Light-Cone Wave Functions of Light Mesons from Light-Front Quantization

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Introduction

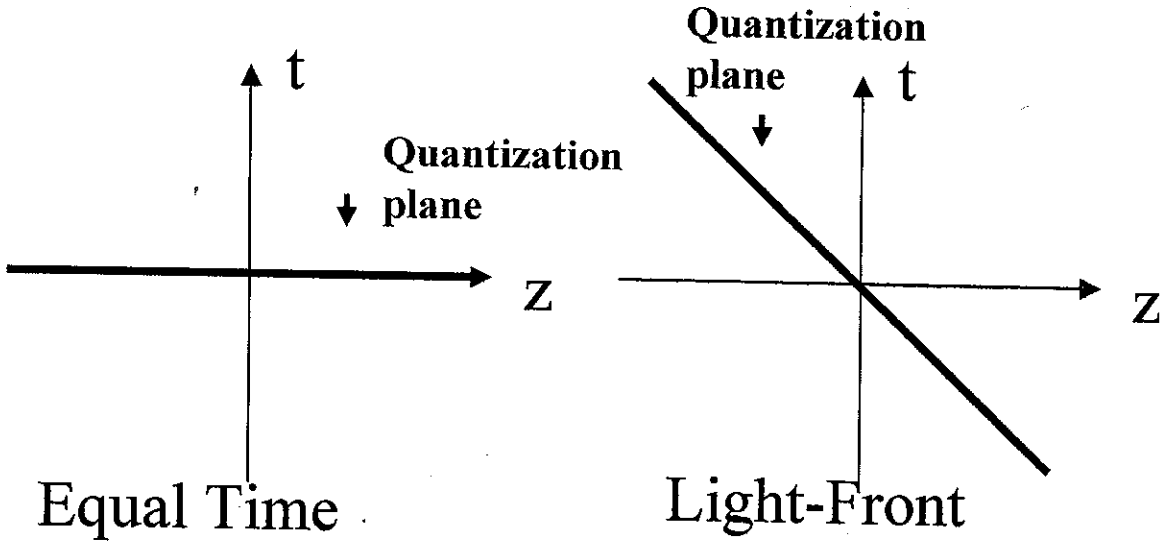
Background and Motivation

Hadron ~ relativistic bound state of quarks and gluons

How to represent the hadron structure

➡ **Light-cone (LC) wavefunction is Best suited**

Quantization condition is imposed on the $x^+ = \frac{t+z}{\sqrt{2}} = \text{const}$ plane

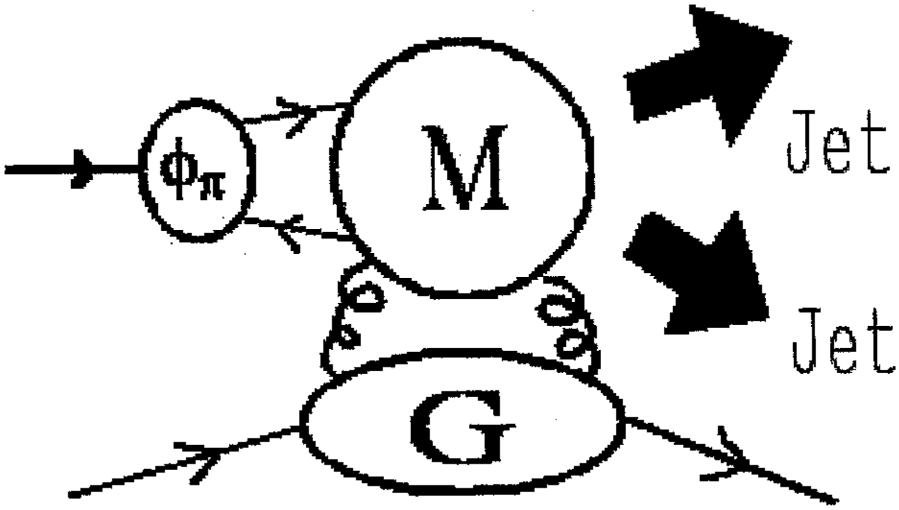


As a result, the Light-Front formalism is invariant (or covariant) under Lorentz-boost transformation. (7 Poincare transformations are preserved)

Current experimental condition

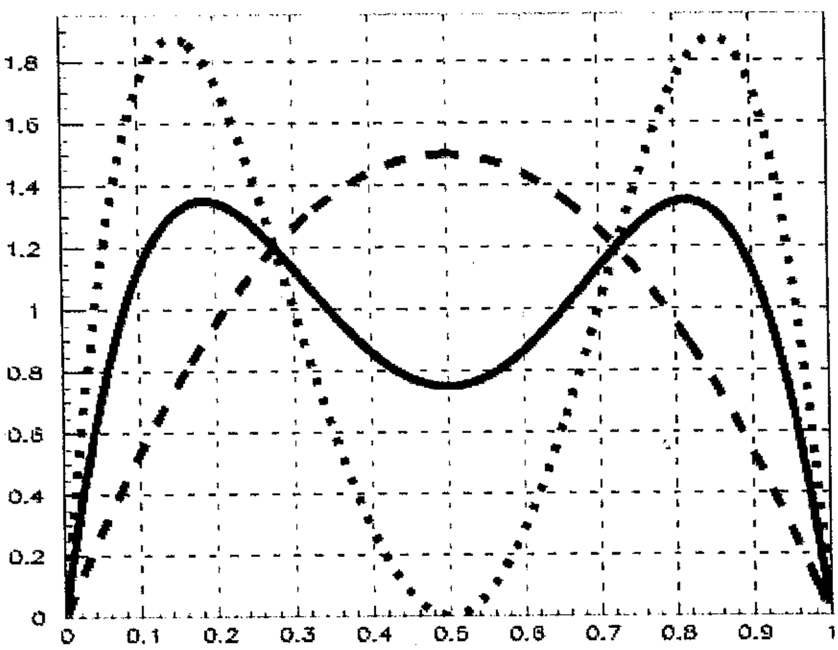
E791 experiment at Fermilab

Pion LC wavefunction is focused



Pion diffractive dissociation

The present experiment is insufficient to distinguish ϕ^{CZ} and ϕ^{ASY}. **We hope future development!**

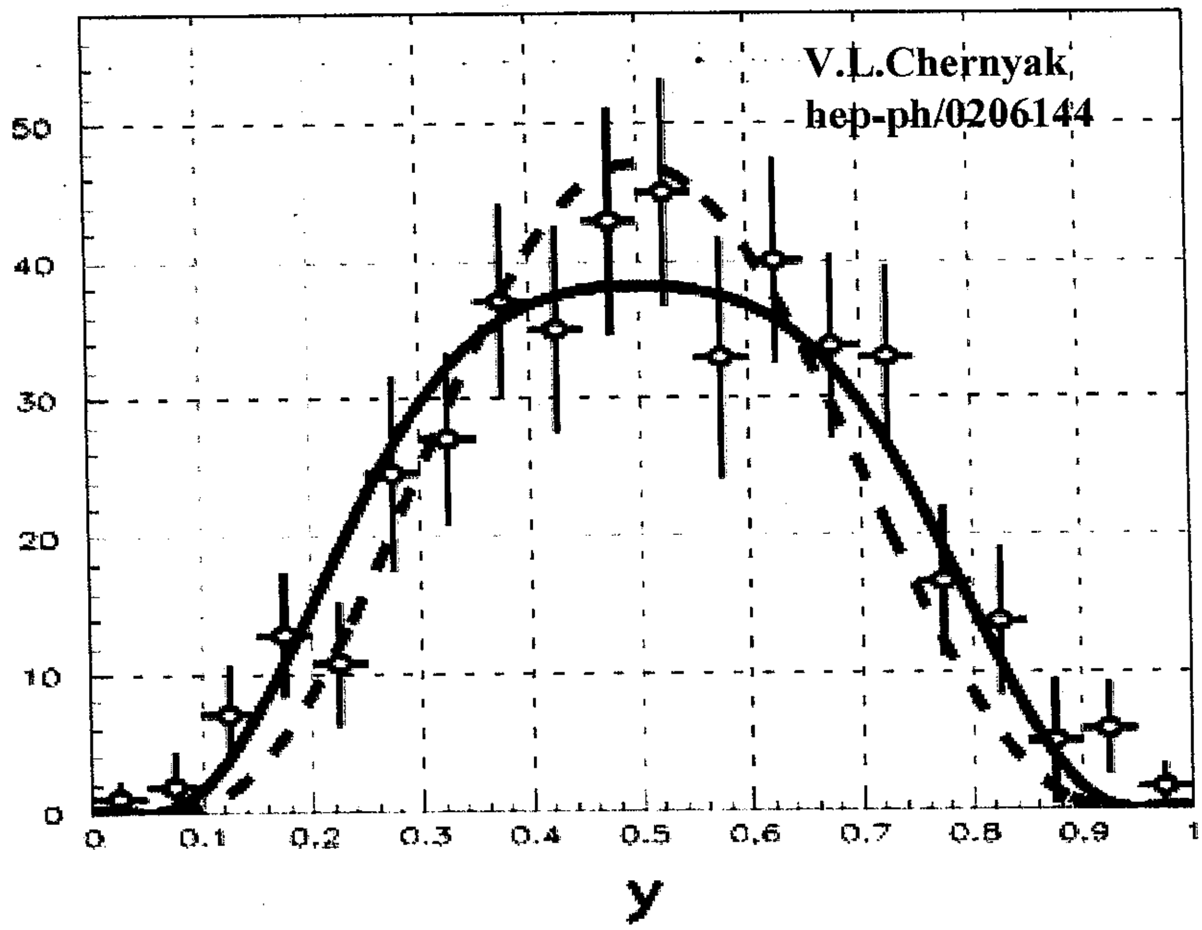


CZ: Chernyak and Zitnitsky

Black and Red Lines

ASY: Asymptotic behavior (Brodsky and Lepage)

Blue Line



Longitudinal momentum distribution of jets.

Open circle is experimental data.

Red Line is estimated from ϕ^{CZ}.

Blue Line is estimated from ϕ^{ASY}.

Actually we could not see the difference between two typical light-cone wavefunctions.

Similar experiment may be planned by using the virtual photon.

Vacuum Triviality and Chiral Symmetry

1.Vacuum triviality

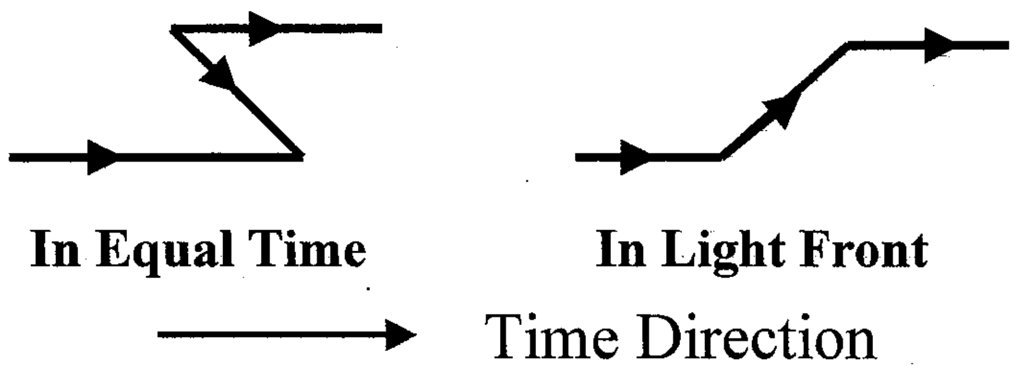
Dispersion Relation $P^2 = 2P^+P^- - P_\perp^2 = M^2$

LF Energy $P^- = \dfrac{P_\perp^2 + M^2}{2P^+}$

Thus $P^- \geq 0 \Longleftrightarrow P^+ \geq 0$

If we neglect $P^+ = 0$ state,

there are no states which are mixed with $|0\rangle$



In Light Front, the vacuum can not creates particles. The vacuum is always trivial! (except for Zero mode)

2. Chiral Symmetry

● Ordinary understanding in Equal-Time Formalism

Chiral Symmetry in QCD

↓

- Dynamically broken down
- Quark masses are generated
- Non-trivial vacuum becomes physical

Hadrons appear as a bound (confined) state.

Especially, pseudo-scalar nonet have roles as Nambu-Goldstone bosons.

This explanation contradicts the trivial vacuum of LF quantization.

To overcome this, one must treat the zero mode.

Result in Nambu-Jona-Lasinio (NJL) Model

NJL Model (with vector coupling)

$$\mathcal{L} = \overline{\Psi}(i\partial\!\!\!/ - m_0)\Psi + \frac{G_1}{2}\left[(\overline{\Psi}\Psi)^2 + (\overline{\Psi}i\gamma_5\Psi)^2\right] - \frac{G_2}{2}\left[(\overline{\Psi}\gamma_\mu\Psi)^2 + (\overline{\Psi}\gamma_\mu\gamma_5\Psi)^2\right]$$

Fermion Constraint

$$i\partial_-\psi_- = (i\gamma_\perp^i\partial_\perp^i + m_0)\frac{1}{2}\gamma^+\psi_+ - \frac{G_1}{2}[\gamma^+\psi_+(\bar{\psi}_+\psi_- + \bar{\psi}_-\psi_+) - i\gamma_5\gamma^+\psi_+(\bar{\psi}_+i\gamma_5\psi_- + \bar{\psi}_-i\gamma_5\psi_+)] + G_2[\psi_-(\bar{\psi}_+\gamma^+\psi_+) + \gamma_5\psi_-(\bar{\psi}_+\gamma^+\gamma_5\psi_+)] + \frac{G_2}{2}[\gamma_\perp^i\gamma^+\psi_+(\bar{\psi}_+\gamma_\perp^i\psi_- + \bar{\psi}_-\gamma_\perp^i\psi_+) - \gamma_\perp^i\gamma_5\gamma^+\psi_+(\bar{\psi}_+\gamma_\perp^i\gamma_5\psi_- + \bar{\psi}_-\gamma_\perp^i\gamma_5\psi_+)]$$

After Bilocalization, we solve this constraint order by order in 1/N expansion.

The Lowest order equation

$$M = m_0 + G_1 N_C M \int \frac{dk^+ d^2k_\perp}{(2\pi)^3} \frac{1}{k^+} \frac{k^+}{|k^+|}$$

diverge !

where $M = m_0 - G_1\langle\overline{\Psi}\Psi\rangle$

There seems to be only a trivial solution $M = 0$ in the chiral limit $m_0 = 0$!

However, if we impose the physical requirement of parity invariant cut-off

$$\frac{k_\perp^2 + M^2}{2\Lambda} < k^+ < \Lambda$$

equation becomes

$$1 = \frac{G_1 N \Lambda^2}{4\pi^2}\left[2 - \frac{M^2}{\Lambda^2} + \frac{M^2}{\Lambda^2}\log\frac{M^2}{2\Lambda^2}\right]$$

and the non-trivial solution can be allowed.

Therefore we can describe the symmetry breaking in LF approach. This was found by K. Itakura and S. Maedan.

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The Hamiltonian is

$$\mathcal{P}^- = \frac{\delta \mathcal{L}}{\delta(\partial_+ \psi_+)} \partial_+ \psi_+ - \mathcal{L}$$

$$= \frac{1}{\sqrt{2}} \psi_+^\dagger (i\gamma_\perp^i \partial_\perp^i + m_0) \gamma^- \psi_- + \psi_-^\dagger \frac{\delta}{\delta \psi_-^\dagger} \mathcal{L}_{\rm int} - \mathcal{L}_{\rm int}$$

$$= \left\{ \frac{1}{2\sqrt{2}} \psi_+^\dagger (i\gamma_\perp^i \partial_\perp^i + m_0) \gamma^- \psi_- + \text{h.c.} \right\}$$

The effect of the interaction is included in the bad component!

(Accidental mechanism in NJL?)

In order to obtain the Hamiltonian, we must solve the constraint equation at the second order O(1/N).

In the Hamiltonian chiral symmetry is broken while the vacuum is trivial.

New Difficulty in G_2 != 0

Divergence of the product of the good component fields in constraint equation.

We replace the term

$$\bar{\psi}_+\gamma^+\psi \Longrightarrow : \bar{\psi}_+\gamma^+\psi :$$

in order to obtain the same gap equation as $G_2 = 0$

In order to derive the hamiltonian, we can not use the hamiltonian of $G_2 = 0$ case.

We must solve again the constraint equation with G_2. This task is very complicated.

Eigenvalue equations

For pion

$$m_\pi^2 \phi_\pi(x, k_\perp) = \frac{k_\perp^2 + M^2}{x(1-x)} \phi_\pi(x, k_\perp^i)$$
$$- \frac{G_1 N \alpha_1}{(2\pi)^3} \frac{1}{x(1-x)} \int_0^1 dy d^2 l_\perp \frac{l_\perp^2 + M^2}{y(1-y)} \phi_\pi(y, l_\perp)$$

For vector meson (transversely polarized)

$$m_\rho^2 \phi_\rho(x, k_\perp) = \frac{k_\perp^2 + M^2}{x(1-x)} \phi_\rho(x, k_\perp^i)$$
$$- \frac{G_2 N \alpha_2}{(2\pi)^3} \frac{1}{x(1-x)}$$
$$\times \int_0^1 dy d^2 l_\perp \frac{M^2 - 2y(1-y) l_\perp^2}{y(1-y)} \phi_\rho(y, l_\perp)$$

Coupling Constants

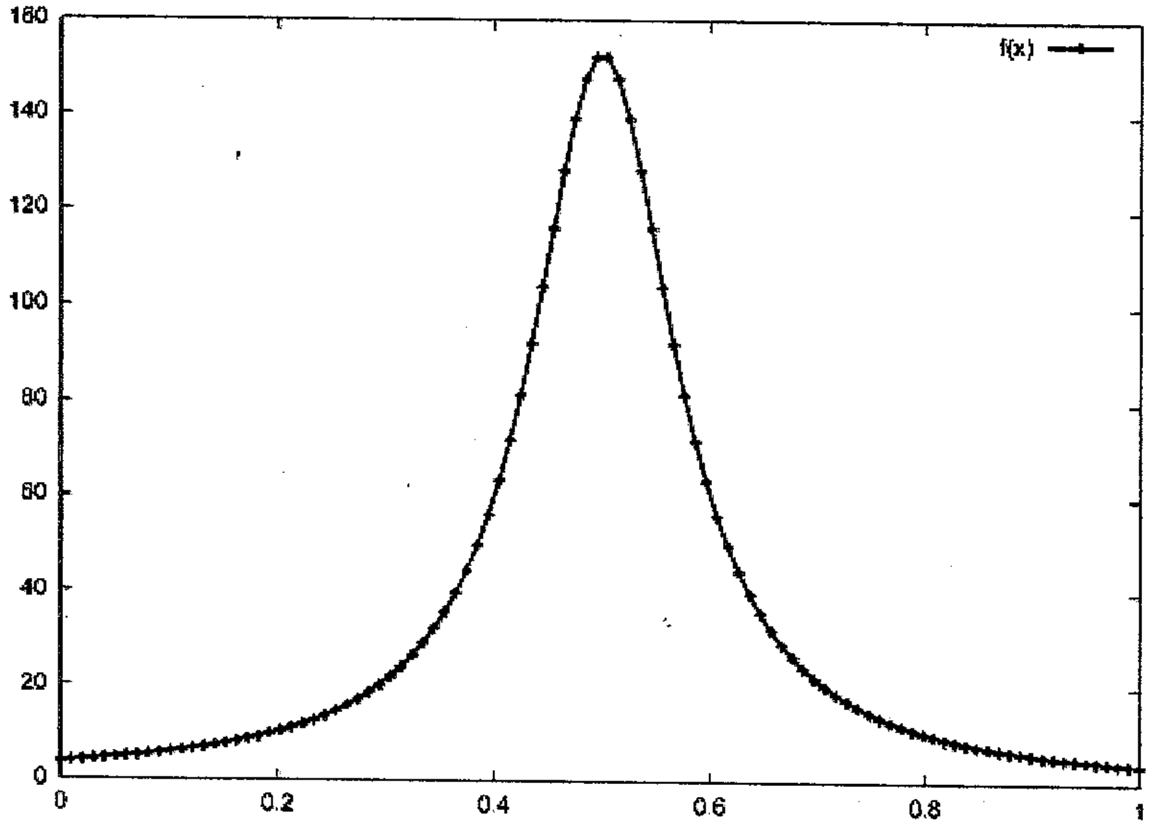
$$\alpha_i = \frac{1}{1 + G_i N \int \frac{d\boldsymbol{q}}{(2\pi)^3} \frac{1}{P^+ - q^+} \frac{q^+}{|q^+|}}$$

LC Wavefunction

Except for the dirac structure and the hadron mass value, the LC wavefunctions in NJL model are same form as

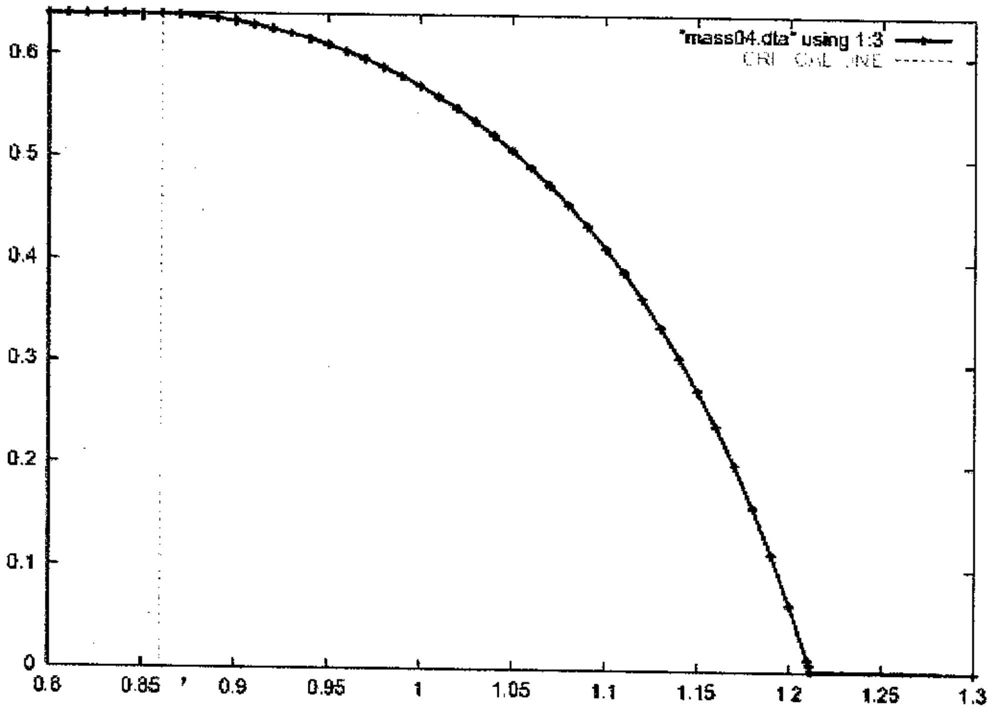
$$\phi(x) \propto \frac{1}{x(1-x) - \dfrac{M^2 + k_\perp^2}{m_H^2}}$$

m_H: Hadron mass.



770MeV rho meson case (M=400MeV)

The vector meson mass square as a function of the strength of the vector interaction.



Green Line indicates the threshold.

X axis G_2/G_1

Y axis m_ρ^2/Λ^2

Mass behavior is consistent with the results from Equal Time quantization (cf. Kugo, PTP55 1976)

Summary

The light-cone wavefunction is a best useful tool to describe the hadron structure and the Light-Front quantization is the optimal approach to obtain it.

One of the difficulties is the apparent inconsistency between the trivial vacuum and the spontaneous chiral symmetry breaking. However, Itakura and Maedan's approach can resolve this problem and now we apply to vector meson case.

We can successfully obtain the bound state solutions for pion, sigma and rho meson state (transeversely polarized). But, to this end, we had to solve the complicated constraint equation by hand. More systematic and more elegant (from the phisical view point) methods should be established for the future work.

END