International Symposium on Hadron Spectroscopy, Chiral Symmetry
and Relativistic Description of Bound Systems
Feb. 24-26, 2003, Ichigaya, Tokyo

Numerical Calculations of Excited Baryon Masses in Lattice QCD

Shoichi Sasaki (Univ.of Tokyo)

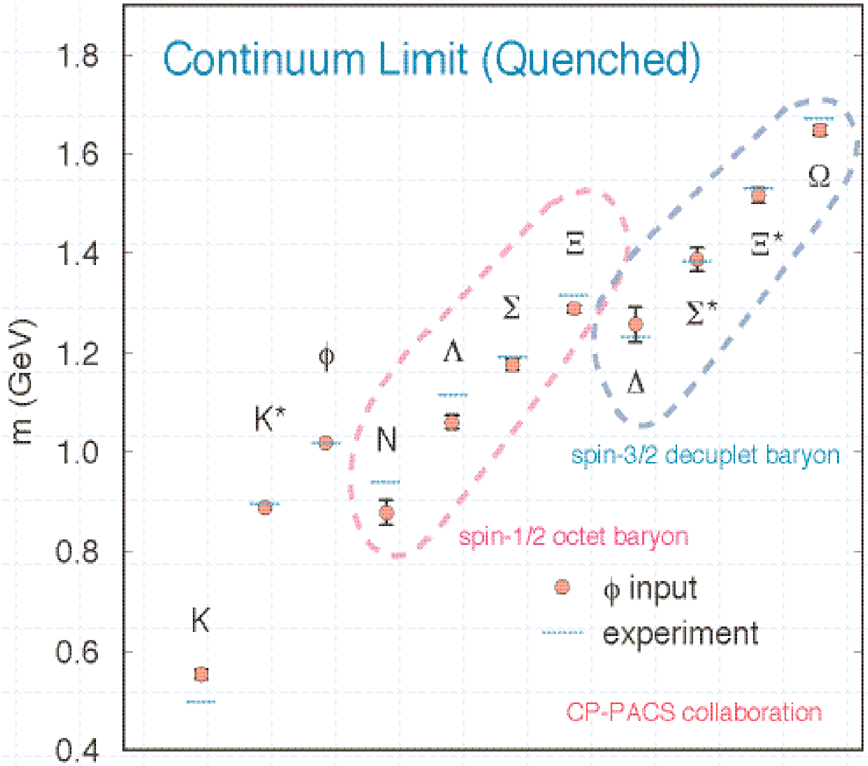
References:

S. Sasaki, T. Blum and S. Ohta, PRD65 (02) 074503

S. Sasaki, K. Sasaki, T. Hatsuda and M. Asakawa, hep-lat/0209059

Hadron Mass Spectrum from Lattice QCD

Experimental results are reproduced to within about 5-10% errors.



CP-PACS collaboration,
Phys. Rev. Lett. 84 (2002) 238

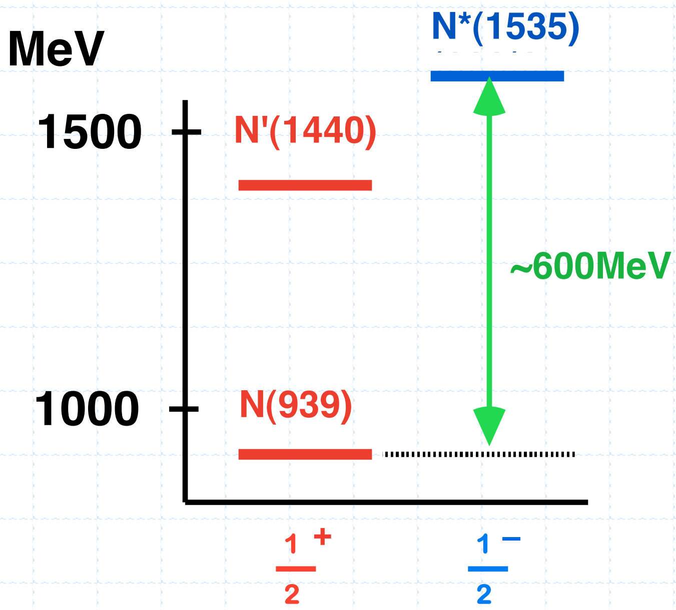
☛ But this success is restricted to ground states, and does not fully apply to excited states

Lattice studies of N* spectrum

- **Lee-Leinweber**
  D□34 action hep-lat/9809095, D234 action, hep-lat/0011060, 0110164.
- **Sasaki-Blum-Ohta (RIKEN-BNL)**
  Domain wall fermion, hep-lat/9909093, Phys. Rev. **D65** (2002) 074503.
- **Richards et al (UKQCD-QCDSF-LHPC)**
  Clover fermion, hep-lat/0011025, Phys. Lett. **B532** (2002) 63.
- **Melnitchouk et al (Adelaide)**
  Fat-link clover fermion, hep-lat/0202022.
- **Nakajima-Matsufuru-Nemoto-Suganuma**
  Clover fermion & anisotropic action, hep-lat/0204014.
- ....

A puzzle in nucleon excited states

Experiment

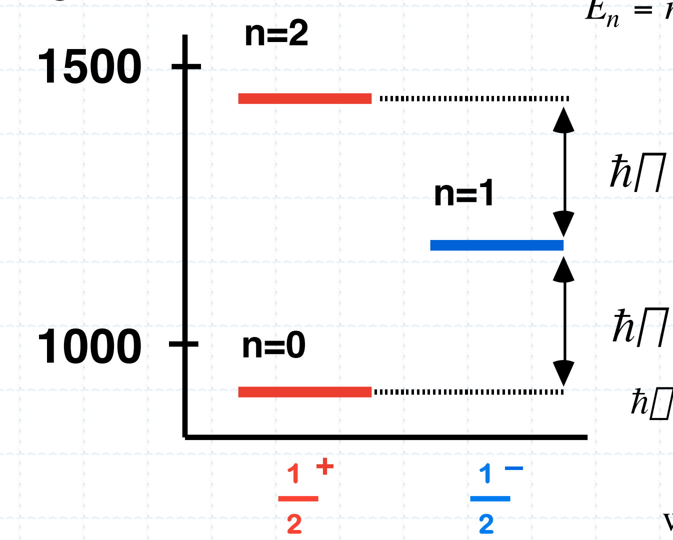


Potential Models

Naive quark model
Bag model

e.g. Harmonic-oscillator potential

$$E_n = m_N + n \cdot \hbar\omega$$



$\hbar\omega \approx 250\text{MeV}$

$\sim r_{\text{RMS}} \approx 0.7\text{fm}$

with $m_Q = 300\text{MeV}$

N < N* < N' (wrong ordering)

☞ Need lattice QCD calculation to resolve it.

How to compute the nucleon mass

Consider time-ordered, 2pt function:

$$\langle N(t)\overline{N}(0)\rangle_{\alpha\beta} = \sum_{\vec{x}} \langle 0|T\{N_\alpha(\vec{x},t)\overline{N}_\beta(\vec{0},0)\}|0\rangle$$

interpolating operator: $N = \epsilon_{abc}q_a q_b q_c$ with J=1/2, I=1/2

<u>The lowest-lying state dominates for large t</u>

$$\langle N(t)\overline{N}(0)\rangle_{\alpha\beta} = \underbrace{(1+\gamma_4)_{\alpha\beta}\theta(t)A_N e^{-M_N t}}_{\text{nucleon}} + \underbrace{(1-\gamma_4)_{\alpha\beta}\theta(-t)A_N e^{+M_N t}}_{\text{anti-nucleon}}$$

Particle contribution:

$$\mathrm{Tr}\{P_+\langle N(t)\overline{N}(0)\rangle\} \sim e^{-M_N t}$$ with the projection operator $P_+ = (1+\gamma_4)/2$

Spin-1/2 interpolating operators

Baryon operators under $SU(2)_L x SU(2)_R$

- ✓belong to the (1/2,0)+(0,1/2) chiral multiplet
- ✓without derivative

$J^P=1/2^+$ operator:

$N_1^+ = \varepsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$ and $N_2^+ = \varepsilon_{abc}(u_a^T C d_b)\gamma_5 u_c$

$J^P=1/2^-$ operator: $N_i^- = \gamma_5 N_i^+$

$N_1^- = \varepsilon_{abc}(u_a^T C\gamma_5 d_b)\gamma_5 u_c$ and $N_2^- = \varepsilon_{abc}(u_a^T C d_b)u_c$

It turns out that $\langle N^-(t)\bar{N}^-(0)\rangle = -\gamma_5\langle N^+(t)\bar{N}^+(0)\rangle\gamma_5$

$$\langle N^+(t)\bar{N}^+(0)\rangle \propto (1+\gamma_4)\theta(t)A_{N_+}e^{-M_{N_+}t} + (1-\gamma_4)\theta(-t)A_{N_+}e^{+M_{N_+}t}$$

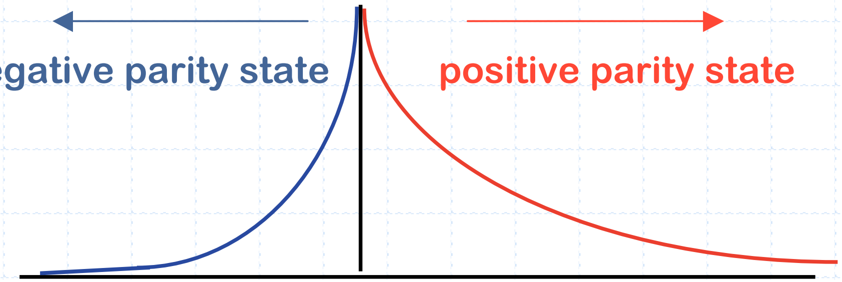
(positive parity)

$$-(1+\gamma_4)\theta(-t)A_{N_-}e^{+M_{N_-}t} - (1-\gamma_4)\theta(t)A_{N_-}e^{-M_{N_-}t}$$

(negative parity)

Wrap-around effect

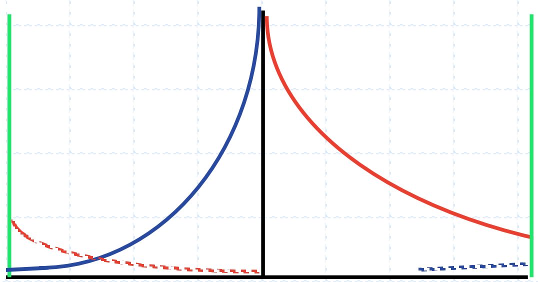
After projection with operator P_+



$$\mathrm{Tr}\left\{P_+\left\langle N^+(t)\overline{N}^+(0)\right\rangle\right\}$$

$$\sim \theta(t)A_{N_+}e^{-M_{N_+}t} - \theta(-t)A_{N_-}e^{+M_{N_-}t}$$

Finite extent T with (anti-) periodic boundary condition



$$b=\begin{cases}+1 & \text{(PBC)}\\ -1 & \text{(APBC)}\end{cases}$$

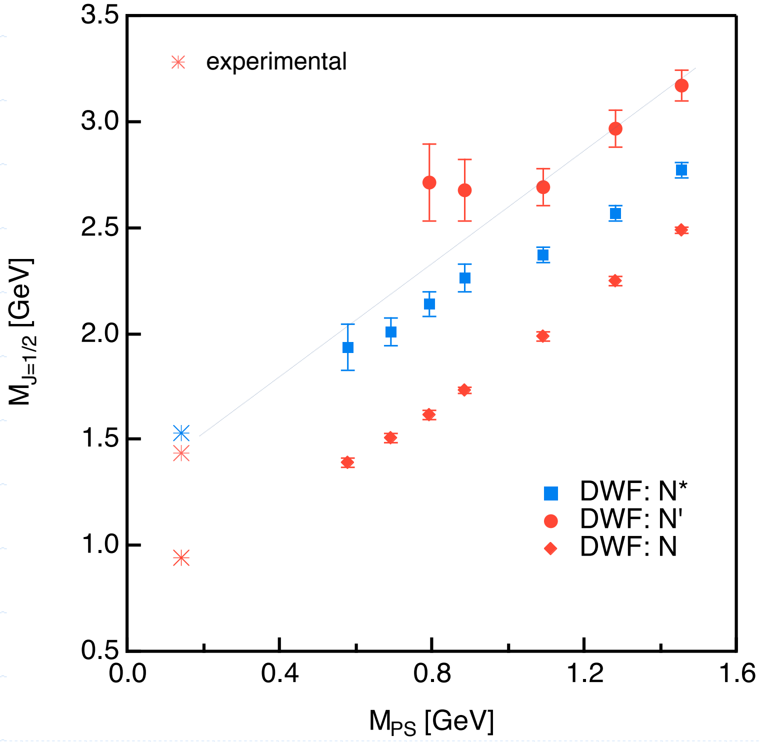
$$\mathrm{Tr}\left\{P_+\left\langle N^+(t)\overline{N}^+(0)\right\rangle\right\}$$

$$\sim \begin{cases} A_{N_+}e^{-M_{N_+}t} - b\cdot A_{N_-}e^{+M_{N_-}(t-T)} & \text{for } T/2 > t > 0 \\ -A_{N_-}e^{+M_{N_-}t} + b\cdot A_{N_+}e^{-M_{N_+}(t+T)} & \text{for } -T/2 < t < 0 \end{cases}$$

☛Necessary to take an appropriate boundary condition

Nucleon excited states in lattice QCD

DWF calculation 16^3x32x16 at beta =6.0



S. Sasaki, T. Blum, S. Ohta, PRD65 (02) 074503

- Large mass splitting of N(939)-N*(1535) is observed.
  - M_N~1.0GeV and M_{N^*}~1.6GeV .
  - good agreement with the experimental value.
- N'(1440) state is heavier than N* state (in the heavy quark region).
  - inverted level order ?
  - missing the Roper N'(1440)?

➜ Possible systematic errors

- ✓ quenching
- ✓ finite lattice spacing
- ✓ non-negligible higher states contribution
- ✓ finite volume effect

To resolve the remaining puzzle

Need information in the lighter quark mass region

I. Analysis to take account of higher lying states

- ☛ Maximum Entropy Method

II. Finite size effect might be serious for excited states

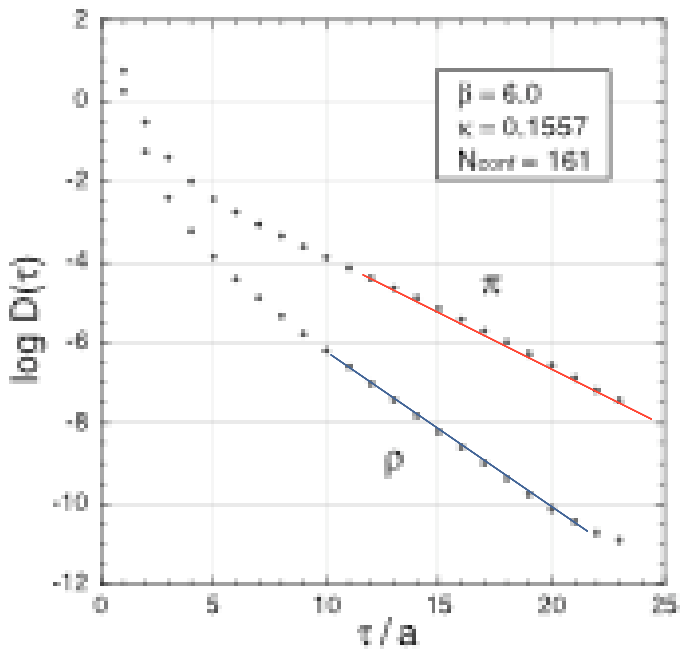
- ☛ Large physical volume > $(2.0\ \mathrm{fm})^3$

III. Similarity of the level pattern is found in the □ channel

- ☛ Not only spin-1/2 baryon but also spin-3/2 baryon

Measure the hadron masses in lattice QCD

Traditional approach: Asymptotic analysis

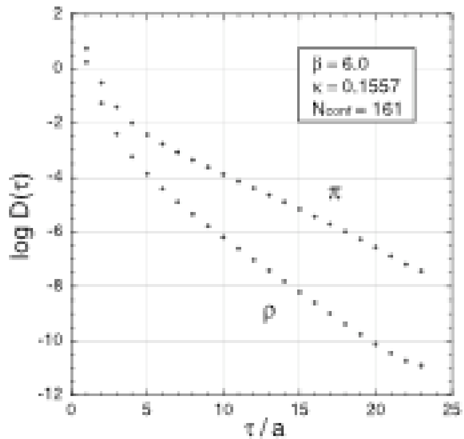


$$D(\tau) = \int d^3x \left\langle h(\vec{x},\tau) h^{+}(0,0) \right\rangle$$

$$= \sum_n \left|\left\langle 0 \middle| h \middle| n \right\rangle\right|^2 e^{-M_n \tau} \xrightarrow{\tau\to\infty} \sim e^{-M_0 \tau}$$

- ✓ Allow us to fit only the large-τ behavior of a hadron correlator
  - → Data truncation
  - → Discard information for exicted states

Spectral functions in lattice QCD



Nakahara, Asakawa, Hatsuda, PRD60 (99) 091503
PPNP46 (01) 459

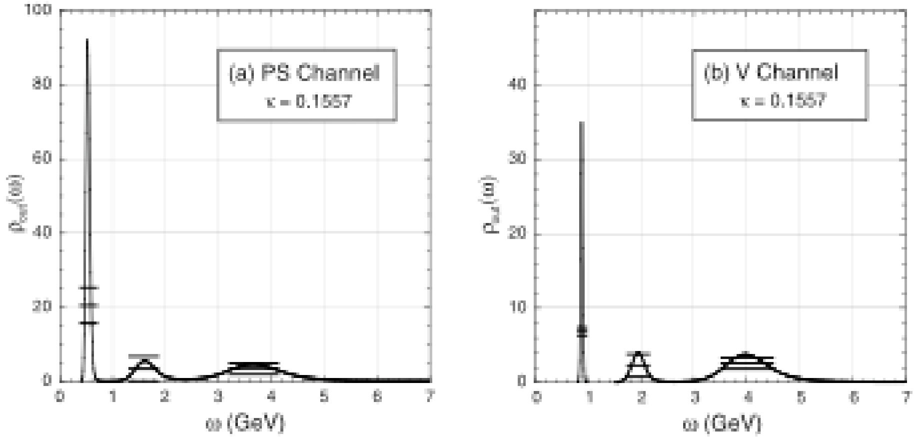
pseudo-scalar, vector, scalar, axial-vector

Yamazaki (CP-PACS collaboration), PRD65 (02) 014501

pseudo-scalar, vector (continuum limit)

Application only for mesons !!

MEM analysis



Maximum Entropy Method (MEM)

- Spectral function A(ω) gives us much information

$$D(\tau) = \int_0^\infty d\omega\, A(\omega) e^{-\tau\omega}$$

Input : $N_\tau \sim O(10)$
Output: $N_\omega \sim O(100 - 1000)$

But, the reconstruction of A(ω) from D(τ) is not unique !

Assume : the probability of A(ω) can be assigned for given D(τ)
➜The most probable A(ω) is inferred from P[A|D]

$$A_{\mathrm{imag}}(\omega) = \int [dA]\, A(\omega) \cdot P[A\,|\,D]$$

Note: the posterior probability P[A|D] is not the conditional probability of D(τ) given A(ω) ; P[D|A] (the likelihood function)

More about MEM

Bayesian statistics

posterior probability — likelihood function

$$P[A|D] = \frac{P[D|A]P[A]}{P[D]} \propto P[D|A]P[A]$$

evidence — prior function

- Central limiting theorem

$$P[D|A] \propto e^{-\chi^2/2}$$

- Information entropy (Shanon-Jaynes)

m(ω) : default model
α : the entropy strength

$$P[A] \propto e^{\alpha S} = \exp\left\{\alpha \int_0^\infty d\omega \left[A(\omega) - m(\omega) - A(\omega)\ln\left(\frac{A(\omega)}{m(\omega)}\right)\right]\right\}$$

[Bryan's method]

To find the maximum of P[A|D] with respect to A(ω) at fixed D(τ),

☛ Minimize $Q=\chi^2/2-\alpha S$ to use singular value decomposition (SVD)

Asakawa, Hatsuda, Nakahara, PPNP46 (01) 459, for more detail

Details of the simulation

Gauge: Standard plaquette action

β=6.0, $a^{-1} \simeq 2.0$ GeV

three lattice sizes 32^3x32, V$\simeq$(3.0 fm)^3, 200 configs

24^3x32, V$\simeq$(2.2 fm)^3, 300 configs

16^3x32, V$\simeq$(1.5 fm)^3, 444 configs

Fermion: Wilson fermions

4 quark masses (M_π / M_ρ = 0.69 – 0.91)

Point source - Point sink

P.B.C. + A.P.B.C. for the temporal direction

Basic result:

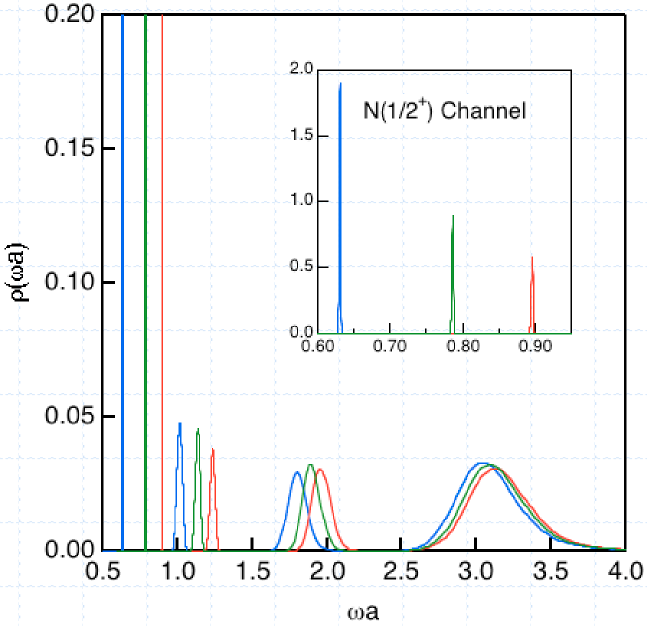
K_c=0.1568(15)

Nucleon and Delta Spectral functions

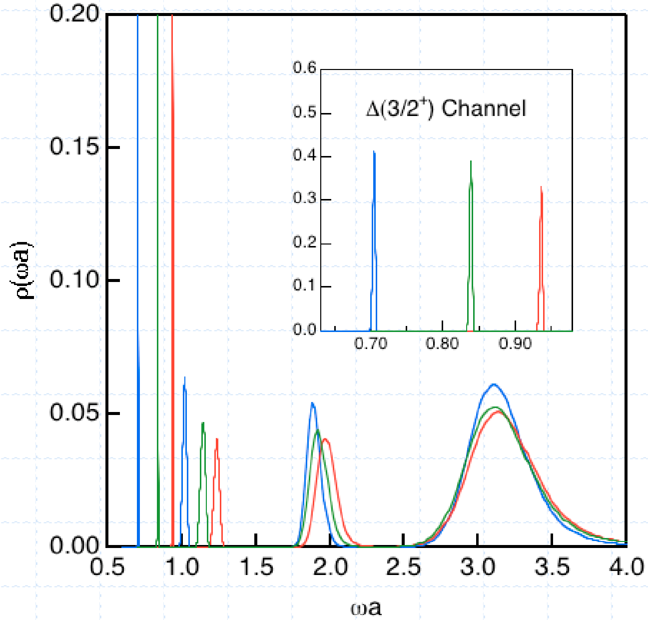
MEM analysis

beta=6.0, V=24^3x32 (Wilson fermions)

$\square_{max}$a~ 2□, $N_{\square}$ ~ O(500), m_0a=4x10^{-3} (N), 8x10^{-3} (□)



Nucleon channel

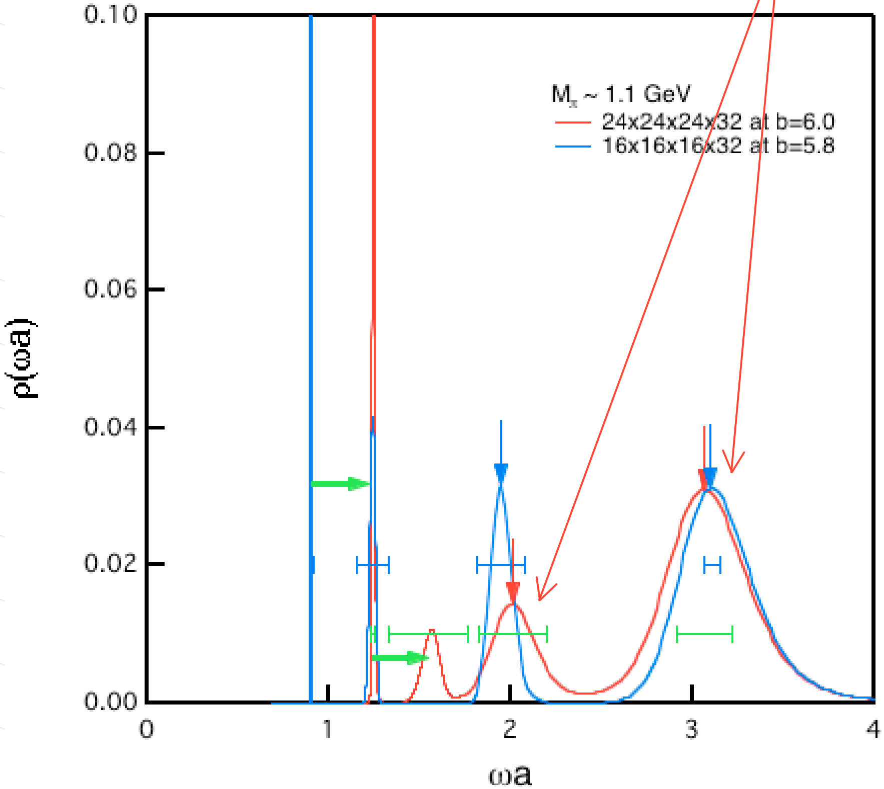


Delta channel

Baryonic bound state of doublers ?

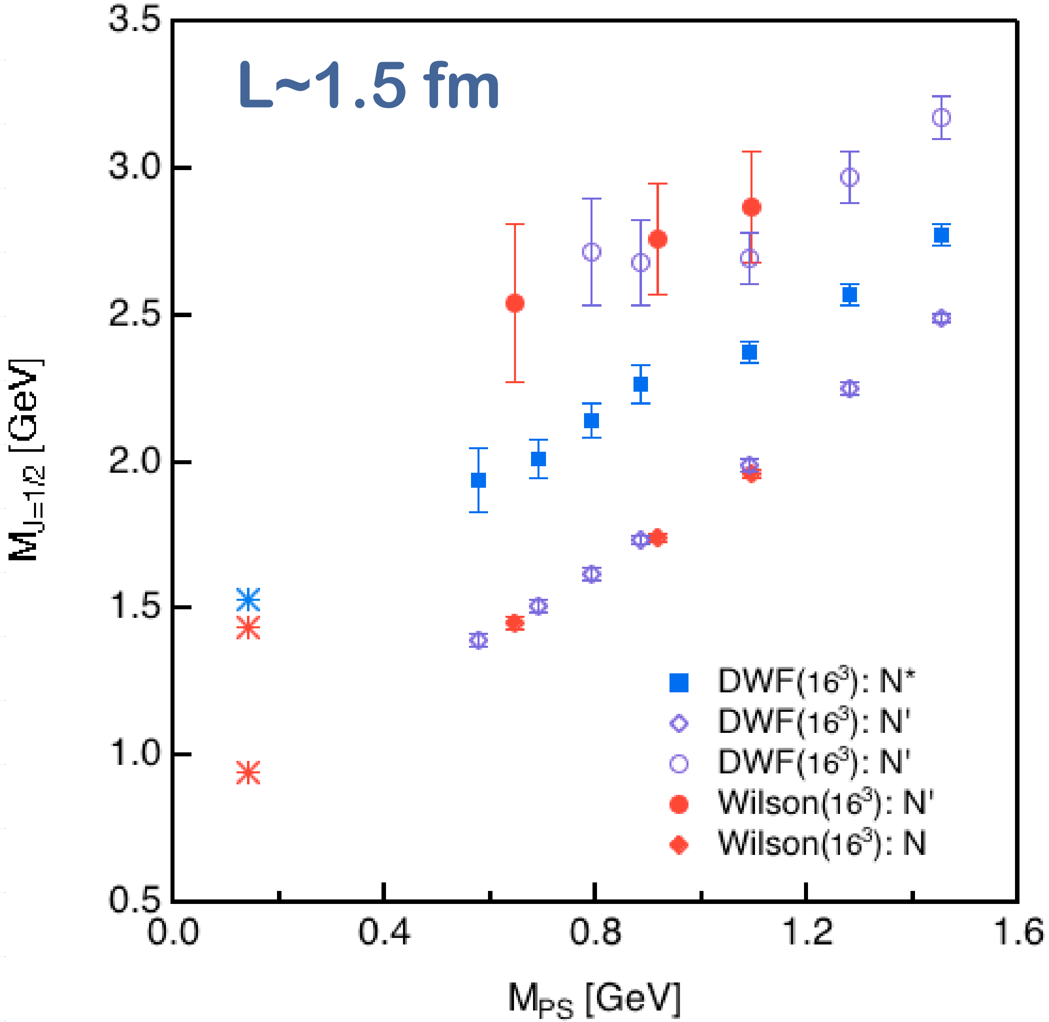
unphysical states

infinitely heavy in the continuum limit (a→0)



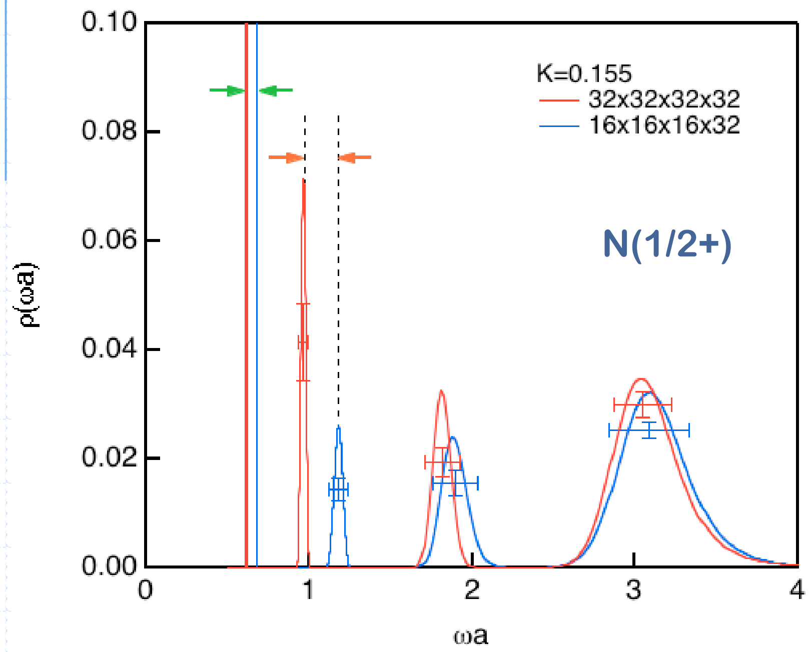
CP-PAC collaboration already found the mesonic bound state of doublers (Yamazaki et al., PRD65 (02) 014501)

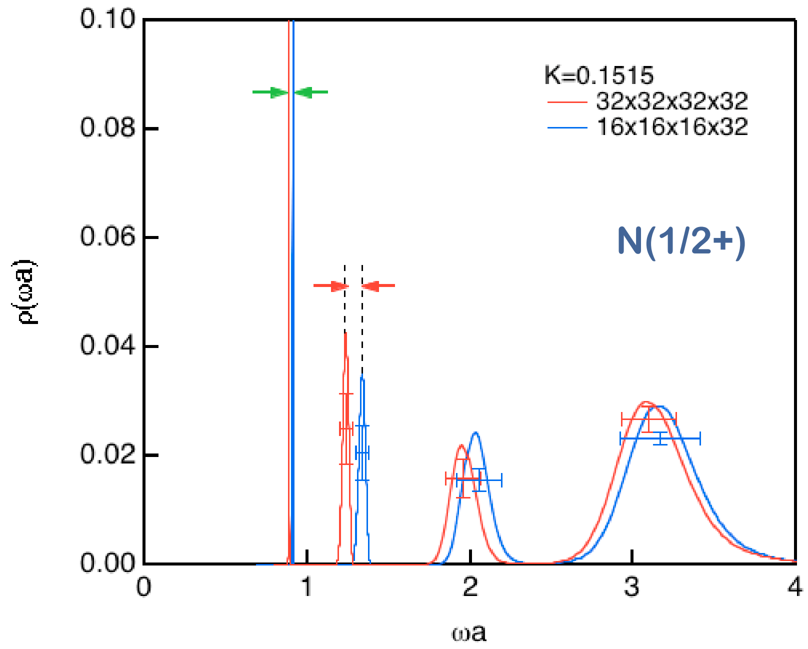
Results for 16^3x32 lattice at □=6.0



Large finite-size effect on the N’ state

The observed size effect is **much larger** for the (physical) **excited state** than for the **ground state**

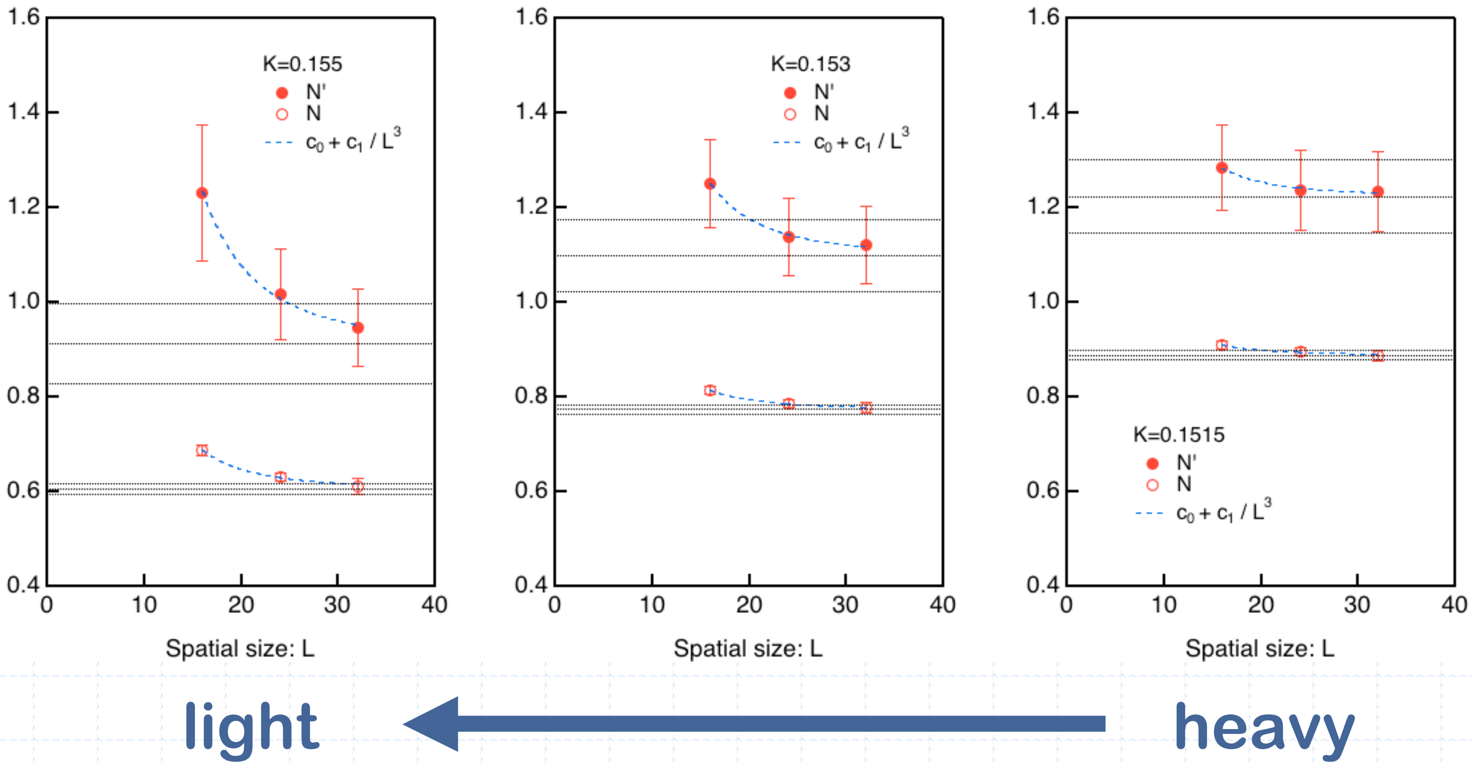




Light ($m_{□}$~0.6GeV) Heavy ($m_{□}$~1.1GeV)

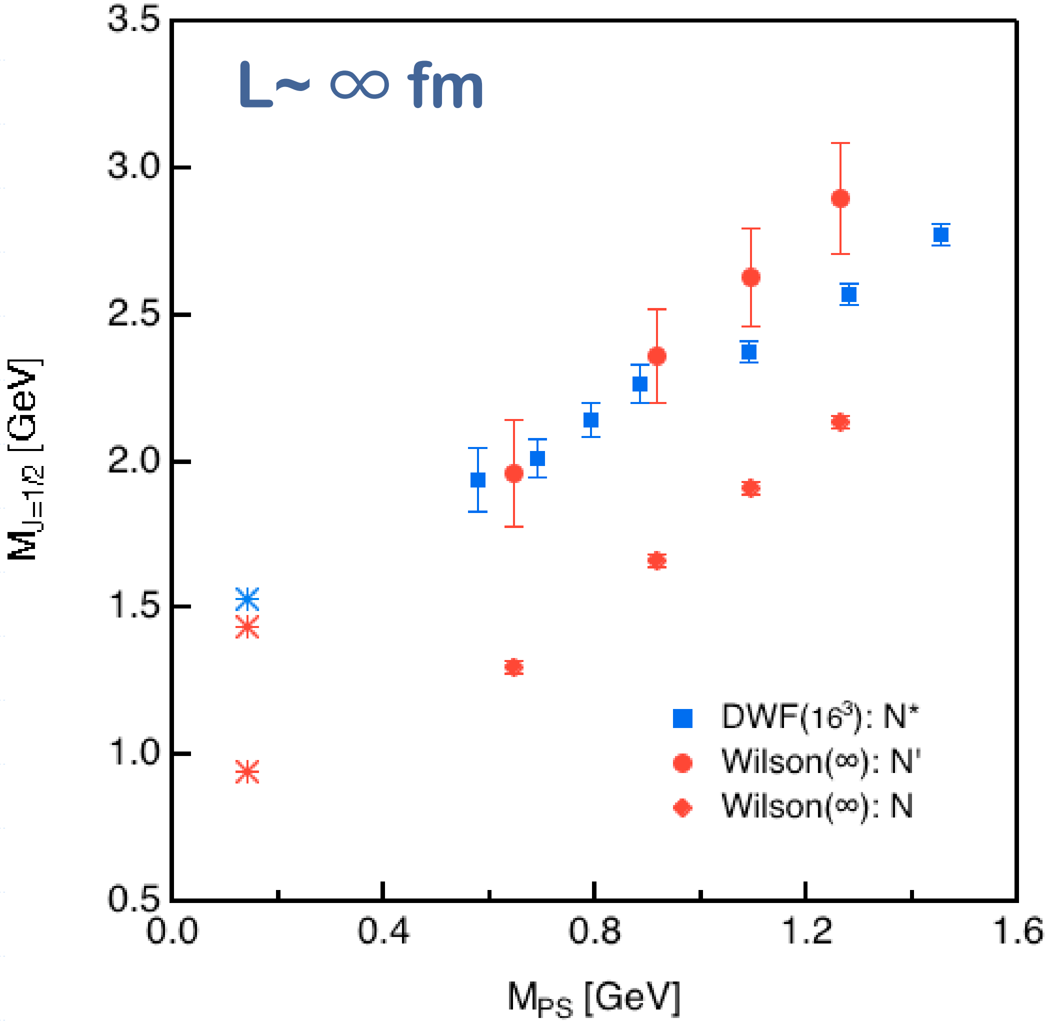
Infinite volume limit

- All data is well fitted by the 1 / L³ curve



☛ The finite-size effect is rather severe in the lighter quark mass region

Level switching between N’ and N*



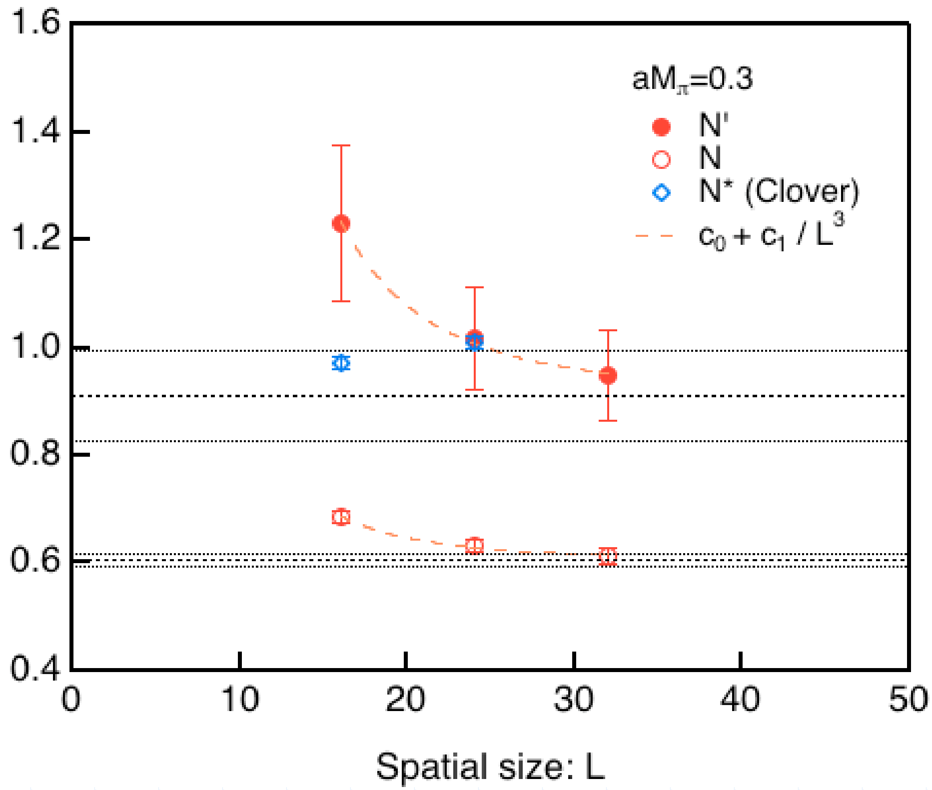
Summary

Maximum entorpy method is applied to quenched baryon spectrum with Wilson fermions

- ❒ Succeed in extracting N and spectral functions
  - ✓ Can extract the mass of first excited states
  - ✓ Find unphysical states around a ~2.0 and 3.0
- ❒ Confirm the large finite-size effect on the N' state in the light quark mass region
  - ✓ Spatial lattice size La ~ 2.2 fm is not large enough
  - ☛ The level switching between N' and N* should happen in lattice simulations with large spatial size La > 3.0 fm
  - ☛ The Roper resonance can be described by the simple three quark excitation of sizable extent

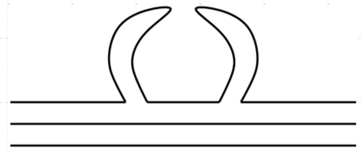
Opposite finite-size effect on N* to N and N’

The N* mass on the larger lattice (L=24) is higher than on the smaller lattice (L=16) by an amount of order 5%. (JLab/UKQCD, 2002)



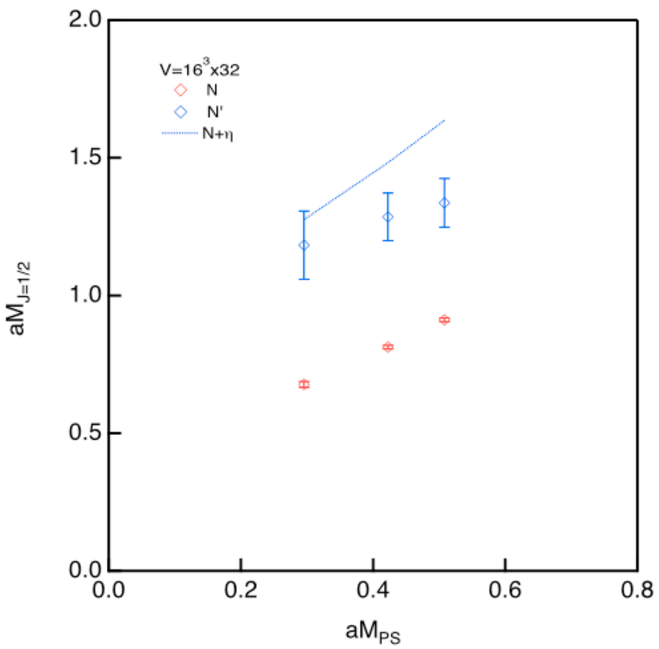
Data for N* (clover fermions at []=6.0) is taken from Phys. Lett. B532 (2002) 63

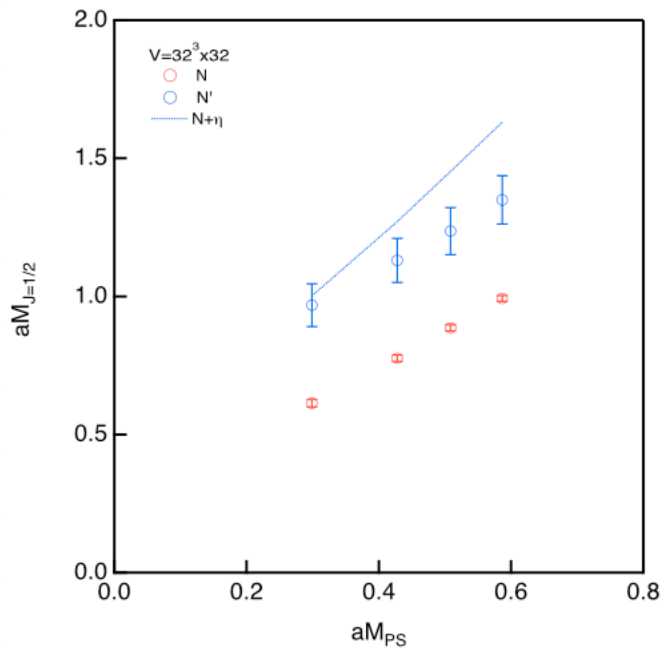
Multi-hadron state???



N-η

No, it can't be !





$$\underline{M_{N\eta} < \sqrt{M_N{}^2 + p_{\min}{}^2} + \sqrt{M_\eta{}^2 + p_{\min}{}^2}} \quad \text{where } p_{\min} = \frac{2\pi}{aL} \quad \text{and} \quad M_\eta = M_\pi$$