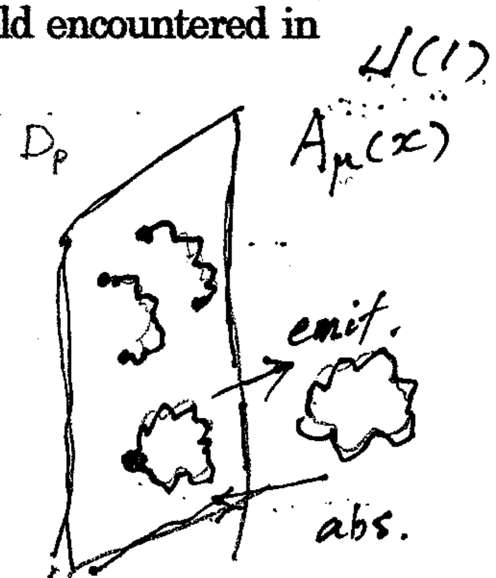
$p=0$ world line
$=1$ 〃 sheet



Under the suitable choice of coordinates in D-dimensional space, the **space-time configuration of a D-p brane** can be described in terms of (D –p –1) coordinates : $X^i(\tau, \sigma_1, \sigma_2, \ldots \sigma_p)$ $i = p+1, p+2, \ldots, D-1$ **which are transverse** to the remaining the so-called **(p+1) world(-volume) coordinates**, $x_\mu = (\tau, \sigma_1, \sigma_2, \ldots \sigma_p)$.

(ii) U(1)-gauge field $A_\mu(x)$ living on a Dp brane and Creation- Annihilation of Closed String

It is quite important to note that **open string** with both ends constrained on the same Dp brane produces **U(1)-gauge field, $A_\mu(x)$** living on the Dp brane, which is one of the **mass-less modes of the open string and regarded as a local component field encountered in Fierz decomposition of bilocal field.**



In addition, it is noticeable that a Dp brane is concerned with **creation and annihilation of closed string** which is made through **the joint of both ends of a open string** on the Dp brane.

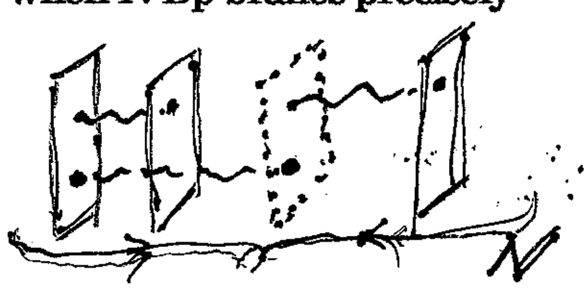
These aspects lead us sometimes to the interesting view: "**Dp brane is a Space-time Wall to the fundamental strings.**

(iii) U(N) Yang-Mills Gauge Fields living on N Dp branes and Noncommutative Position Coordinates of Dp branes

If there exist N Dp brans, it becomes possible that open strings have **their ends** constrained on **different D branes** and each U(1) gauge **field** on the respective Dp branes tends to produce *NxN U(N)* **Yang-Mills Gauge Fields, <m | A μ (x) | n >, when *N* Dp branes precisely** are on top of each other.

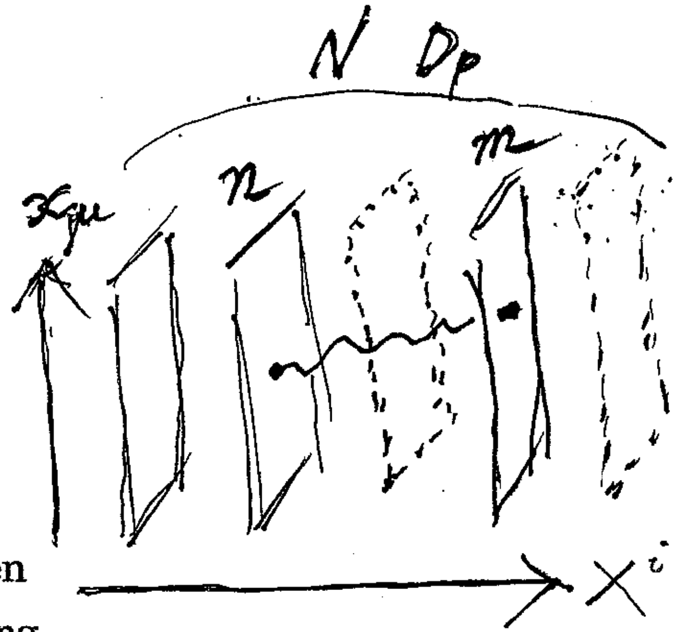
$U(N)$: $N \times N$

$\langle m | A_\mu(x) | n \rangle$



Furthermore, in this case, there arises a very interesting fact in describing the position coordinates of N **Dp branes:** Indeed, each **Dp coordinate** $X^{i}(x_{\mu})$ given in (i), tends to $N \times N$ **matrix form** like *U(N)* Gauge Field;

$$< m \,|\, X^{i}(x_{\mu}) \,|\, n > \quad m,n = 1,2,\ldots,N$$



Diagonal part, $< n \,|\, X^{i}(x_{\mu}) \,|\, n >$

may be naturally interpreted to describe **the position coordinates of n-th Dp brane** and **Nondiagonal part;**

$$< m \,|\, X^{i}(x_{\mu}) \,|\, n > \quad (m \neq n)$$

is explained due to **the interaction** between **m-th** and **n-th** Dp branes by the open string.

In this way, we encounter **the idea of the noncommutative position coordinates of Dp branes (Witten, 1995),** that is, **Entirely Unexpected Idea from original string theories.**

(iv) p-brane Democracy and Superalgebra

Dp brane so far explained might be seen still somewhat **secondary existence**, in comparison with the **fundamental strings.** But it is quite important to see that **their existence is well-founded in the frame work of Superalgebra**, such like **Osp(1 | 32),** in accordance with the idea of the so-called **p-brane democracy** (Townzend, 1995).

As was stated in (ii), D-p branes **interacts with open and closed strings.** The interactions take place through a local **tensor currents or** charges constructed by $X^i(x_\mu)$

$$\begin{array}{ccccc} \text{D-p brane} & \rightarrow & J_{\mu 0,\ \mu 1\mu 2\ldots\mu p}(x) & \rightarrow & Z_{\mu 1\mu 2\ldots\mu p} \\ (X^i(x_\mu)) & & (\text{current}) & & (\text{charge}) \end{array}$$

that is, **Currents (Charges) of anti-symmetric tensor character of order** $p+1$ (.p) **which interact** with **various local gauge fields** (Fierz's local component fields) **supplied** by massless modes of Superstrings ($G\mu\nu$, $B\mu\nu$, Φ; $C\mu$, $A\mu\nu\sigma$, ϕ, $B'\mu\nu$, $C\mu\nu\sigma\rho$), familiar in **supergravity theories.**

The so-called **p-brane democracy** is guaranteed by the fact that the above charges, $Z_{\mu 1\mu 2\ldots\mu p}$ **constructed by p-branes** in general, together with **Supercharges**, constitute a **specific Superalgebra,** like

$$\{Q_\alpha,\ Q_\beta\} = \Sigma_p\ (\Gamma^{\mu 1\mu 2\ldots\mu p})_{\alpha\beta}\ Z_{\mu 1\mu 2\ldots\mu p},$$

$$[\,Q_\alpha,\ Z_{\mu 1\mu 2\ldots\mu p}\,] = \cdots Q, \quad [Z\ldots,\ Z\ldots] = Z\ldots$$

What types of Superalgebra should be chosen is a central concern in seeking for **M-theory, F-theory or S-theory** possibly lying behind the present Superstring theories. **It can be regarded in a sense as a fundamental set of observables of space-time (Einstein,1930). Ortho-symplectic group** Osp(1|32), for instance, is recently noticed by many authors as a promising candidate for M-theory, where **32** is the maximal number of Supercharges.

$ Do brane and Yukawa's Elementary Domain

In the preceding arguments, (i)~(iv), I tried to summarize the remarkable aspects of D branes, **with a certain expectation that the objects might be one of the promising candidates to reach the goal which Yukawa aimed through his elementary domain, but did not achieve.** At this point, I should like to notice the next paper:

M Theory as A Matrix Model: A Conjecture

(Banks, Fischer, Shenker, Susskind; 1996)

In this paper, one finds that **Do brane with p=0, sometimes called D-**

particle, is regarded as the fundamental constituents of Superstrings, and the supersymmetric matrix quantum mechanics of N –body system of D-particles is proposed through the following Lagrangian:

$$L \sim \int d\tau \ \{\Sigma_{nm,i} \ \ 1/2 \langle n|\dot{X}^{i}(\tau)|m\rangle\langle m|\dot{X}^{i}(\tau)|n\rangle \ + \ \ldots\},$$

defined on the 11-dimensional light-cone frame. τ : light cone time

At a glance, D-particle seems to be a usual point-like particle, if we neglect the nondiagonal part of $X^i(\tau)$, the first term in L just being kinetic terms of N-body system of D-particles. It recalls to us the fundamental constituents which we have seen in the early string model as an infinite limit of multi-local field.

However, one has to remember (i) that D-particle is defined as a constraint on the ends of open string, and thus can never be separable from its background, just like the exciton or urciton.

Furthermore, one should remember that in the N D-particles system, the position coordinates as a whole must be described by N x N noncommutative matrices, as seen in (iii).

These facts naturally lead us, [redacted] beyond the above idea of noncommutative position coordinates of D-particles, to the idea of noncommutative space-time, that is, a definite departure from the continuous space-time itself, which was seriously sought in the Yukawa's elementary domain.

$ Snyder-Yang's Quntized Space-time (1947)

At this point, one should recall the idea of Snyder-Yang's Quantized space-time (1947), sometimes remarked above. Needless to say, the Snyder-Yang's theories were proposed with the aim of solving ultraviolet divergence by virtue of the quantized or discrete space-time in place of the naïve cutoff procedure.

It is quite important to note that, among many recent proposals of noncommutative space-time algebra, Snyder-Yang's space is discrete, but Lorentz-covariant, as was emphasized by Yang (1965). (i)

In addition, it should be pointed out that especially Yang's space-time algebra is deeply related to (Euclidean) Conformal Algebra and to de Sitter (dS) algebra, all of which have the common symmetry group (see Appendix). Indeed, the latter symmetry is noticed remarkably in the recent M-theory, the so-called dS / CFT correspondence or holographic hypothesis, as explained later. (ii)

Taking into consideration these points, I attempted recently **relativistic second quantization of the above quantum mechanics of D-particles system**, which makes possible **the creation-annihilation of D-particles**, by introducing second-quantized D-particle field defined on the Yang's quantized space-time (hep-th/0002001).

$ Yang's Space-time Algebra

D-dimensional Yang's quantized space-time and momentum are introduced via the dimensional contraction of SO (D+1,1) angular momentum operator, Σ_{MN}, with two extra dimensions, a and b ;

$$X_\mu \equiv \lambda \Sigma_{\mu a}, \qquad P_\mu \equiv 1/R\ \Sigma_{\mu b}, \qquad (\mu = 1,2,\ldots, D)$$

$$\text{where} \quad \Sigma_{MN} \equiv i\,(Y_M \partial/\partial Y_N - Y_N \partial/\partial Y_M). \qquad M, N = (\mu, a, b)$$

and $\{Y_M\}$ are (D+2)- dimensional Yang's parameters with $M = (\mu, a, b)$, μ=D being a time-like and *a*, *b* two space-like extra dimensions. One finds that

$$[X_\mu, X_\nu] = i\lambda^2 \quad \Sigma_{\mu\nu},$$

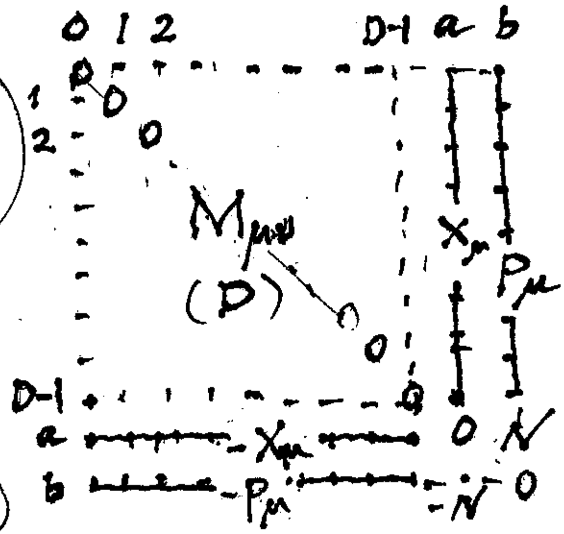
$$[P_\mu, P_\nu] = i/R^2\ \Sigma_{\mu\nu}$$

Σ_{MN}

and spatial components, Xi and Pi have discrete eigenvalues, in units of λ and $1/R$.

minimum

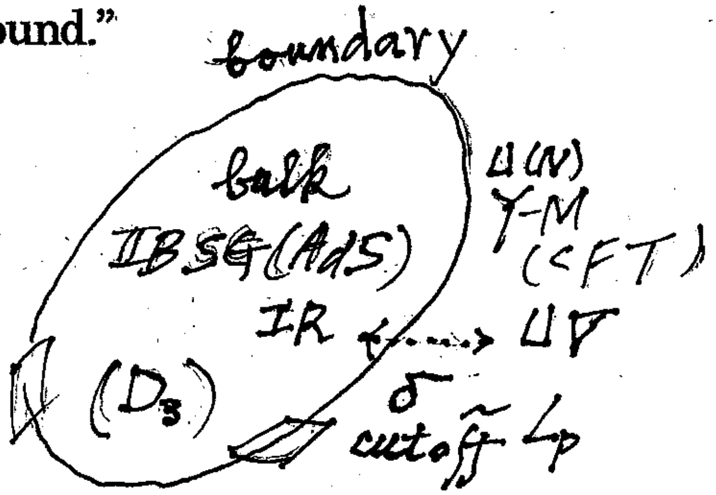
$\Longrightarrow$ (B)



$ M-Theory and Divergence Problem
---Holographic Hypothesis and IR / UV Connection

Now I would like to proceed to the final topics, the **DivergenceProblem**, which was a central concern of Yukawa and much discussed in the recent Superstring theory, in the form of **IR/UV** connection or the **holographic renormalization.** The **holographic hypothesis** is nicely expressed by Susskind- Witten(1998) in the following way:

"**A macroscopic (bulk) region** of space and every thing inside it can be represented by a **boundary theory living on the boudary of the region.**" "Furthermore, **the boundary theory should not contain more than one degree of freedoms per Planck area.** ...**One might imagine that the boundary theory is cutoff or discrete so that the information density is bound.**"



In the above argument, **Type IIB superstring Theory with gravity (AdS 5) in a bulk,** and **U(N) Super Yang-Mills Theory on the boundary (Conformal field theory)** are nicely set up through the mediation of **D3 branes** (Maldacena, 1997), and **a single cutoff parameter** δ **leads us in a unified way to the regularization of both the infrared divergence in the bulk theory and the ultraviolet divergence in the boundary conformal theory,** that is, to the regularized Bekenstein-Hawking area-entropy relation, the so-called IR/UV connection or holographic renormalization. → $N_{dof} = A/4G$

Concluding Remarks

(I) D0 branes and Yukawa's Elementary Domain

I have pointed out several important results in the recent rapid development of superstring theory or M-theory, which seem to me deeply related to **the fundamental problems of space-time.** Especially I noticed **the concept of D0 branes** as the fundamental constituent of superstring, **in close accord with Yukawa' s Elementary Domain.**

(II) Divergence Problem in Superstring Theory

On the other hand, however, we very often encounter **various kinds of Divergences related to Local Fields as massless modes of superstrings.** At this point, we wonder why the ultraviolet divergences appear even in the superstring theory, while many authors assert that this problem already disappeared by virtue of nonlocality and super-symmetry intrinsic to the superstring theory.

This fact recall us **the Yukawa's argument on the Fierz's local component fields in bilocal field theory,** that is, "Possible divergences accompanied with local component fields might be ultimately solved by taking into account **the total effects of infinite number of local component fields."** **The naïve cutoff or regularization procedure discussed, for instance, in the so-called UV/IR connection in Superstring theory** (see Susskind-Witten,1998), might be understood as **an effective substitute for such ultimate or exact calculations.**

(III) Yang's Quantized Space-Time and Cutoff

I would like to emphasize again **a possibility of the field theory on Yang's quantized space-time** : (i) It has **the Common Symmetry to the dS/ CFT correspondence** and (ii) **Discrete structure of its space-time and momentum may provide a theoretical ground for the unified cutoff of UV- IR divergences.** In order to arrive at the consistent divergence-free theory, however, many problems must be left in the future.

Appendix Yang's Quantized Space-time Algebra and dS / CFT Correspondence

Although the group-theoretical origin of dS/CFT correspondence is well- known, in this Appendix we show that the extended Snyder-Yang's quantized space-time algebra explained in the text also relates to the (Euclidean) conformal algebra, through the different representation bases of the same SO(D+1,1) symmetry.

As was shown in the text, the space-time coordinates and momentum operators in Yang's quantized space-time algebra, are given by

$$X\mu \quad \equiv \lambda \quad \Sigma\mu a, \tag{A.1}$$

$$P\mu \quad \equiv \quad 1/R \ \Sigma\mu b, \tag{A.2}$$

with suitable units λ and R , together with the reciprocal operator between them,

$$N \ \equiv \lambda \quad \Sigma \ ab \tag{A.3}$$

and D-demensional angular momentum,

$$M\mu\ \nu \ \equiv \Sigma\mu\ \nu. \tag{A.4}$$

Here, ΣMN with M, N = (μ ,a, b) is angular momentum operator of S(D+1,1) expressed in terms of **Yang's (D+2)-dimensional parameter space {YM}:**

$$\Sigma MN \equiv i\,(YM\,\partial/\partial YN \ - \ YN\,\partial/\partial YM\,), \tag{A. 5}$$

$$M,\ N=(\mu,a,b)$$

with μ=1,2,..., D, μ=D being a time-like, *a* and *b* two space-like extra dimensions.

As was emphasized by Yang, it is quite important to see that space-components (μ=i) of Xμ and Pμ:

$$X_i = \lambda \Sigma_{ia} = i \lambda (Y_i \partial/\partial Y_a - Y_a \partial/\partial Y_i), \qquad (A.1)'$$

$$P_i = 1/R\ \Sigma_{ib} = i/R\ (Y_i \partial/\partial Y_b - Y_b \partial/\partial Y_i), \qquad (A.2)'$$

with i=1,2,...D-1, have discrete eigenvalues under units of λ and $1/R$, respectively, and contrarily time-components ,

$$X_o (\equiv -i X_D) = \lambda \Sigma_{oa}$$

$$= i\lambda (Y_o \partial/\partial Y_a + Y_a \partial/\partial Y_o), \qquad (A.1)''$$

$$P_o (\equiv -i P_D) = 1/R\ \Sigma_{ob}$$

$$= i/R (Y_o \partial/\partial Y_b + Y_b \partial/\partial Y_o) \qquad (A.2)''$$

with $Y_o = -i Y_D$, have continuous eigenvalues.

Now let us examine the D-dimensional Euclidean conformal algebra derived from the same SO(D+1,1). In fact, in this case, choosing 0 and b directions as the **extra two dimensions** with opposite metric signature, their generators { D', P' u, K' u, M' uv} with u, v= (a, i=1,2,..., D-1) are given by

$$D' \equiv \Sigma_{ob} = i (Y_o \partial/\partial Y_b + Y_b \partial/\partial Y_o), \qquad (A.6)$$

$$P'_u + K'_u \equiv \Sigma_{ou} = i (Y_o \partial/\partial Y_u + Y_u \partial/\partial Y_o), \qquad (A.7)$$

$$P'_u - K'_u \equiv \Sigma_{bu} = i (Y_b \partial/\partial Y_u - Y_u \partial/\partial Y_b), \qquad (A.8)$$

together with

$$M'_{uv} \equiv \Sigma_{uv} = i\ (Y_u \partial/\partial Y_v - Y_v \partial/\partial Y_u). \qquad (A.9)$$

From the above expresssions, one sees that P' u + K' u and D' have continuous eigenvalues and P' u – K' u and M' uv have discrete eigenvalues. Consequently, as was pointed out in the text, the momentum operators P' u (K' u), which are commutative among themselves, have continuous eigenvalues. This remarkable fact in contrast to Yang's algebra can be seen, for instanse, in the following form,

$$P'_i = i/2 \{ Y_o \partial/\partial Y_i + Y_i \partial/\partial Y_o + Y_b \partial/\partial Y_i - Y_i \partial/\partial Y_b \}. \qquad (A.10)$$

Finally let us examine D+1 dimensional **de Sitter Algebra** $\mathbf{dS_{D+1}}$, which is also concerned with the same SO(D+1,1) and may be well related to the Yang's quantized space-time algebra, likely to the conformal algebra familiar in the holographic hypothesis. In fact, the generators of $\mathbf{dS_{D+1}}$ are defined by

$$\underline{P}\alpha \equiv 1/R \quad \Sigma\, \alpha b, \qquad (A.11)$$

$$\underline{M}\alpha\,\beta \quad \equiv \Sigma\, \alpha\,\beta \qquad (A.12)$$

with $\alpha, \beta =$ (a, $\mu = 1,2,\ldots,D$). They are directly expressed in terms of those of Yang's algebra ($X\mu$, $P\mu$, $M\mu\,\nu$, N) in the following form:

$$\underline{P}\alpha : \quad \underline{P}\mu (= 1/R \;\; \Sigma\,\mu b) = P\mu,$$

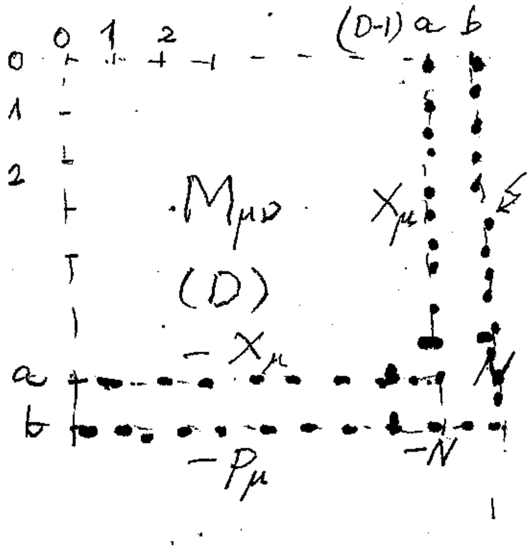
$$\underline{P}a\, (= 1/R \;\; \Sigma ab) = 1/(R\lambda)N. \qquad (A.13)$$

$$\underline{M}\alpha\,\beta : \; \underline{M}\mu\,\nu \;\; (= \Sigma\,\mu\,\nu) = M\mu\,\nu,$$

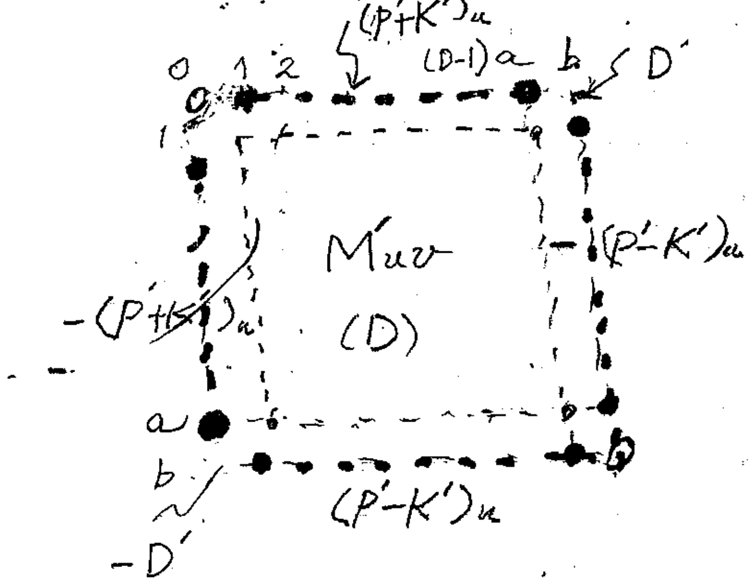
$$\underline{M}\mu a = -\underline{M}a\mu (= \Sigma\,\mu a) = \;\; (1/\lambda)X\mu. \qquad (A.14)$$

$SO(D+1,1): \Sigma_{MN}$ $\quad (M,N) = \overbrace{0, \underbrace{1, 2 \cdots D-1, a}_{(\mu,\nu)}}^{(\mu,\nu)}, b$ $\quad (\alpha, \beta)$

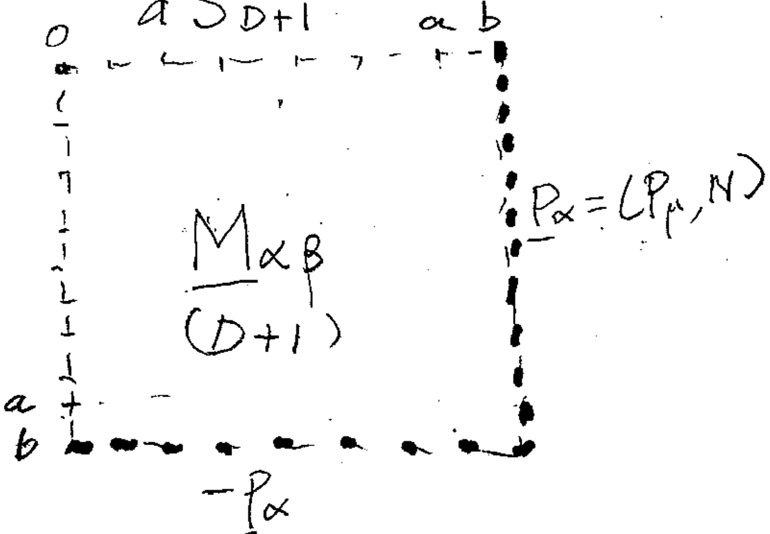
Yang$_D$



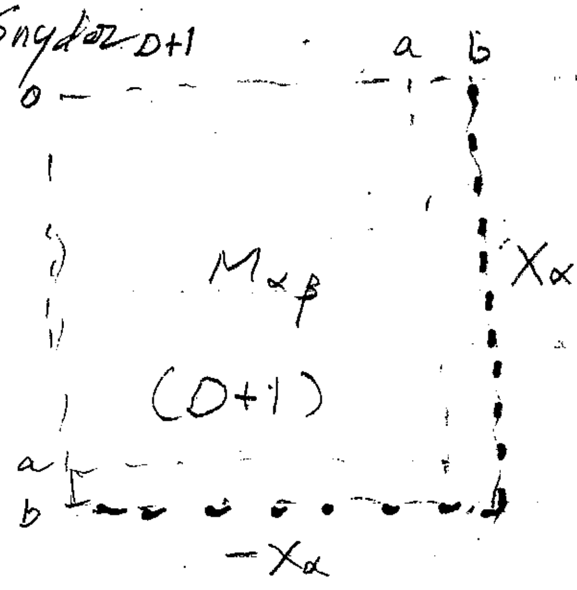
(E) Conformal$_D$



dS_{D+1}



Snyder$_{D+1}$



gravity dS_{D+1} / CFT_D correspondense

dS_{D+1} / Yang_D " ?

complementary

in Holographic hypothesis