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Bi-Local Theories

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Content

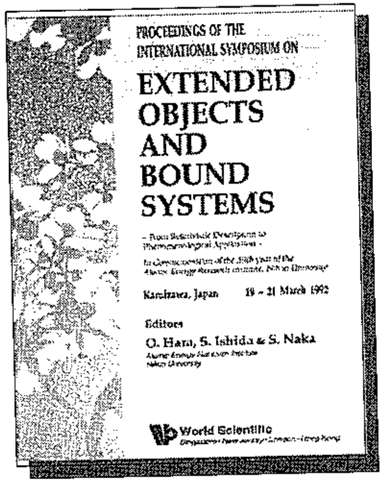
§1 Introduction

Karuizawa,

Japan March,1992

Extended Objects and Bound Systems

… in commemoration of the 35th year of the Atomic Energy Research Institute, Nihon University



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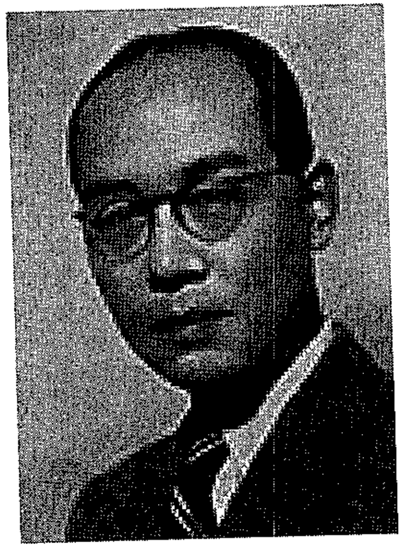
Tokyo, Japan February, 2003

Hadron Spectroscopy, Chiral Symmetry and Relativistic Description of Bound Systems

Nihon-U & KEK

In 1947, Yukawa discussed an equation for matter fields under various force fields. Then, he suggested that if the material particles have a space-like extension, then force-field potentials, say electromagnetic potential A_μ, are not necessarily functions of x^μ alone, but may depend on p^μ also.



$\cdots$ he introduced the non-commutativity of vector a field $A_\mu A_\nu \neq A_\nu A_\mu$.

In 1948, Yukawa wrote a paper entitled "Possible Types of Nonlocalizable Fields. In this paper, Yukawa considered a scalar-zero-mass field U depending on both of (x_μ, p_μ) and discussed a possibility to introduce a constant with a dimension of length into the theory associated with the relative coordinates $\bar{x} = x' - x''$ in the matrix element $< x'|U|x'' >$.

With $\bar{x}$ and the center of mass coordinate $X = (x' + x'')/2$, Yukawa's matrix field, the bi-local field, can be written as $\Phi(X, \bar{x}) =< x'|U|x'' >$. In the next papers, adapting "Born's reciprocity(p $\leftrightarrow$ x symmetry) "as guiding principle, Yukawa determined the field equations of U field to the following form:

$$\left.\begin{array}{c} [p^\mu, [p_\mu, U]] - m^2c^2U = 0 \\ [x^\mu, [x_\mu, U]] + \lambda^2 U = 0 \\ [p^\mu, [x_\mu, U]] = 0 \end{array}\right\} \quad (1)$$

; that is, he set

$$\left.\begin{array}{c} (P^\mu P_\mu - m^2c^2)\Phi = 0 \\ (\bar{x}^\mu \bar{x}_\mu + \lambda^2)\Phi = 0 \\ P^\mu \bar{x}_\mu \Phi = 0 \end{array}\right\}, \quad (2)$$

where P and $\bar{p}$ are the momenta conjugate to X and $\bar{x}$ respectively. The first of Eq.(2) is Klein-Gordon equation for a mass m field. The second and the third of Eq.(2) mean that the bi-local system has a space-like extension λ in the rest frame of the system.

critical comments

- The masses of all spin states are degenerate to m.
- The λ can be removed by a canonical transformation. (Hara-Shimazu,1950).

In answer to these claims

Yukawa said "this is certainly not true, whenever we take into account the interaction between two non-local fields ". Then, Yukawa discarded the second of Eq.(2) and extended the master wave equation to the form

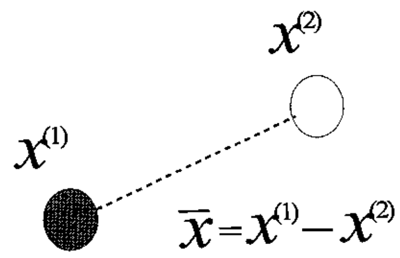
$$(P^{\mu}P_{\mu} - M^2(P, \bar{p}, \bar{x}))\Phi = 0 \qquad (3)$$

In 1953, Yukawa proposed the following M^2 operator:

$$M^2(P, \bar{p}, x) = \alpha(\bar{p}^{\mu}\bar{p}_{\mu} + \kappa^2\bar{x}^{\mu}\bar{x}_{\mu}) \qquad (4)$$

In this case, the mass square of the bi-local system becomes a linear function of their spin eigenvalue, which is preferable from phenomenological point of view.

Yukawa called this type of non-local fields as "*four-dimensional oscillator model for the elementary prticles*".



$\cdots$ as a result, the bi-local system had the character of relativistic potential approach to the two-particle bound system.

Yukawa also studied the physical state condition to eliminate the relative time of the system, since such a degree of freedom causes either an infinite degeneracy of mass eigenvalues or an indefinite metric Hilbert space.

What kind of condition did he set?

As an available physical state condition for the bi-local system with the four-dimensional oscillator potential, Yukawa set $P \cdot a^{\dagger}|\Phi_{phy}\rangle=0$.

P.S.C.

Yukawa (1953)

$$P^{\mu}a_{\mu}^{\dagger}|\Phi_{phy}\rangle = 0$$

$\downarrow$

ground state

$$a_0^{\dagger}|0\rangle = a_i|0\rangle = 0$$

$\downarrow$

definite metric

Takabayasi(1964)

$$P^{\mu}a_{\mu}|\Phi_{phy}\rangle = 0$$

$\downarrow$

ground state

$$a_0|0\rangle = a_i|0\rangle = 0$$

$\downarrow$

indefinite metric

$$a_{\mu} = \sqrt{\frac{1}{2\kappa}}(\kappa\bar{x}_{\mu} + i\bar{p}_{\mu}), \quad a_{\mu}^{\dagger} = \sqrt{\frac{1}{2\kappa}}(\kappa\bar{x}_{\mu} - i\bar{p}_{\mu})$$

$$[a_{\mu}, {a_{\nu}}^{\dagger}] = -g_{\mu\nu}$$

classical action

$$S = \int d\tau \sum_{i=1}^{2} m(\bar{x}) \sqrt{\dot{x}^{(i)2}} = \int d\tau \frac{1}{2} \sum_{i=1}^{2} \left\{ \frac{\dot{x}^{(i)2}}{e_i} + e_i m^2(\bar{x}) \right\}$$

$$\left(\begin{array}{c} m(\bar{x}) = \kappa \bar{x}^2 + const. \\ e_i; \text{einbein} \sim m(\bar{x})^{-1} \sqrt{\dot{x}^{(i)2}} \end{array} \right)$$

$$\Downarrow$$

$$\frac{1}{4} P^2 + \bar{p}^2 + m(\bar{x})^2 = 0 \rightarrow \text{master wave equation}$$

$$P_\mu \bar{p}^\mu = 0 \rightarrow \langle P_\mu \bar{p}^\mu \rangle = 0 \rightarrow \begin{cases} P^\mu a_\mu^\dagger |\Phi\rangle = 0 \\ P^\mu a_\mu |\Phi\rangle = 0 \end{cases}$$

§2 Development of Bi-Local Theories in Our Group (~ 1980)

- □ Elimination of constraints
- □ Interaction of bi-local fields
  - form factor -
- □ Interaction of bi-local fields
  - three vertex -
- □ Bi-local theories of spinning particles

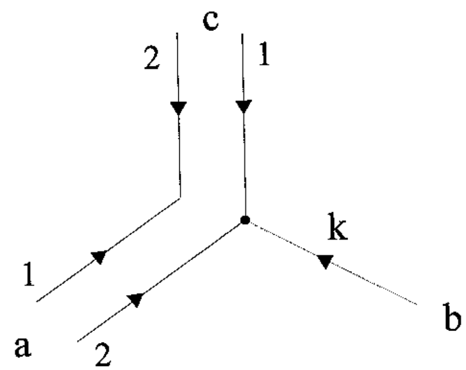
Interaction of bi-local fields with external fields
-form factor-

$$V(x^{(2)}) = g e^{ik\cdot x^{(2)}}$$

$$\Downarrow$$

$$V_{phy}(x^{(2)}) = g e^{ik\cdot [x^{(2)}]}$$

($[\cdots]$: projection to physical subspace)

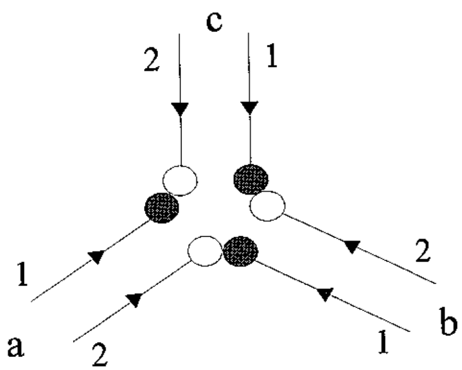


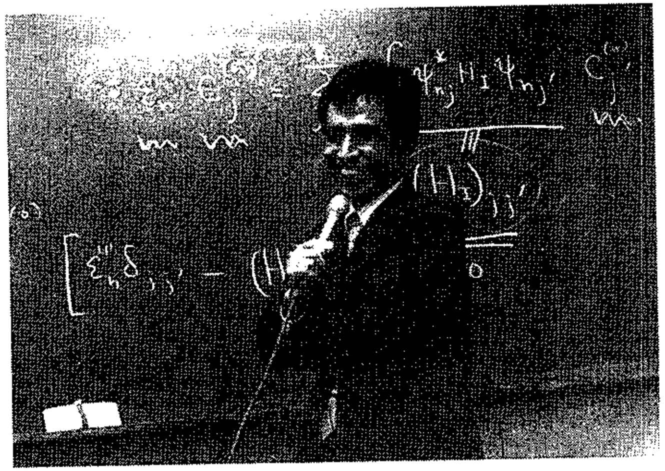
ground state form factor: $\langle 0|vertex|0\rangle$

vertex	indefinite metric	definite metric
V	$e^{t/(16\kappa)}$	$\dfrac{1}{1-\frac{t}{2m^2}} e^{\frac{t/(16\kappa)}{1-t/(2m^2)}}$
V_{phys}	$\sim e^{t\cdot\pi^2/(64\kappa)}$ for $-\frac{t}{4m^2} \gg 1$	

$(\ t = -k^2\ )$

Interaction of bi-local fields - three vertex-



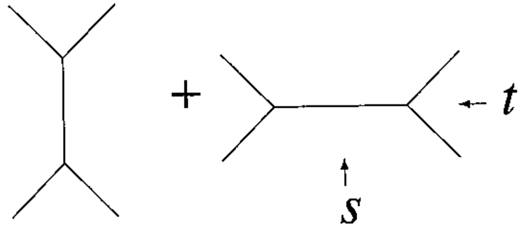


$$\left.\begin{array}{l}[x_\mu^{(1)}(b) - x_\mu^{(2)}(a)]|V\rangle = 0 \\ [p_\mu^{(1)}(b) + p_\mu^{(2)}(a)]|V\rangle = 0\end{array}\right\} (a, b, c \text{ cyclic})$$

$$\Downarrow$$

$$|V_{phys}\rangle = \Lambda_a \Lambda_b \Lambda_c |V\rangle$$

$$(P \cdot a\Lambda = 0)$$



exp. increase Regge

§3 Recent Topics

A Bi-Local Field Eq. out of Bethe-Salpeter Eq.

B Bi-Local Higgs-Like Fields Based on Non-Commutative Geometry

C q-Deformed Bi-Local System

A Bi-Local Field Eq. out of Bethe-Salpeter Eq.

(P. T. P., **103**(2000),847)

B.S	$\Leftrightarrow$	B. L.
$\Phi_{BS}(x_1, x_2) = G_1 G_2 K \Phi_{BS}$		$(G_1^{-1} + G_2^{-1} + V)\Phi_{BL}(x_1, x_2) = 0$ $G_i^{-1} = p_i^2 - m^2, \ (i = 1, 2)$

<u>classical Lagrangian model</u>

$$S = \int d\tau \left\{ \frac{1}{2} \sum_{i=1}^{2} \left(\frac{\dot{x}_i^2}{e_i} + e_i m^2 \right) + \sqrt{e_1 e_2 K(\bar{x})} \right\}$$

$$\downarrow$$

$$(p_1^2 - m^2) - \sqrt{\frac{e_2}{e_1} K(\bar{x})} = 0$$

$$(p_2^2 - m^2) - \sqrt{\frac{e_1}{e_2} K(\bar{x})} = 0$$

$\longrightarrow$ B. L. $\quad (e_1 = e_2, \ \sqrt{K} = \frac{1}{2} V)$

$$\downarrow$$

B. S. $\quad (p_1^2 - m^2)(p_2^2 - m^2) = K(\bar{x})$

$$\left\{\sum_{i=1}^{2} G_i^{-1} - \left(\sum_{i=1}^{2} G_i\right) K\right\} \Phi_{BS} = 0$$

$$G_i^{-1} = \tfrac{1}{4}P^2 + (-1)^i P\cdot\bar{p} + \bar{p}_{\parallel}^2 + \bar{p}_{\perp}^2 - m^2 + i\epsilon, \ (i = 1, 2)$$

$$\Downarrow$$

Assumption : instantaneous int.

$$K = iU^{-1}\left\{\delta\left(\frac{P\cdot\bar{x}}{P^2}\right) V(\bar{x}_{\perp})\right\} U$$

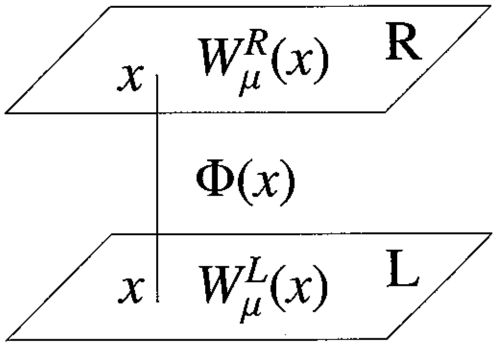
$$U = \exp\left[-i\bar{p}_{\parallel}^2 \left(\bar{p}_{\perp}^{-2}\bar{p}_{\perp}^{\mu}, \bar{x}_{\mu}\right)_w\right]$$

$$\Downarrow \quad \Phi'_{BS} = \delta\left(\tfrac{P\cdot\bar{x}}{P^2}\right) U\Phi_{BS}$$

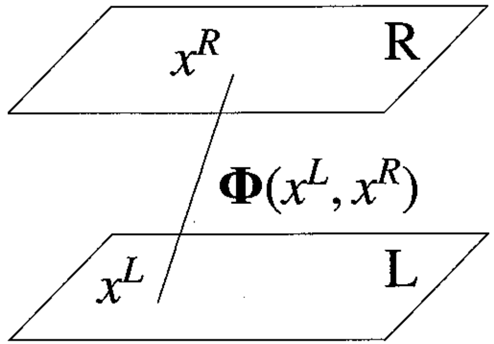
$$\left\{\frac{1}{2} + 2(\bar{p}_{\perp}^2 - m^2) + V(\bar{x}_{\perp})\right\} \Phi'_{BS} = 0$$

$$\textit{norm}: \ \langle\Phi'_{BS}| \left\{ \sqrt{P^2}\,\delta(X_{\parallel} - \tau)\,\delta\left(\frac{P\cdot\bar{p}}{P^2}\right)\right\}_w |\Phi'_{BS}\rangle$$

B Bi-Local Higgs-Like Fields Based on Non-Commutative Geometry (P. T. P., **103**(2000),411)



Higgs fields in NCG (Connes,1990)



Higgs fields in bi-local NCG

$$D_\mu = \begin{pmatrix} D_\mu^L & 0 \\ 0 & D_\mu^R \end{pmatrix}$$

$$D_\eta = \partial_\eta - ig\begin{pmatrix} 0 & \phi \\ \phi^* & 0 \end{pmatrix} = ig\begin{pmatrix} 0 & \Phi \\ \Phi^* & 0 \end{pmatrix}$$

$$\eta = \frac{1}{M}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \partial_\eta \eta = -i$$

$$\mathcal{L} = -\tfrac{1}{4}\mathrm{Tr}F_{AB}^\dagger F^{AB} + \bar{\Psi}(\gamma^\mu i D_\mu + i\kappa D_\eta)\Psi$$

$F_{\mu\nu}^2 \cdots$ kinetic terms for $W_\mu^{L/R}$
$F_{\eta\mu}^2 \cdots$ kinetic terms for ϕ, ϕ^*
$F_{\eta\eta}^2 \cdots$ potential terms for ϕ, ϕ^*

$$S \sim -\frac{1}{4}\int d^4x \mathrm{tr} \boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu} + \int d^4x \bar{\boldsymbol{\Psi}} i\gamma^{\mu}(\partial_{\mu} - ig\mathbf{W}_{\mu})\boldsymbol{\Psi}$$

$$+ \int d^4x^L \int d^4x^R \mathrm{tr}(\partial_{\mu}\Phi_{LR}\partial^{\mu}\Phi_{RL} + 4\bar{\partial}_{\mu}\Phi_{LR}\bar{\partial}^{\mu}\Phi_{RL})$$

$$-2g^2 \int d^4x^L \int d^4x^R \mathrm{tr}\left(\Phi_{LR}\Phi_{RL} - \frac{1}{g^2}M^2\right)^2$$

$$+g\kappa \int d^4x^L \int d^4x^R (\bar{\boldsymbol{\Psi}}^L \Phi_{LR} \boldsymbol{\Psi}^R + \bar{\boldsymbol{\Psi}}^R \Phi_{RL} \boldsymbol{\Psi}^L)$$

$$\boldsymbol{\Phi}_{LR} = U_L \Phi_{LR} U_R^{\dagger}, \quad \Phi_{LR} = \frac{1}{g}M + (\sigma_{LR} + i\pi^a_{LR}\tau_a)$$

$$U_{L/R} = P\exp\left\{ig \int_{(C_{L/R})}^{x^{L/R}} dz^{\mu}\mathbf{W}_{\mu}\right\}$$

$$M^2 \propto \bar{x}^2 \Rightarrow COQM$$

C q-Deformed Bi-Local System

q-deformation

$$[a, a^\dagger] = 1, \quad (N = a^\dagger a)$$

$$\Downarrow$$

$$a_q a_q^\dagger - q^\alpha a_q^\dagger a_q = q^{-\alpha(N+\beta)}$$

mapping $a \to a_q = a\sqrt{\frac{[N]_q}{N}}$

$$a_q^\dagger a_q = [N]_q = \frac{\sinh[\alpha(N+\beta)\log q]}{\sinh[\alpha \log q]}$$ $\cdots$ eigenvalues deviate from equal spacing

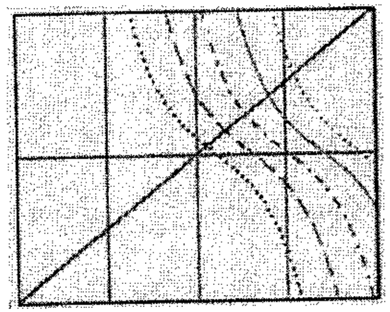
If we apply this deformation to relative coordinates of the four-dimensional oscillator model, what can we expect? In particular, will anything happen for $\beta \propto P^2$?

wave equation of q-deformed bi-local system

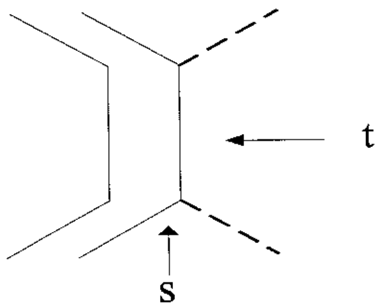
P.T.P. **109**(2003),103

$$\left(\frac{1}{4}P^2 + 2\kappa\{a^\dagger_{q\mu}, a^\mu_q\} + const.\right)\psi = 0$$

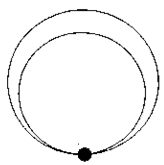
$$\{a^\dagger_{q\mu}, a^\mu_q\} = \frac{\sinh[\alpha(\frac{1}{2}\{a^\dagger_\mu, a^\mu\} - \gamma P^2)\log q]}{\sinh[\frac{1}{2}\alpha \log q]}$$



- s-t symmetry will arise for large s (or t) providing $\gamma = e^{\log q}/16\kappa\alpha \log q$



- finite loop diagram



$\cdots$these results are preliminary and are under investigation.

§4 Concluding Remarks

- The relativistic description of extended objects is not simply an effective approach to the bound systems of local fields. Indeed, there are other viewpoints of extended objects which are supposed to be fundamental in nature. For example, the relativistic strings, the fields in non-commutative world, the matrix (bi-local) fields and so forth should be considered to belong to this category. Hence, the bi-local fields are still interesting subject to study.

  In any case, we should keep in our mind that there may be other ways following which the more basic change of the fundamental concepts have to be required.

- ⋯ I am tired ⋯ Please leave me alone!