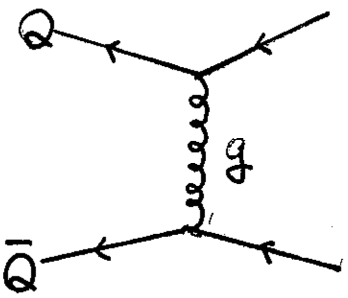
§1. Introduction

We examine the static QCD potential and bottomonium spectrum within the non-relativistic boundstate theory based on perturbative QCD.

Historically perturbative QCD predictions were unsuccessful.

- QCD Potential

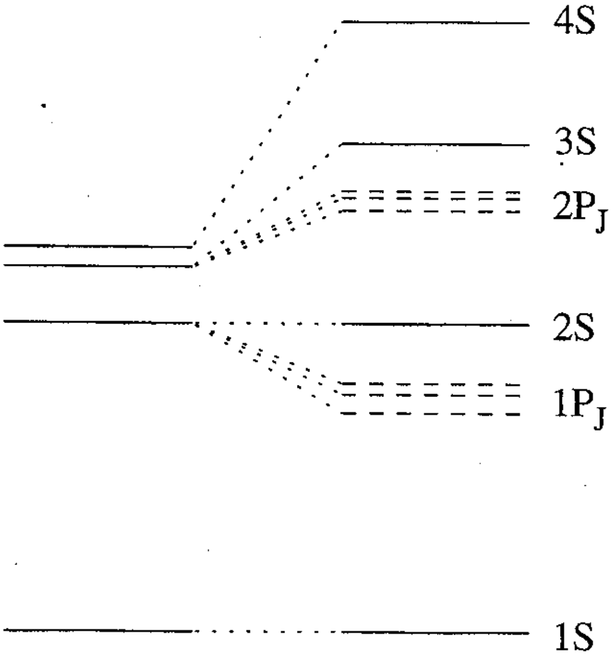
$$V_{QCD}(r) \simeq -\frac{4}{3}\frac{\alpha_s}{r}$$



- Phenomenological Potential Models

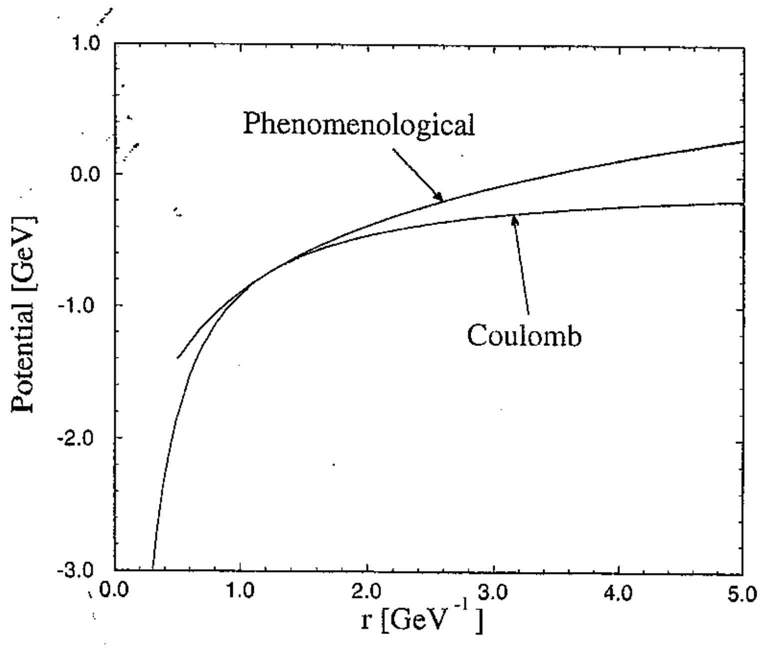
$$\hat{H} = \frac{\vec{p}^{\,2}}{m_Q} + V_{pheno}(r) \quad ; \quad V_{pheno}(r) \simeq -\frac{a}{r} + \sigma r$$

$$0.5\ \mathrm{GeV}^{-1} \lesssim r \lesssim 5\ \mathrm{GeV}^{-1}$$



Coulomb Spectrum

Bottomonium Spectrum (exp.)



Accurate perturbative computations of QCD potential / heavy quarkonium spectrum thanks to recent theoretical progress:

(1) Complete Next-to-Next-to-Leading Order (N^2LO) corrections

$$V_{QCD}(r) = -\frac{4}{3}\frac{\alpha_s}{r}\left(1 + \star\alpha_s + \underline{\star}\alpha_s^2\right), \quad E_n = -\frac{4}{9n^2}\alpha_s^2 m\left(1 + \star\alpha_s + \underline{\underline{\star}}\alpha_s^2\right)$$

Pineda, Ynduráin
Peter
Schröder

(2) Renormalon cancellation

$\Rightarrow$ Drastic improvement of convergence, if we express the energy levels in the $\overline{MS}$ mass instead of the pole mass.

Hoang, Smith, Stelzer, Willenbrock
Beneke

e.g.

- $\Upsilon(1S)$ $\mu = 2.49$ GeV, $\alpha_s(\mu) = 0.274$

$$M_{\Upsilon(1S)} = 9.94 - 0.17 - 0.20 - 0.30 \text{ GeV} : \text{Pole-mass scheme}$$
$$= 8.41 + 0.84 + 0.20 + 0.013 \text{ GeV} : \overline{MS} \text{ scheme}$$

- $\Upsilon(2S)$ $\mu = 1.09$ GeV, $\alpha_s(\mu) = 0.433$

$$M_{\Upsilon(2S)} = 9.94 - 0.10 - 0.19 - 0.45 \text{ GeV} : \text{Pole-mass scheme}$$
$$= 8.41 + 1.46 + 0.093 + 0.009 \text{ GeV} : \overline{MS} \text{ scheme}$$

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§2. QCD Potential

"Renormalon" = Perturbative uncertainty originating from IR gluons
$(\lambda_g \sim \Lambda_{QCD}^{-1})$.

- $\mathcal{O}(\Lambda_{QCD})$ renormalon contained in $V_{QCD}(r)$ gets cancelled in the total energy of a static $Q\bar{Q}$ pair:

$$E_{tot}(r) = 2\, m_{pole} + V_{QCD}(r)$$

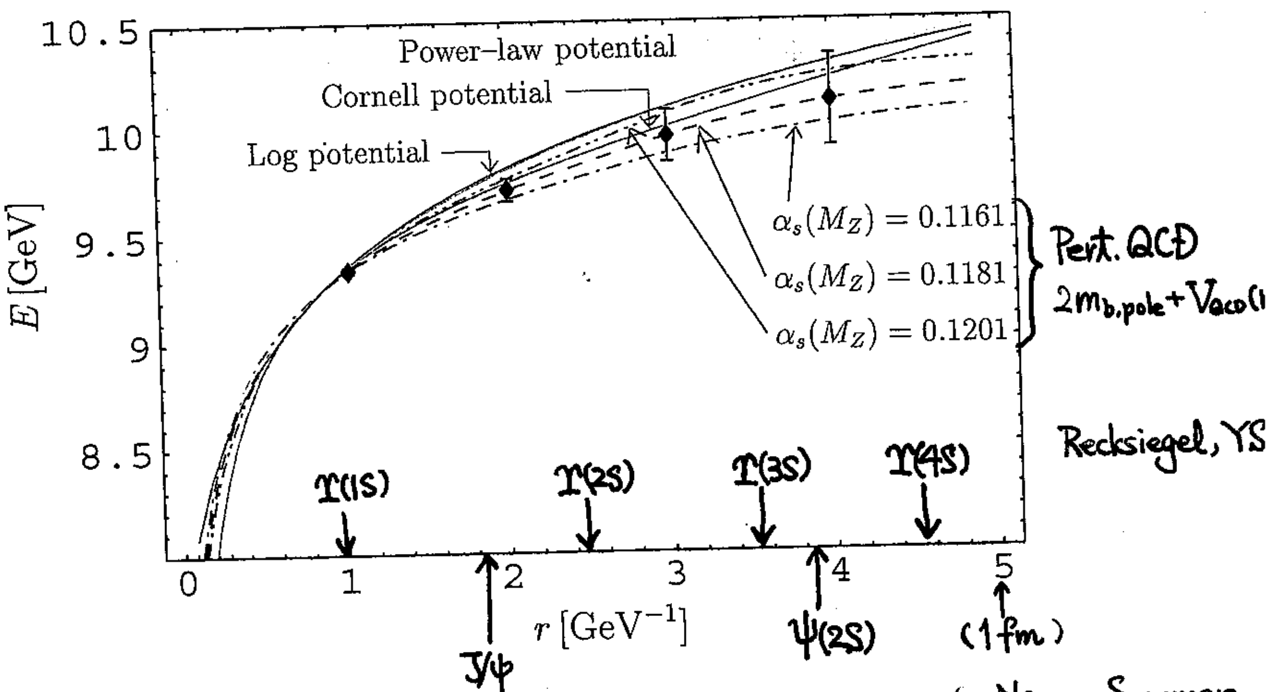
if m_{pole} is re-expressed in terms of the $\overline{MS}$ mass $\bar{m} \equiv m_{\overline{MS}}(m_{\overline{MS}})$:

$$m_{pole} = \bar{m}\ (1 + \star\alpha_s + \star\alpha_s^2 + \cdots)$$

$\Rightarrow$ Remaining uncertainty $\sim \mathcal{O}(\Lambda_{QCD}^3 r^2) \ll \mathcal{O}(\Lambda_{QCD})$

Phenomenological Potentials vs. $E_{tot}(r) = 2m_{b,pole} + V_{QCD}(r)$ up to $\mathcal{O}(\alpha_s^3)$

- $\bar{m}_b = 4.190$ GeV
- $n_\ell = 4$ ($\bar{m}_c = 1.243$ GeV, $\bar{m}_u = \bar{m}_d = \bar{m}_s$



Recksiegel, YS

※ Also good agreement with lattice calculations has been confirmed in the case $n_\ell = 0$ (quenched approx.).

Necco, Sommer
Pineda
Lee
Recksiegel, YS

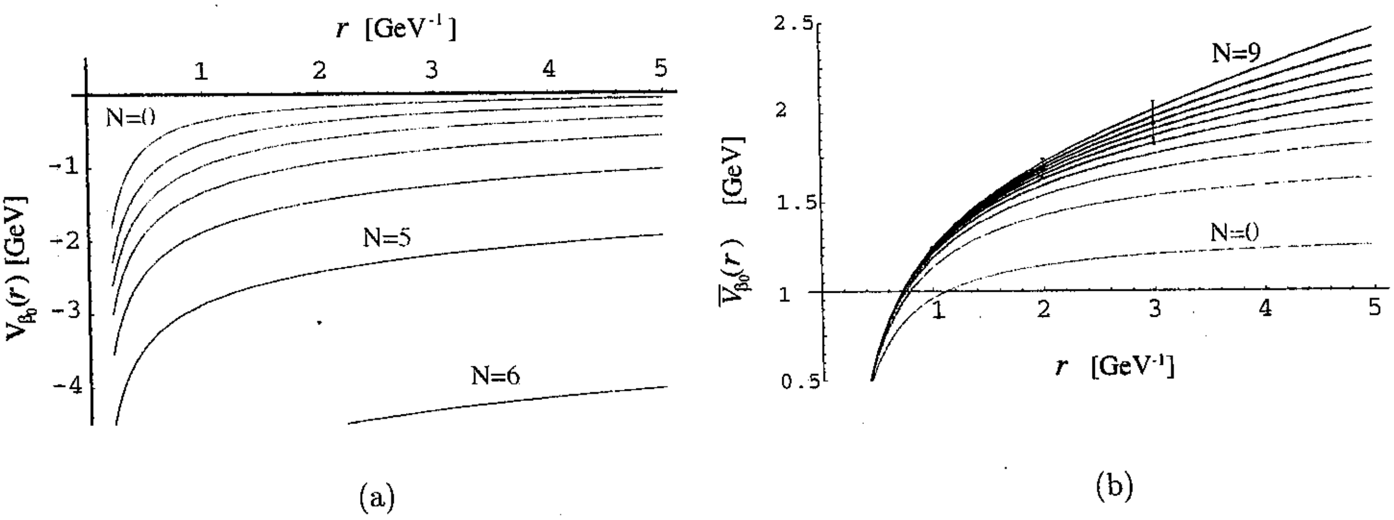


Figure 1: The QCD potential in the large-β_0 approximation truncated at $O(\alpha_S^{N+1})$ term. We set μ = 2.49 GeV, n_l = 4 and $\alpha_S(\mu)$ = 0.273 [corresponding to $\alpha_S^{(5)}(M_Z)$ = 0.1181]. (a) Before subtraction of the leading renormalon. (b) After subtraction of the leading renormalon.

hep-ph/0104259

Why does $V_{QCD}(r)$ become steeper than the Coulomb potential at large r ?

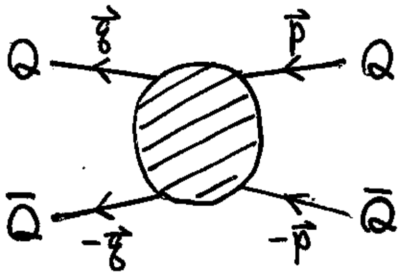
The interquark force $|F(r)|$ gets stronger than the Coulomb force due to the running of $\alpha_S(\mu)$:

$$F(r) \equiv -\frac{d}{dr} V_{QCD}(r) \equiv -C_F \frac{\alpha_F(\mu = 1/r)}{r^2}$$

$$-C_F \frac{\alpha_0}{r^2} \quad \text{Coulomb}$$

§3. Bottomonium Spectrum

Matching pert. QCD to potential-NRQCD (effective theory)



$$= \langle \vec{q}, -\vec{q} \,|\, \frac{1}{E - \hat{H} + i\epsilon} \,|\, \vec{p}, -\vec{p} \rangle$$

Hamiltonian up to N^2LO:

$$\hat{H}_0 = \frac{\vec{p}^{\,2}}{m} - C_F \frac{\alpha_s}{r}$$

$$\hat{H}_1 = - C_F \frac{\alpha_s}{r} \left(\frac{\alpha_s}{4\pi}\right) \left[\beta_0 \log(\mu'^2 r^2) + a_1 \right]$$

$$\hat{H}_2 = - \frac{\vec{p}^{\,4}}{4m^3} - C_F \frac{\alpha_s}{r} \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\beta_0^2 \left\{ \log^2(\mu'^2 r^2) + \frac{\pi^2}{3} \right\} + (\beta_1 + 2\beta_0 a_1) \log(\mu'^2 r^2) + a_2 \right]$$

$$+ \frac{\pi C_F \alpha_s}{m^2} \delta^3(\vec{r}) + \frac{3 C_F \alpha_s}{2 m^2 r^3} \vec{L} \cdot \vec{S} - \frac{C_F \alpha_s}{2 m^2 r} \left(\vec{p}^{\,2} + \frac{1}{r^2} r_i r_j p_j p_i \right)$$

$$- \frac{C_A C_F \alpha_s^2}{2 m r^2} - \frac{C_F \alpha_s}{2m^2} \left[\frac{\vec{S}^2}{r^3} - 3 \frac{(\vec{S} \cdot \vec{r})^2}{r^5} - \frac{4\pi}{3} (2\vec{S}^2 - 3) \delta^3(\vec{r}) \right]$$

$$\left(\begin{array}{l} C_F = \frac{4}{3}, \; C_A = 3, \; \mu' = \mu e^{\gamma_E} \\ m \equiv m_{pole} \end{array} \right)$$

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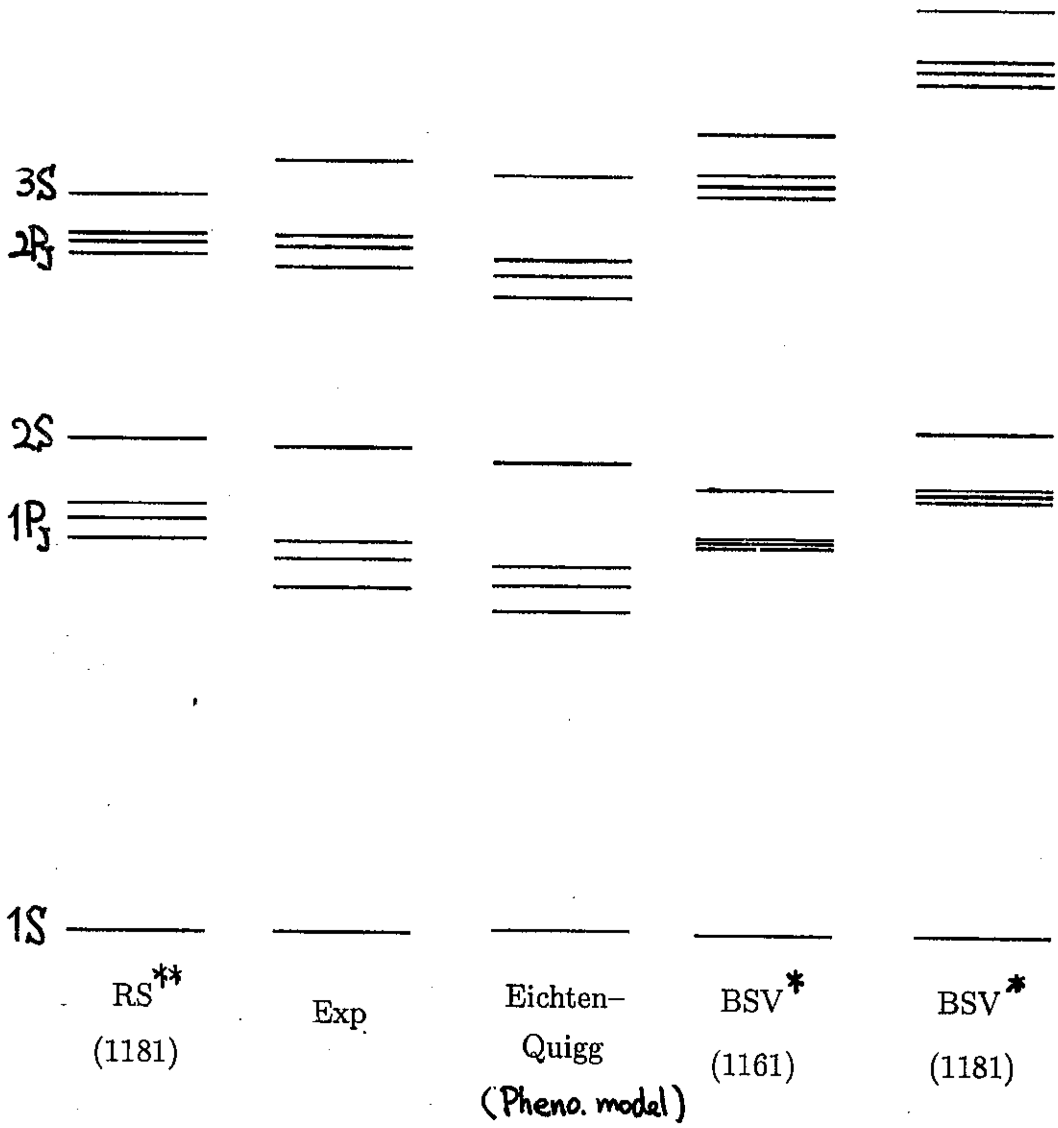
$\Rightarrow$ Compute energy eigenvalues in expansions in α_s.

Bottomonium spectrum up to $n=3$ states



* Fixed-order pQCD predictions up to $\mathcal{O}(\alpha_s^4 m)$ for the input $\alpha_s(M_Z) = 0.1161$ and 0.1181.

Brambilla, Y.S., Vairo

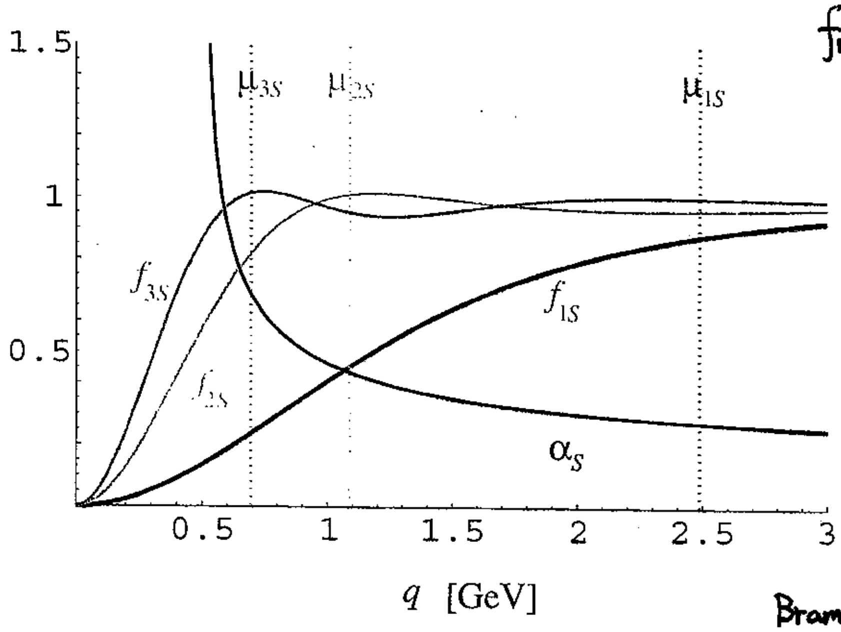
** Pert. QCD prediction including part of higher-order corrections, in particular the full $\mathcal{O}(\alpha_s^5 m)$ corrections to the fine structure ($\alpha_s(M_Z) = 0.1181$, $\mu = 3\,\mathrm{GeV}$).

Recksiegel, Y.S.

§4. Physical Picture

$$E_n \approx \langle n | 2m_{b,\text{pole}} + V_{QCD}(r) | n \rangle \approx 2\overline{m}_b + \int \frac{d^3\vec{q}}{(2\pi)^3} C_F \frac{4\pi\alpha_s(q)}{q^2} f_n(q)$$

$f_n(q)$: support function



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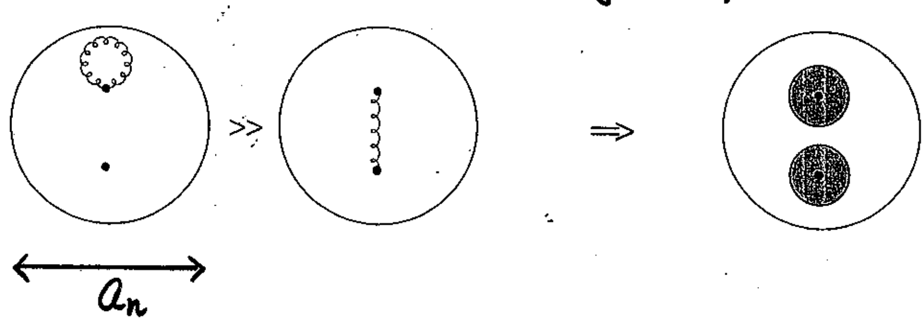
Recksiegel, YS

Interpretation

(I) $E_n \approx 2\overline{m}_b +$ (self-energies $1/\overline{m}_b \lesssim \lambda_g \lesssim a_n$)



(II) Level spacings increase at higher levels (as compared to Coulomb levels) because self-energies grow rapidly as the boundstate size a_n increases. $1/\overline{m}_b \lesssim \lambda_g \lesssim a_n$

§5. Conclusions

After incorporating renormalon cancellation $\Lambda_{QCD} \to \Lambda_{QCD} \times \left(\frac{\Lambda_{QCD}}{\alpha_s m}\right)^2$

- $2m_{pole} + V_{QCD}(r)$ agrees well with $\begin{cases} \text{phenomenological potentials.} \\ \text{lattice data.} \end{cases}$

- Bottomonium levels up to $n = 3$ (principal quantum number) can be computed reliably in pert. QCD (potential-NRQCD) and are consistent with the experimental values <u>within estimated uncertainties.</u>

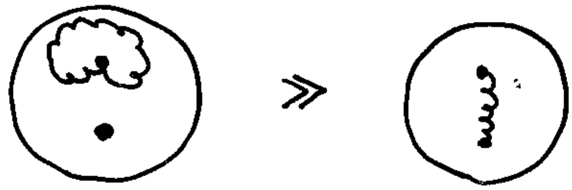
- Charmonium & B_c levels $\Rightarrow$ only $1S_J$ levels can be computed reliably

- Bottom & charm quark $\overline{MS}$ masses $\longleftarrow$ $\Upsilon(1S)$ & J/ψ

$$m_b^{\overline{MS}}(m_b^{\overline{MS}}) = 4192 \pm 20\,[\alpha_s(M_Z)] \pm 25\,[h.o.] \pm 3\,[m_c] \text{ MeV}$$

$$m_c^{\overline{MS}}(m_c^{\overline{MS}}) = 1243 \pm 15\,[\;''\;] \pm 50\,[\;''\;] \text{ MeV}$$

- New physical picture on the composition of the bottomonium masses and the interquark force $F(r)$.



- For accurate predictions of fine structure and S-P splittings, inclusion of N^3LO corrections is mandatory.