Sigma meson in the nuclear force

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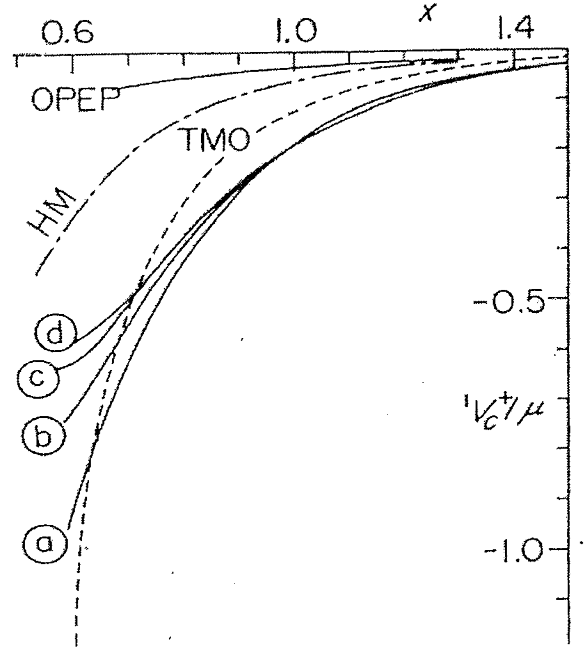
Can the nuclear potential be derived from the first principle?

- In 1950's and 60's, the nuclear potential had been studied expensively, in which Taketani's idea played an important roll. In his idea the nuclear force is divided into three parts according to the range.
- The outermost region is described by the one-pion exchange, and innermost region is treated by phenomenological way. On the other hand in the middle region, the two-pion exchange is responsible to the nuclear potential, and which is supposed to be calculated from the interaction Lagrangian $\mathcal{L}_{\pi N}$.
- The figure is $V_c(r)$ of the singlet even state, in which the 2π exchange curve TMO is the standard perturbation calculation (static) and HM is the result with full recoil considered. OPEP is the 1π exchange contribution.



- On the other hand, curves a (0.30), b (0.25), c (0.20) and d (0 ~ 0.15) are the one boson exchange potentials whose parameters are fitted to the phase shift data. The four curves differ by the hard core radii (whose values are written in the bracket.)
- Large attractive potential must be missing from the TMO and HM potentials, and therefore these potentials can not reproduce the phase shift data well.
- Let us consider whether the inclusion of the strong Van der Waals potential resolves the difficulty.

Strong Van der Waals interaction (2)

- $$\frac{\left|<1|R^3 H'|0>\right|^2}{\Delta E_1} \le C \le \frac{<0|(R^3 H')^2|0>}{\Delta E_1}$$

or $$\frac{2}{3}\frac{*e^4}{\Delta E_1} a_1^2 a_2^2 \le C \le \frac{2}{3}\frac{*e^4}{\Delta E_1} <r_1^2><r_2^2>$$

- $< r^2 >$ is the ordinary mean square radius of the ground state, whereas a^2 is the square of the dipole transition amplitude from the ground state to the first excited state. They are defined by:

$$<r^2> = \int d^3 r \; \Psi^*_{0,0,0} r^2 \Psi_{0,0,0} \quad \textit{and}$$

$$a^2 = 3|\hat{z}|^2 \quad \textit{with} \quad \hat{z} = \int d^3 r \; \Psi^*_{1,1,0}(r \cos\theta)\Psi_{0,0,0}$$

- “Proposal to observe strong Van der Waals interaction in the nuclear force” ., T.Sawada , Phys.Lett. B43, 517, (1973)

Relation between the extra singularity of the amplitude at t=0 and the behavior of the long range potential at R ~ ∞

- In order to confirm the existence of the long range force unambiguously, it is necessary to observe the singularity of the amplitude A(s,t) at t=0, which is the characteristic feature of the long range interaction.
- We shall derive the relation between the parameters in $V(R) \sim -C/R^\alpha$ and in the threshold behavior of the spectral function $A_t(s,t)=\pi C' t^\gamma + \ldots$. They are $\alpha = 2\gamma+3$ and $C = C' \, 2\Gamma(\alpha-1)/m^2$.
- In order to obtain these relations we must represent rV(r) by the form of the Laplace transformation:

$$V(r) = -\frac{1}{\pi m^2}\frac{1}{r}\int_0^\infty dt' w(t') e^{-r\sqrt{t'}}$$

- Since the change of the weight function at finite t' does not alter the asymptotic behavior of the potential, we can in particular choose $w(t) = \pi C' t^\gamma$, then

$$V(r) = -\frac{2C'}{m^2}\Gamma(2\gamma+2)\frac{1}{r^{2\gamma+3}}$$

- If we compute Fourier transformation of both sides of the Laplace representation, we obtain the integral representation of the amplitude in the Born approximation

$$A^{(B)}(s,t) = \frac{1}{\pi}\int_0^\infty \frac{w(t')}{t'-t}dt'$$

- Therefore the weight w(t) is nothing but the imaginary part of the amplitude A(s,t) of the first Born approximation. So the relations among the parameters come out. Since the power of the next Born term is $2\gamma+1$, the term of power γ is the leading singularity.

The extra singularity

- For the spectral function π C't^ γ , the singular behavior of the scattering amplitude A(s,t) is

$$A(s,t) = -\frac{\pi}{\sin \pi\gamma} C'(-t)^{\gamma} + \cdots$$

, where the dots means the background regular function of t. If we remember t= - 2ν (1- z) , the extra singularity at t=0 occurs at z=1 and also at ν =0 when the variables (ν , z) are used in stead of (s ,t).

- For fixed ν the singular behavior (1-z)^ γ must occur. On the other hand, behavior ν^ γ must appear whenever we compute the angular integration with a weight u(z). In particular when u(z) is a Legendre function, a partial wave amplitude will appear.

- Since for the Van der Waals potential of the London type α=6 and therefore γ=3/2, we expect to observe ν √ν behavior in the partial wave amplitude $a_\ell(\nu)$. We shall analyze only the S and P waves, because the very low energy phase shift data of the higher partial waves are not available. The once subtracted amplitudes defined by

$$a_0^{once}(\nu) = \frac{a_0(\nu) - a_0(0)}{\nu} \quad and \quad a_1^{once}(\nu) = \frac{a_1(\nu)}{\nu}$$

are more convenient to analyze, because the singular behavior becomes ν^ (γ-1) and becomes easier to observe. For γ=3/2 we expect to observe a cusp √ν at ν=0. Moreover for S-wave the coefficient of √ν is positive when the force is attractive.

Removal of the nearby cut

- In order to see the extra singularity C'' √ν as clear as possible, it is necessary to remove the nearby singularities of the background regular function and to prepare the domain of analyticity as wide as possible.
- The unitarity cut is removed by introducing a function

$$K_\ell^{once}(\nu) = a_\ell^{once}(\nu) - \frac{1}{\pi}\int_0^L d\nu' \frac{\operatorname{Im} a_\ell(\nu')}{\nu'(\nu'-\nu)}$$

$$where \quad \operatorname{Im} a_\ell(\nu) = \frac{\sqrt{m^2+\nu}}{\sqrt{\nu}} \sin^2 \delta_\ell(\nu)$$

in which the upper bound L of the integration is chosen at 7.2 where the inelastic channel opens.

- The nearest left hand cut is that of the one-pion exchange, which starts from ν= -1/4. This cut is removed if we subtract the amplitude of the one-pion exchange (OPE):

$$a_\ell^{once,1\pi}(\nu) = \left(\frac{1}{4}\frac{g^2}{4\pi}\right)\frac{1}{\nu}\left(\frac{1}{2\nu}Q_\ell\left(1+\frac{1}{2\nu}\right) - \delta_{\ell,0}\right)$$

- Since the nearby cuts are removed and the two-pion exchange spectrum starts at ν= -1, and whose threshold behavior is (-1- ν)^3/2, we can expect that the background function is almost constant and has small slope. Next figure is a plot of a function:

$$-\overline{K}_0^{once} = -\left(K_0^{once}(\nu) - a_0^{once,1\pi}(\nu)\right)$$

against T_{lab}, in which the energy dependent S-wave phase shift data of Nimegen group are used. We can clearly see a cusp pointing upward, which means the long range force of the attractive sign.

Conclusion and prospect

- The curve with a cusp is fitted by a spectral function with 3 parameters:

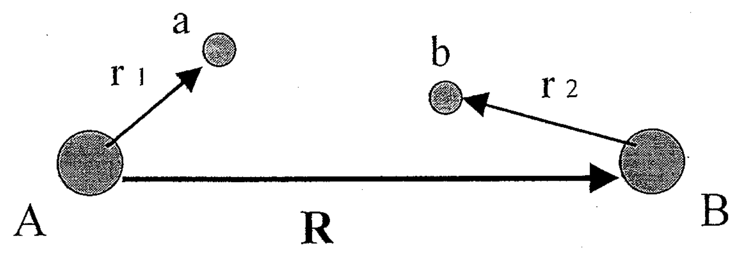
$$A_t(s,t) = \pi C' t^{\gamma} e^{-\beta t}$$

, the result of the fit of the 3 parameters is
  $\gamma=1.54$, $\beta=0.063$ and $C'=0.18$
  in the unit of $\mu=1$.
- In terms of the potential $V \sim -C/r^{\wedge}\alpha$, these values mean $\alpha=6.09$ and $C=0.18$, and which indicates that reasonably strong Van der Waals force is acting between nucleons.
- Therefore it is important to construct nuclear potential anew based on the Van der Waals interaction plus the short range potential arising from the exchanges of mesons.

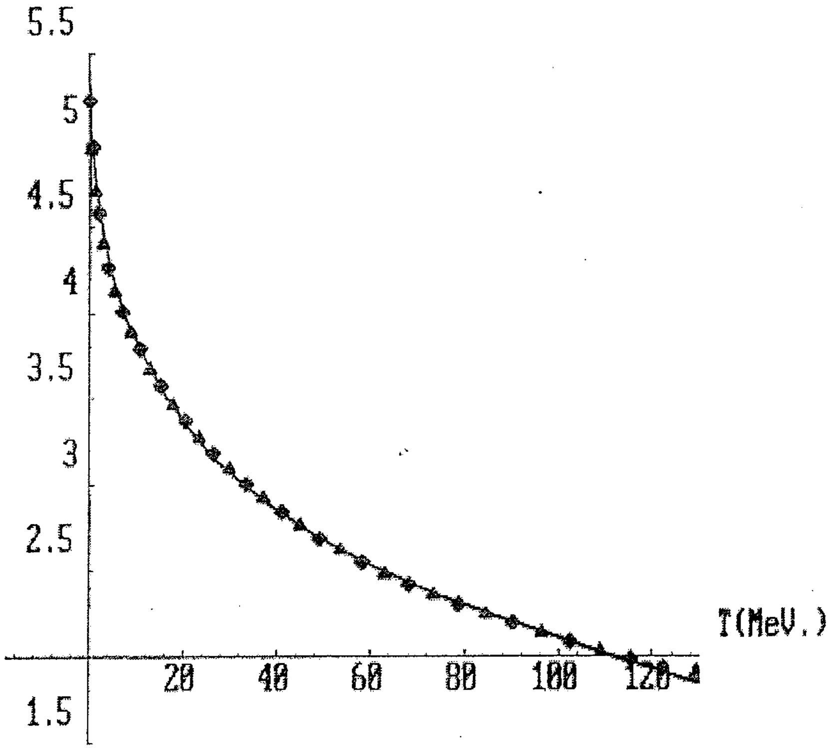
- Since the order of magnitude of the strength C of the Van der Waals potential is $C \sim (2/3)({}^*e^2)^2 a_1^2 a_2^2/\Delta E_1$, for QCD for which ${}^*e^2=0.3$, C becomes 0.003 even if we choose the maximum values $a \sim 1/2$. So the QCD gives too small value to C.
- On the other hand for the magnetic monopole model where ${}^*e^2=137/4$, C becomes the observed value for the radius $a \sim 1/5$ in the unit of $\mu=1$.
- Therefore the magnetic monopole model of hadron is the most favorite model of the composite hadron.
- My dream is to confirm the dyon model by observing the monopoles of opposite sign fuse to form a meson.

Possible Van der Waals interaction in the nuclear force

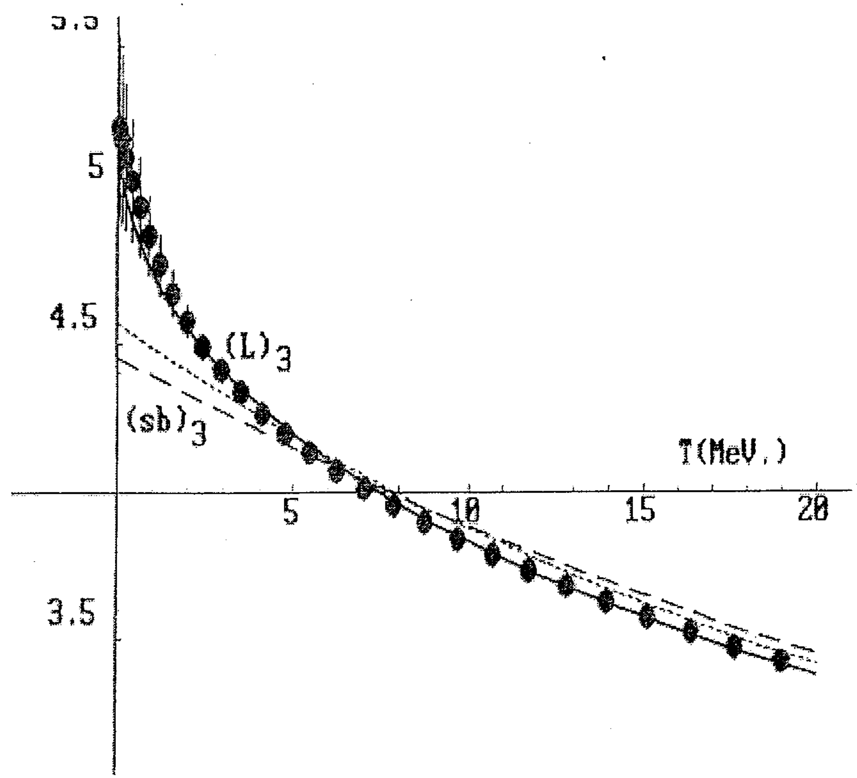
- In 1960's our picture of the hadron changed from elementary particle to composite particle with the Coulombic constructive force.
- Since the mechanism of appearance of the Van der Waals force is universal, we can expect such a force between hadrons as well as between ordinary atoms. Only difference is the strength C of the potential V(R) ~ - C / R^6+ …
- Order of magnitude of the strength C is C ≈ (2/3) $(^*e^2)^2\, \bar{a}_1^2\, \bar{a}_2^2 / \Delta E_1$, where $(^*e^2)$ is the coupling constant of the basic Coulomb interaction and ΔE_1 is the first excitation energy. $\bar{a}_1^2$ and $\bar{a}_2^2$ are the mean square radii the composite hadrons, whose values are $(1/2)^2 \sim (1/6)^2$ in the unit of $\mu=1$.



- $$H' = {}^*e^2\left(\frac{1}{R_{AB}} + \frac{1}{R_{ab}} - \frac{1}{R_{Ab}} - \frac{1}{R_{Ba}}\right)$$
  $$\rightarrow \frac{{}^*e^2}{R^3}(x_1x_2 + y_1y_2 - 2z_1z_2)$$
- $$-V(R) = \sum_n{}' \frac{(H')_{0n}(H')_{n0}}{E_n - E_0}$$
- Since each term is positive definite and the denominator increases with n, we can obtain the upper and lower bound of the strength C. In the estimation of the upper bound, the closure relation is used.



5.5
5
4.5
4
3.5
3
2.5
1.5
T(MeV.)
20
40
60
80
100
120



5
4.5
3.5
(L)$_3$
(sb)$_3$
T(MeV.)
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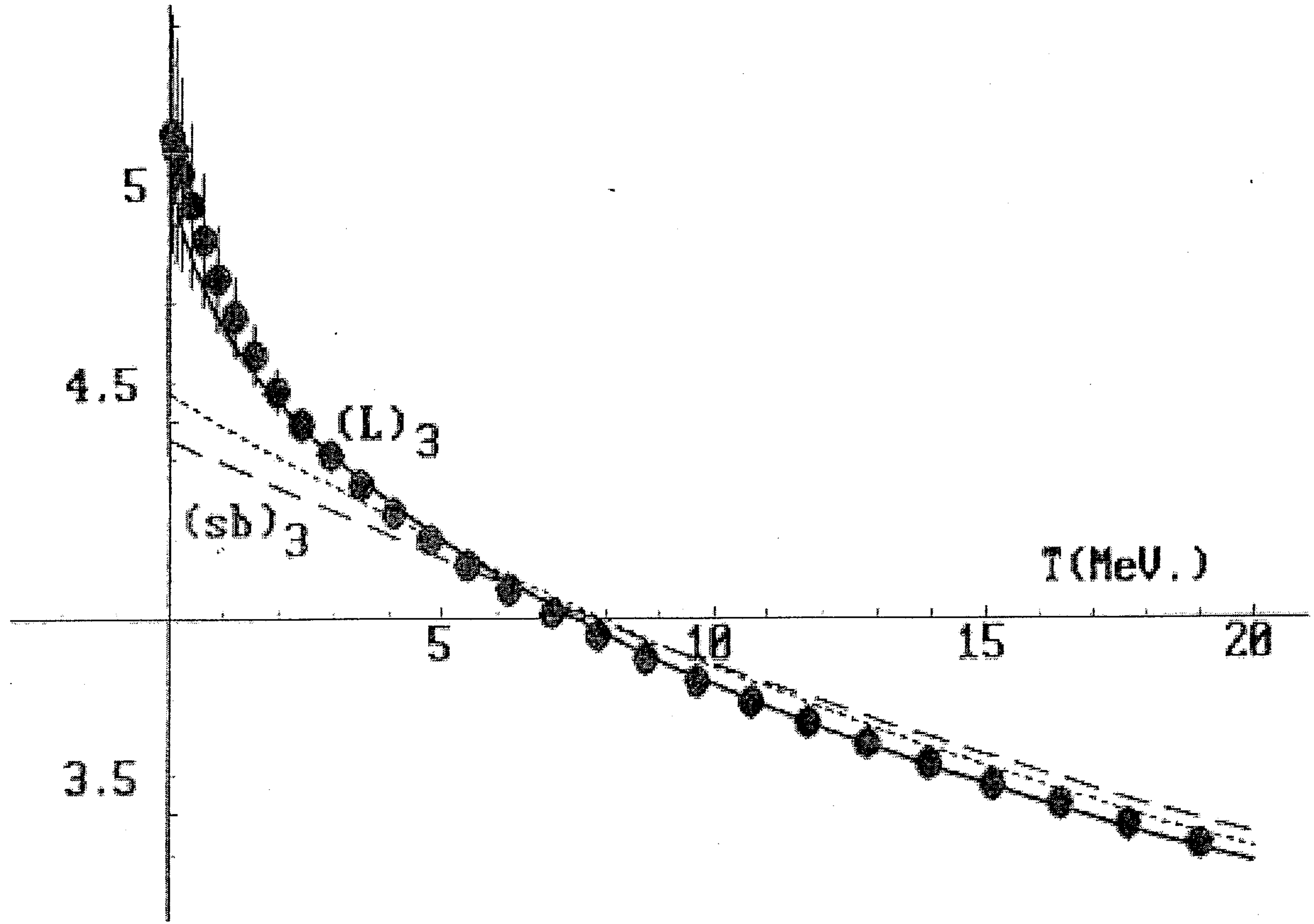
20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