Ground State Baryons in Covariant Level Classification Scheme

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I. Introduction

Covariant Level Classification Scheme of Hadrons presented by Shin Ishida

$$\tilde{U}(12)_{SF} \otimes O(3,1)_L \longleftarrow \underset{\text{NROM}}{SU(6)_{SF} \otimes O(3)_L}$$

Hadron WF : <u>Tensors</u> in $\tilde{U}(12)_{SF} \otimes O(3,1)_L$ space

$$\left(\begin{array}{ll} \underline{\text{Meson} \quad \Phi_A{}^B(x,y)} \sim \psi_A(x)\bar{\psi}^B(y) & A=(\overset{\text{spinor}}{\alpha}, \overset{\text{flavor}}{a}) \text{ etc.} \\ \underline{\text{Baryon} \quad \Phi_{A_1A_2A_3}(x_1,x_2,x_3)} \sim \psi_{A_1}(x_1)\psi_{A_2}(x_2)\psi_{A_3}(x_3) & \end{array} \right.$$

$\Longrightarrow$ Existence of

Chiral Particle/States Out of Non-Rel. Scheme

Ground State Baryons

$\tilde{U}(12)_{SF}$ **$SU(6)_{SF}$**

$$(12 \times 12 \times 12)_{sym} = 364 \longleftarrow (6 \times 6 \times 6)_{sym} = 56$$

$$= 182_B \oplus 182_{\bar{B} \text{ or } "\gamma_5 B"}$$

Parity

$$182_B \left[\begin{array}{lllll} 56^{\oplus} & \Delta^{\oplus}_{\frac{3}{2}} & N^{\oplus}_{\frac{1}{2}} & & \\ 70 & \Delta_{\frac{1}{2}} & N_{\frac{3}{2}} & N_{\frac{1}{2}} & \Lambda_{\frac{1}{2}} \\ 56' & \Delta'_{\frac{3}{2}} & N'_{\frac{1}{2}} & & \end{array} \right. \qquad 56^{\oplus} \quad \Delta^{\oplus}_{\frac{3}{2}} \quad N^{\oplus}_{\frac{1}{2}}$$

Chiral States: Properties ?

Magnetic Moment

One Pion Emission Decay

II. Spinor-Flavor Wave Function

(Constituent Dirac Spinor)

$$\psi_{q,\alpha}(X) = \sum_{\mathbb{P}} \Big[e^{iP\cdot X} \underset{u_\alpha(P)}{W^{(+)}_{q,\alpha}(P)} + e^{-iP\cdot X} \underset{u_\alpha(-P)}{W^{(-)}_{q,\alpha}(P)} \Big]$$

$$\overline{\psi}_{\bar{q}}^{\alpha}(X) = \sum_{\mathbb{P}} \Big[e^{iP\cdot X} \underset{\overline{v}_{\bar{q}}^{\alpha}(P)}{\overline{W}_{\bar{q}}^{(+)\alpha}(P)} + e^{-iP\cdot X} \underset{\overline{v}_{\bar{q}}^{\alpha}(-P)}{\overline{W}_{\bar{q}}^{(-)\alpha}(P)} \Big]$$

$$\mathbb{P} = \mathbb{O} \qquad W_q^{(+)} = \begin{pmatrix} \chi \\ 0 \end{pmatrix} \ \rho_3 = +1 \qquad W_q^{(-)} = \begin{pmatrix} 0 \\ \chi \end{pmatrix} \ \rho_3 = -1$$

$$\overline{W}_{\bar{q}}^{(+)} = (0, -\chi'^{\dagger}) \ \bar{\rho}_3 = +1 \qquad \overline{W}_{\bar{q}}^{(-)} = (\chi'^{\dagger}, 0) \ \bar{\rho}_3 = -1$$

(Baryon Spinor)

$$W^{(B)}_{\alpha\beta\gamma}(P) = W_{q,\alpha}(P)\, W_{q,\beta}(P)\, W_{q,\gamma}(P)$$

$$\mathbb{P} = \mathbb{O} \quad (\rho_3^{(1)}, \rho_3^{(2)}, \rho_3^{(3)}) = \left.\begin{matrix} (+ & + & +) \\ (+ & + & -) \\ (+ & - & -) \end{matrix}\right\} \text{chiral states}$$

$$W^{(\overline{B})}_{\alpha\beta\gamma}(P) = W^{(B)} \{ W_{q,\alpha} \to W_{\bar{q},\alpha} \}$$

(Composition rule of $|F\rangle, |\rho\rangle, |\sigma\rangle$ WF)

WF is fully-symmetric in $\underset{\text{flavor}}{|F\rangle} \otimes \underbrace{|\rho\rangle \otimes |\sigma\rangle}_{\text{Dirac spinor}}$ space

$$|F_1 F_2\rangle_S = \begin{cases} |F_1\rangle_S \; |F_2\rangle_S \\ |F_1\rangle_A \; |F_2\rangle_A \\ \frac{1}{\sqrt{2}}\left(|F_1\rangle_\alpha |F_2\rangle_\alpha + |F_1\rangle_\beta |F_2\rangle_\beta\right) \end{cases}$$

$|F\rangle_S \sim uuu,\ \frac{1}{\sqrt{3}}(uud+udu+duu)$, etc. $\sim \Delta$-decouplet

$|F\rangle_A \sim -\frac{1}{\sqrt{6}}\epsilon_{abc} \qquad a=u,d,s,\cdots \qquad \sim \Lambda$-singlet

$|F\rangle_\alpha \sim -\frac{1}{\sqrt{6}}\left(-N_b{}^d\epsilon_{dca} + N_c{}^d\epsilon_{dab}\right)$: $(bc) \sim N$-octet

$|F\rangle_\beta \sim \frac{1}{\sqrt{2}} N_a{}^d\epsilon_{dbc}$: $[bc] \sim N$-octet

$|\sigma\rangle_S$: spin $\frac{3}{2}$

$|\sigma\rangle_{\alpha,\beta}$: spin $\frac{1}{2}$ $\qquad$ No $|\sigma\rangle_A$

$$N_a{}^d = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

$\lvert\rho, \overset{\rho_3}{\overset{\shortparallel}{\tfrac{3}{2}}}\rangle_S = \lvert{+}{+}{+}\rangle_\rho$: NR states	$\lvert\rho, -\frac{3}{2}\rangle_S = \lvert{-}{-}{-}\rangle_\rho$
$\lvert\rho, -\frac{1}{2}\rangle_S = \frac{1}{\sqrt{3}}\left(\lvert{+}{-}{-}\rangle_\rho + \lvert{-}{+}{-}\rangle_\rho + \lvert{-}{-}{+}\rangle_\rho\right)$	$\lvert\rho, \frac{1}{2}\rangle_S = \frac{1}{\sqrt{3}}\left(\lvert{+}{+}{-}\rangle_\rho + \lvert{+}{-}{+}\rangle_\rho + \lvert{-}{+}{+}\rangle_\rho\right.$
$\lvert\rho, -\frac{1}{2}\rangle_\alpha = \frac{1}{\sqrt{6}}\left(-\lvert{-}{-}{+}\rangle_\rho - \lvert{-}{+}{-}\rangle_\rho + 2\lvert{+}{-}{-}\rangle_\rho\right)$	$\lvert\rho, \frac{1}{2}\rangle_\alpha = \frac{1}{\sqrt{6}}\left(\lvert{+}{+}{-}\rangle_\rho + \lvert{+}{-}{+}\rangle_\rho - 2\lvert{-}{+}{+}\rangle_\rho\right)$
$\lvert\rho, -\frac{1}{2}\rangle_\beta = \frac{1}{\sqrt{2}}\left(-\lvert{-}{-}{+}\rangle_\rho + \lvert{-}{+}{-}\rangle_\rho\right)$	$\lvert\rho, \frac{1}{2}\rangle_\beta = \frac{1}{\sqrt{2}}\left(\lvert{+}{+}{-}\rangle_\rho - \lvert{+}{-}{+}\rangle_\rho\right)$
Positive Parity	Negative Parity

$$364 \;=\; 182\,(B) \;\oplus\; 182\,(\text{"}\gamma_5 B\text{"} \sim \bar{B})$$

		Positive Parity	Negative Parity
		$E^{\oplus}$	$E^{\ominus}$
56_E	$\Delta_{\frac{3}{2}}$	$\lvert\rho\ \tfrac{3}{2}\rangle_s \lvert\Delta\rangle_s \lvert\sigma\rangle_s$	$\lvert\rho -\tfrac{3}{2}\rangle_s \lvert\Delta\rangle_s \lvert\sigma\rangle_s$
	$N_{\frac{1}{2}}$	$\lvert\rho\ \tfrac{3}{2}\rangle_s \tfrac{1}{\sqrt{2}}(\lvert N\rangle_\alpha\lvert\sigma\rangle_\alpha + \lvert N\rangle_\beta\lvert\sigma\rangle_\beta)$	$\lvert\rho -\tfrac{3}{2}\rangle_s \tfrac{1}{\sqrt{2}}(\lvert N\rangle_\alpha\lvert\sigma\rangle_\alpha + \lvert N\rangle_\beta\lvert\sigma\rangle_\beta)$
		$F^{\oplus}$	$F^{\ominus}$
56_F	$\Delta'_{\frac{3}{2}}$	$\lvert\rho -\tfrac{1}{2}\rangle_s \lvert\Delta\rangle_s \lvert\sigma\rangle_s$	$\lvert\rho\ \tfrac{1}{2}\rangle_s \lvert\Delta\rangle_s \lvert\sigma\rangle_s$
	$N'_{\frac{1}{2}}$	$\lvert\rho -\tfrac{1}{2}\rangle_s \tfrac{1}{\sqrt{2}}(\lvert N\rangle_\alpha\lvert\sigma\rangle_\alpha + \lvert N\rangle_\beta\lvert\sigma\rangle_\beta)$	$\lvert\rho\ \tfrac{1}{2}\rangle_s \tfrac{1}{\sqrt{2}}(\lvert N\rangle_\alpha\lvert\sigma\rangle_\alpha + \lvert N\rangle_\beta\lvert\sigma\rangle_\beta)$
		$G^{\oplus}$	$G^{\ominus}$
70_G	$\Delta_{\frac{1}{2}}$	$\lvert\Delta\rangle_s \tfrac{1}{\sqrt{2}}(\lvert\rho -\tfrac{1}{2}\rangle_\alpha\lvert\sigma\rangle_\alpha + \lvert\rho -\tfrac{1}{2}\rangle_\beta\lvert\sigma\rangle_\beta)$	$\lvert\Delta\rangle_s \tfrac{(-1)}{\sqrt{2}}(\lvert\rho\ \tfrac{1}{2}\rangle_\alpha\lvert\sigma\rangle_\alpha + \lvert\rho\ \tfrac{1}{2}\rangle_\beta\lvert\sigma\rangle_\beta)$
	$N_{\frac{3}{2}}$	$\tfrac{1}{\sqrt{2}}(\lvert\rho -\tfrac{1}{2}\rangle_\alpha\lvert N\rangle_\alpha + \lvert\rho -\tfrac{1}{2}\rangle_\beta\lvert N\rangle_\beta)\lvert\sigma\rangle_s$	$\tfrac{(-1)}{\sqrt{2}}(\lvert\rho\ \tfrac{1}{2}\rangle_\alpha\lvert N\rangle_\alpha + \lvert\rho\ \tfrac{1}{2}\rangle_\beta\lvert N\rangle_\beta)\lvert\sigma\rangle_s$
	$N_{\frac{1}{2}G}$	$\tfrac{1}{2}\big[\lvert\rho -\tfrac{1}{2}\rangle_\alpha(-\lvert N\rangle_\alpha\lvert\sigma\rangle_\alpha + \lvert N\rangle_\beta\lvert\sigma\rangle_\beta) + \lvert\rho -\tfrac{1}{2}\rangle_\beta(\lvert N\rangle_\alpha\lvert\sigma\rangle_\beta + \lvert N\rangle_\beta\lvert\sigma\rangle_\alpha)\big]$	$\tfrac{(-1)}{2}\big[\lvert\rho\ \tfrac{1}{2}\rangle_\alpha(-\lvert N\rangle_\alpha\lvert\sigma\rangle_\alpha + \lvert N\rangle_\beta\lvert\sigma\rangle_\beta) + \lvert\rho\ \tfrac{1}{2}\rangle_\beta(\lvert N\rangle_\alpha\lvert\sigma\rangle_\beta + \lvert N\rangle_\beta\lvert\sigma\rangle_\alpha)\big]$
	$\Lambda_{\frac{1}{2}}$	$-\lvert\Lambda\rangle_A \tfrac{1}{\sqrt{2}}(-\lvert\rho -\tfrac{1}{2}\rangle_\alpha\lvert\sigma\rangle_\beta + \lvert\rho -\tfrac{1}{2}\rangle_\beta\lvert\sigma\rangle_\alpha)$	$\lvert\Lambda\rangle_A \tfrac{(+1)}{\sqrt{2}}(\lvert\rho\ \tfrac{1}{2}\rangle_\alpha\lvert\sigma\rangle_\beta + \lvert\rho\ \tfrac{1}{2}\rangle_\beta\lvert\sigma\rangle_\alpha)$

WF at rest frame $\mathbb{P}=0$: $\lvert F\rho\sigma\rangle = \lvert F\rho\sigma(v_0)\rangle$

$$\frac{P_{0\mu}}{M} = v_{0\mu} = (0\,0\,0 ; i)$$

(WF in general frame)

$$\lvert F\rho\sigma(v)\rangle = B(v)\lvert F\rho\sigma\rangle$$

$$B(v) = B(v)^{(1)}\, B(v)^{(2)}\, B(v)^{(3)} \quad : \text{Lorentz booster}$$

$$B(v)^{(i)} = ch\theta + \rho_1^{(i)}\sigma_z^{(i)}\, sh\theta \qquad ch\theta = \sqrt{\frac{E+m}{2m}} = \sqrt{\frac{\omega+1}{2}}$$

$$v_\mu = (0\ 0\ \omega_3 ; i\omega) \qquad sh\theta = \sqrt{\frac{E-m}{2m}} = \sqrt{\frac{\omega-1}{2}}$$

$$2\, ch\theta\, sh\theta = \frac{\mathbb{P}}{m}$$

$$\begin{pmatrix}\chi\\0\end{pmatrix} \longrightarrow B(v)\begin{pmatrix}\chi\\0\end{pmatrix} = \begin{pmatrix}ch\theta\ \chi\\ sh\theta\ \sigma_z\chi\end{pmatrix} = u(v)$$

(Magnetic Moment)

$$3 \langle F\rho\sigma(\uparrow) | \mu_8^{(1)} \sigma_z^{(1)} | F\rho\sigma(\uparrow) \rangle \qquad \mu_8 = \begin{pmatrix} \mu_u & & \\ & \mu_d & \\ & & \mu_s \end{pmatrix} \qquad \mu_u = \frac{e_u}{2m_u}$$

		$\bar{p}p$	$\bar{n}n$	$\bar{\Lambda}\Lambda$	$\bar{\Lambda}\Sigma^0$	$\bar{\Sigma}^+\Sigma^+$	$\bar{\Sigma}^-\Sigma^-$	$\bar{\Sigma}^0\Sigma^0$	$\bar{\Xi}^-\Xi^-$	$\bar{\Xi}^0\Xi^0$
	$\lvert N\rangle_\alpha$	$\frac{\mu_u+2\mu_d}{3}$	$\frac{2\mu_u+\mu_d}{3}$	$\frac{\mu_u+\mu_d}{2}$	$\frac{\mu_u-\mu_d}{-2\sqrt{3}}$	$\frac{\mu_u+2\mu_s}{3}$	$\frac{\mu_d+2\mu_s}{3}$	$\frac{\mu_u+\mu_d+4\mu_s}{6}$	$\frac{2\mu_d+\mu_s}{3}$	$\frac{2\mu_u+\mu_s}{3}$
	$\lvert N\rangle_\beta$	μ_u	μ_d	$\frac{\mu_u+\mu_d+4\mu_s}{6}$	$\frac{\mu_u-\mu_d}{2\sqrt{3}}$	μ_u	μ_d	$\frac{\mu_u+\mu_d}{2}$	μ_s	μ_s
NR	$\lvert N_E^{56}\rangle$	$\frac{4\mu_u-\mu_d}{3}$	$\frac{-\mu_u+4\mu_d}{3}$	μ_s	$\frac{\mu_u-\mu_d}{\sqrt{3}}$	$\frac{4\mu_u-\mu_s}{3}$	$\frac{4\mu_d-\mu_s}{3}$	$\frac{2\mu_u+2\mu_d-\mu_s}{3}$	$\frac{-\mu_d+4\mu_s}{3}$	$\frac{-\mu_u+4\mu_s}{3}$
NR	Theor.	$\boxed{2.793}$	-1.862	$\boxed{-0.613}$	1.61	2.69	-1.04	0.825	-0.507	-1.44
NR	Exp.	2.793	-1.913	-0.613(4)	1.61(8)	2.46(1)	-1.16(3)		-0.651(3)	-1.25(1)
hiral	$\lvert N_F^{56'}\rangle$	$\frac{4\mu_u-\mu_d}{3}$	$\frac{-\mu_u+4\mu_d}{3}$	μ_s	$\frac{\mu_u-\mu_d}{\sqrt{3}}$	$\frac{4\mu_u-\mu_s}{3}$	$\frac{4\mu_d-\mu_s}{3}$	$\frac{2\mu_u+2\mu_d-\mu_s}{3}$	$\frac{-\mu_d+4\mu_s}{3}$	$\frac{-\mu_u+4\mu_s}{3}$
hiral	Theor.	2.793	-1.862	-0.613	1.61	2.69	-1.04	0.825	-0.507	-1.44
hiral	$\lvert N_G^{70}\rangle$	$\frac{2\mu_u+\mu_d}{3}$	$\frac{\mu_u+2\mu_d}{3}$	$\frac{\mu_u+\mu_d+\mu_s}{3}$	0	$\frac{2\mu_u+\mu_s}{3}$	$\frac{2\mu_d+\mu_s}{3}$	$\frac{\mu_u+\mu_d+\mu_s}{3}$	$\frac{\mu_d+2\mu_s}{3}$	$\frac{\mu_u+2\mu_s}{3}$
hiral	Theor.	0.931	0	0.106	0	1.037	-0.825	0.106	-0.719	0.212
hiral	$\lvert N_{\frac{3}{2}G}^{70}\rangle$	$2\mu_u+\mu_d$	$\mu_u+2\mu_d$	$\mu_u+\mu_d+\mu_s$	0	$2\mu_u+\mu_s$	$2\mu_d+\mu_s$	$\mu_u+\mu_d+\mu_s$	$\mu_d+2\mu_s$	$\mu_u+2\mu_s$
hiral	Theor.	2.793	0	0.318	0	3.11	-2.48	0.318	-2.16	0.636

the same

Table VI. Magnetic moment of nucleon octets: Unit is the nuclear magneton μ_N. $\mu_d = -\mu_u/2$ is assumed. The values with underlines, which are used as inputs, give $\mu_u = 1.862\mu_N$ and $\mu_s = -0.613\mu_N$.

		$\bar{\Delta}^{++}\Delta^{++}$	$\bar{\Delta}^+\Delta^+$	$\bar{\Delta}^0\Delta^0$	$\bar{\Delta}^-\Delta^-$	$\bar{\Sigma}^+\Sigma^+$	$\bar{\Sigma}^-\Sigma^-$	$\bar{\Sigma}^0\Sigma^0$	$\bar{\Xi}^-\Xi^-$	$\bar{\Xi}^0\Xi^0$	$\bar{\Omega}^-\Omega^-$
NR	$\lvert \Delta_{\frac{3}{2}E}^{56}\rangle$	$3\mu_u$	$2\mu_u+\mu_d$	$\mu_u+2\mu_d$	$3\mu_d$	$2\mu_u+\mu_s$	$2\mu_d+\mu_s$	$\mu_u+\mu_d+\mu_s$	$\mu_d+2\mu_s$	$\mu_u+2\mu_s$	$3\mu_s$
NR	Theor.	5.59	2.79	0	-2.79	3.11	-2.48	0.318	-2.16	0.636	-1.84
NR	Exp.	$3.7 \sim 7.5$									
hiral	$\lvert \Delta_{\frac{3}{2}F}^{56'}\rangle$	$3\mu_u$	$2\mu_u+\mu_d$	$\mu_u+2\mu_d$	$3\mu_d$	$2\mu_u+\mu_s$	$2\mu_d+\mu_s$	$\mu_u+\mu_d+\mu_s$	$\mu_d+2\mu_s$	$\mu_u+2\mu_s$	$3\mu_s$
hiral	Theor.	5.59	2.79	0	-2.79	3.11	-2.48	0.318	-2.16	0.636	-1.84
hiral	$\lvert \Delta_{\frac{1}{2}G}^{70}\rangle$	μ_u	$\frac{2\mu_u+\mu_d}{3}$	$\frac{\mu_u+2\mu_d}{3}$	μ_d	$\frac{2\mu_u+\mu_s}{3}$	$\frac{2\mu_d+\mu_s}{3}$	$\frac{\mu_u+\mu_d+\mu_s}{3}$	$\frac{\mu_d+2\mu_s}{3}$	$\frac{\mu_u+2\mu_s}{3}$	μ_s
hiral	Theor.	1.86	0.931	0	-0.931	1.037	-0.825	0.106	-0.719	0.212	-0.613

the same

Table VII. Magnetic moment of Δ decouplets: Unit is the nuclear magneton μ_N. $\mu_d = -\mu_u/2$ is assumed. The $\mu_u = 1.862\mu_N$ and $\mu_s = -0.613\mu_N$ are used.

the same as the prediction by NRQM $SU(6)_{SF}$

Chiral $56'$ has the same M.M. as $56^{\oplus}$.

70_G different from

		$\bar{\Lambda}^-\Lambda^-$
Chiral	$\lvert \Lambda_{\frac{1}{2}G}^{70}\rangle$	$\frac{\mu_u+\mu_d+\mu_s}{3}$
Chiral	Theor.	0.106

Table VIII. Magnetic moment of Λ singlet

(WF mixing and Parity of Chiral States)

$SU(6)_{SF}$ good $\Leftarrow$ (Magnetic Moment, F/D ratio of N-octet

$$56^{\oplus} = c_1\, 56_E^{\oplus} + s_1\, 56_F^{\oplus} \qquad |c_1|^2 + |s_1|^2 = 1$$

$$\left.\begin{aligned} 56' &= -s_1^* 56_E + c_1^* 56_F \\ 70 &= 70_G \end{aligned}\right\} \text{chiral states}$$

Candidates for Chiral States

$N^{\oplus}(1440)$ $\qquad$ $\Lambda^{\ominus}(1405)$

$\Longrightarrow$ Two Possibilities

(Possibility A)

$56'^{\oplus}$, $70^{\ominus}$

$\cup$ $\qquad$ $\cup$

$N(1440)$ $\qquad$ $\Lambda(1405)$

singlet ~~octet~~

$\Uparrow$

Promising!

(Possibility B)

$56'^{\ominus}$, $70^{\oplus}$

$\cup$ $\qquad$ $\cup$

$\Lambda(1405)$ $\qquad$ $N(1440)$

octet

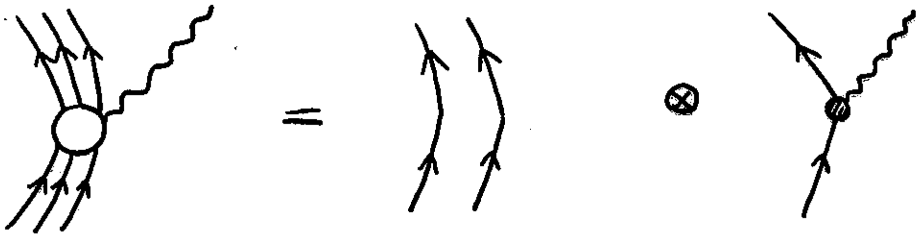
difficulties :

- Negative parity Octet in low mass, but No $N^{\ominus}$ below 1.5 GeV in exp.
- $\Gamma^{\text{theor.}}_{\Lambda(1405)\to\Sigma\pi} = 0$ but $\Gamma^{\text{exp}} = 50$ MeV

III. Properties of Ground State Baryons

(Interactions among hadrons)   by Shin Ishida

Interaction vertex = (Spectator) Overlapping interaction ⊗ Elementary quark interaction



$$\overline{B}^{A_1 A_2 D}(P_2)\, B_{A_1 A_2 C}(P_1)\, \overline{M}_D{}^C(q)$$

$$= (\overline{\Psi}^{A_1} \psi_{A_1})\, (\overline{\Psi}^{A_2} \psi_{A_2}) \otimes \overline{\Psi}^D \psi_C\, \overline{M}_D{}^C(q)$$

Non-symmetric

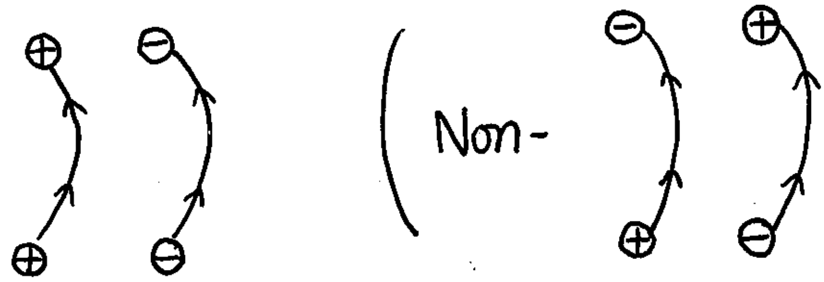
↓ Chiral symmetric

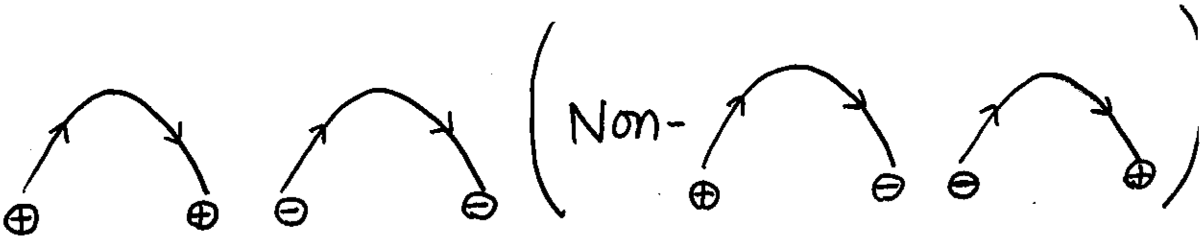
$$(\overline{\Psi}^{A_1} \psi_{A_1}) = \overline{\Psi}(v_2)\, \overset{(0)}{\gamma}\, \psi(v_1) \;;\quad \overset{(0)}{\gamma} = \frac{-i\hat{v}_2\cdot\gamma - i\hat{v}_1\cdot\gamma}{2}$$

$$\hat{v}_\mu = \begin{cases} v_\mu & (\text{for} \\ -v_\mu & (\text{for} \end{cases}$$

$\Longrightarrow$ $\boxed{\rho_3 - \text{line rule}}$

ρ_3 at respective rest frame conserves on spectator quark line





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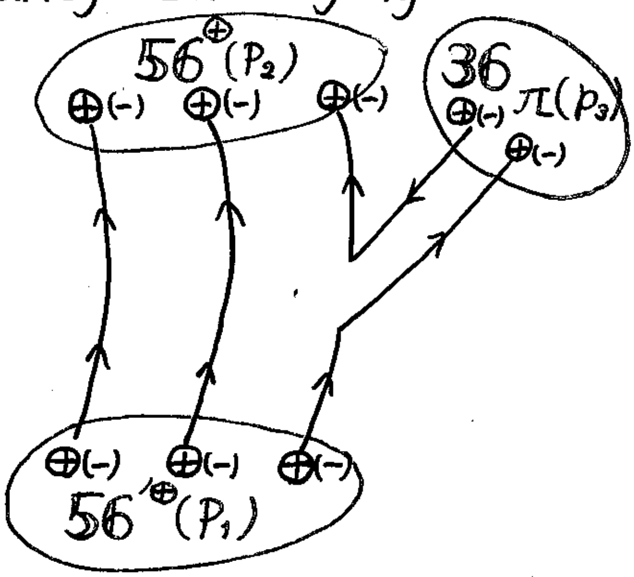
(Mechanism of Pion Emission)

Strong Interaction : is presumably

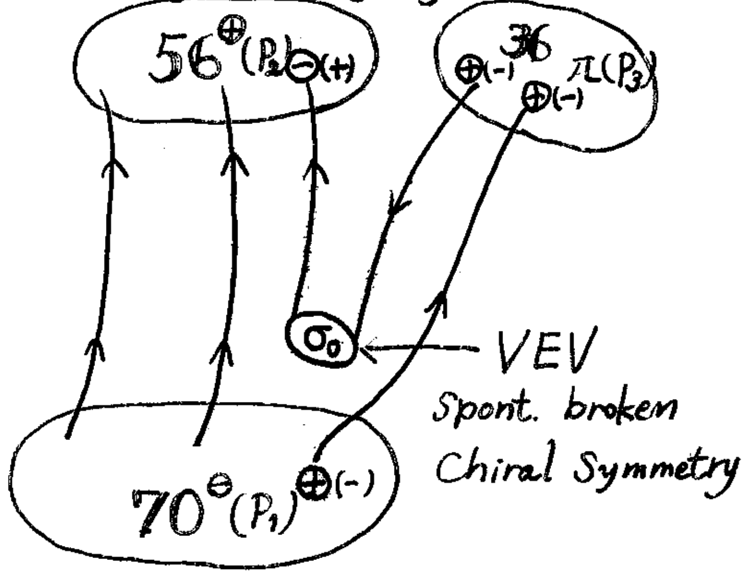
spontaneously broken **Chiral Symmetric Overlapping Interactions**

$\Longrightarrow$ Concrete form of effective One Pion Emission vertex derived!

Parity - Unchanging



Parity - Changing



$$3g\langle \overline{F\rho\sigma(v_2)}|(\gamma^{(0)} i\gamma_5 \phi \overset{(0)}{\gamma})^{(1)}(\gamma^{(0)})^{(2)}(\overset{(0)}{\gamma})^{(3)}|F\rho\sigma(v_1)\rangle,$$
$$= 3g\langle F\rho\sigma|(\sigma_z \rho_3 i\phi)^{(1)}|F\rho\sigma\rangle$$
$$\times\, ch^2\theta_2\, sh(\theta_2+\theta_3)\, ch\,\theta_3$$

P-wave decay

$$3g_\pi\langle \overline{F\rho\sigma(v_2)}|(\overset{(0)}{\gamma} S \overset{(0)}{\gamma} i\gamma_5 \phi \overset{(0)}{\gamma})^{(1)}(\overset{(0)}{\gamma})^{(2)}(\overset{(0)}{\gamma})^{(3)}|\rangle$$
$$= 3g_\pi\langle F\rho\sigma|(i\gamma_5 S\phi)^{(1)}|F\rho\sigma\rangle$$
$$\times\, ch^3\theta_2\, ch^2\theta_3$$

S-wave decay

$(56^{\oplus} \longrightarrow 56^{\oplus} + 36\,(\pi))$

πNN-vertex $\qquad\qquad 56 = c_1\, 56_E + s_1\, 56_F$

$$\mathcal{M} = 3ig\left(|c_1|^2 - \tfrac{1}{3}|s_1|^2\right) s_{\mathrm{II}} \underbrace{\left[-\tfrac{1}{3}\alpha + \beta\right]}_{\cup} \underbrace{ch^2\theta_2\; sh(\theta_2+\theta_3)\; ch\theta_3}_{\shortparallel}$$

$$\left.\begin{aligned} \alpha &= {}_{\alpha}\langle \bar{N} | \phi^{(\iota)} | N \rangle_{\alpha} \\ \beta &= {}_{\beta}\langle \bar{N} | \phi^{(\iota)} | N \rangle_{\beta} \end{aligned}\right\} \Rightarrow \frac{5\sqrt{2}}{9}\, \bar{\psi}\, \boldsymbol{\tau}\cdot\boldsymbol{\pi}\, \psi \qquad \left(\frac{ch^3\theta_2}{2m_\pi} + \frac{ch\theta_2\, ch^2\theta_3}{2M_2}\right)\mathbb{P}$$

$$\mathcal{L}^{\mathrm{exp}}_{\pi NN} = g_{\pi NN}\, \bar{u}(P_2)\, i\gamma_5\, u(P_1)\; \bar{\psi}\, \boldsymbol{\tau}\cdot\boldsymbol{\pi}\, \psi$$

$$= i\, 2 s_{\mathrm{II}}\, g_{\pi NN}\, \bar{\psi}\, \boldsymbol{\tau}\cdot\boldsymbol{\pi}\, \psi\; sh\theta_2 = \frac{\mathbb{P}}{2M_2}$$

$$\Longrightarrow \quad g_{\pi NN} = 13.45 = \underbrace{g\left(|c_1|^2 - \tfrac{1}{3}|s_1|^2\right)}_{\shortparallel} \frac{5}{3\sqrt{2}}\left(\frac{M_N}{m_\pi} + 1\right)$$

1.462 : common coupling constant.

$$\Gamma_{B' \to B\pi} = \boxed{f^2}\; \frac{\mathbb{P} M_B}{2\pi M_{B'}} \left| \underline{g\left(|c_1|^2 - \tfrac{1}{3}|s_1|^2\right)} \right|^2 \left(ch^2\theta_2\; sh(\theta_2+\theta_3)\; ch\,\theta_3\right)^2$$

	f	Γ_{theor} /MeV	$\Gamma_{\mathrm{exp.}}$ /MeV
$\Delta \to N\pi$	$\sqrt{\frac{4}{3}}$	81.7	115 ~ 125
$\Sigma^*(1385) \to \Lambda\pi$	$\sqrt{\frac{2}{3}}$	30.7	33
$\to \Sigma\pi$	$\frac{2}{3}$	4.4	4
$\Xi^*(1530) \to \Xi\pi$	$\sqrt{\frac{2}{3}}$	11.6	9.1 ± 0.5

$\uparrow$

$SU(6)_{SF}$

the same as the predictions by NRQM.

(Candidates for $56^{\oplus}$ chiral states) PDG'02

$N^{\oplus}(939)$	$\Delta^{\oplus}(1232)$	$\Lambda^{\oplus}(1116)$	$\Sigma^{\oplus}_{\frac{1}{2}}(1191)$	$\Xi^{\oplus}_{\frac{1}{2}}(1318)$
$\boxed{N^{\oplus}_{\frac{1}{2}}(1440)}$	$\boxed{\Delta^{\oplus}_{\frac{3}{2}}(1600)}$	$\Lambda^{\ominus}(1405)$	$\Sigma^{*\oplus}_{\frac{3}{2}}(1385)$	$\Xi^{*\oplus}_{\frac{3}{2}}(1530)$
$N^{\ominus}_{\frac{3}{2}}(1520)$	$\Delta^{\ominus}_{\frac{1}{2}}(1620)$	$\Lambda^{\ominus}_{\frac{3}{2}}(1520)$	$\boxed{\Sigma^{\oplus}_{\frac{1}{2}}(1660)}$	$\Xi\ (1690)$
$N^{\ominus}_{\frac{1}{2}}(1535)$	$\Delta^{\ominus}_{\frac{3}{2}}(1700)$	$\boxed{\Lambda^{\oplus}_{\frac{1}{2}}(1600)}$	$\Sigma^{\ominus}_{\frac{3}{2}}(1670)$	$\Xi^{\ominus}_{\frac{3}{2}}(1820)$
$N^{\ominus}_{\frac{1}{2}}(1650)$	$(\Delta^{\oplus}_{\frac{1}{2}}(1750))$ $(\Delta^{\ominus}_{\frac{1}{2}}(1900))$	$\Lambda^{\ominus}_{\frac{1}{2}}(1670)$	$\Sigma^{\ominus}_{\frac{1}{2}}(1750)$	$\Xi\ (1950)$
$N^{\ominus}_{\frac{5}{2}}(1675)$	$\Delta^{\oplus}_{\frac{5}{2}}(1905)$	$\Lambda^{\ominus}_{\frac{3}{2}}(1690)$	$\Sigma^{\ominus}_{\frac{5}{2}}(1775)$	$\Xi\ (2030)$
$N^{\oplus}_{\frac{5}{2}}(1680)$	$\Delta^{\oplus}_{\frac{1}{2}}(1910)$	$\Lambda^{\ominus}_{\frac{1}{2}}(1800)$	$(\Sigma^{\ominus}_{\frac{3}{2}}(1580))$ $(\Sigma^{\ominus}_{\frac{1}{2}}(1620))$	
$N^{\ominus}_{\frac{3}{2}}(1700)$	$\Delta^{\oplus}_{\frac{3}{2}}(1920)$	$\Lambda^{\oplus}_{\frac{1}{2}}(1810)$		$\Omega^{\oplus}_{\frac{3}{2}}(1672)$
$N^{\oplus}_{\frac{1}{2}}(1710)$	$\Delta^{\ominus}_{\frac{5}{2}}(1930)$ $(\Delta^{\ominus}_{\frac{3}{2}}(1940))$	$\Lambda^{\oplus}_{\frac{5}{2}}(1820)$	$\Sigma^{\oplus}_{\frac{5}{2}}(1915)$	$\Omega\ (2250)$
$N^{\oplus}_{\frac{3}{2}}(1720)$	$\Delta^{\oplus}_{\frac{7}{2}}(1950)$	$\Lambda^{\ominus}_{\frac{5}{2}}(1830)$	$\Sigma^{\ominus}_{\frac{3}{2}}(1940)$	
$N^{\ominus}_{\frac{7}{2}}(2190)$		$\Lambda^{\oplus}_{\frac{3}{2}}(1890)$ $(\Lambda^{\oplus}_{\frac{7}{2}}(2020))$ $\Lambda^{\ominus}_{\frac{7}{2}}(2100)$	$\Sigma^{\oplus}_{\frac{7}{2}}(2030)$	

Mass formula for $56'^{\oplus}$ chiral states

$$\begin{pmatrix} 1.44^2 - 0.939^2 \\ = 1.19 \end{pmatrix} \approx \begin{pmatrix} 1.6^2 - 1.232^2 \\ = 1.04 \end{pmatrix} \approx \begin{pmatrix} 1.6^2 - 1.116^2 \\ = 1.32 \end{pmatrix} \approx \begin{pmatrix} 1.66^2 - 1.191^2 \\ = 1.337 \end{pmatrix}$$

$$\ll \ 2\Omega = 2.3\ \mathrm{GeV}^2$$

$$\left(\Omega = 1.15\ \mathrm{GeV}^2 \leftarrow (\text{Regge slope})^{-1}\right)$$

(Properties of $56^{\oplus}$ Chiral States)

$56'^{\oplus}(B') \to 56^{\oplus}(B) + 36(M)$

$$\Gamma_{B'\to B\pi} = \boxed{f^2}\ \frac{|\mathbf{p}| M_B}{2\pi M_{B'}} \left| g c_1^* s_1^* \right|^2 \left(ch^2\theta_2\ sh(\theta_2+\theta_3)\, ch\,\theta_3 \right)^2$$

P-wave decay

B'	BM	f	Γtheor /MeV	Γexp. /MeV	
$N^{\oplus}(1440)$	$N\pi$	$\frac{10\sqrt{2}}{3\sqrt{3}}$	227.5	~227.5	((250~450) × (60~70)%)
	$\Delta\pi$	$\frac{16}{3\sqrt{3}}$	13	~87.5	(× (20~30)%)
$\Lambda(1600)$ (1560 ~1700)	$\Sigma\pi$	$\frac{4\sqrt{2}}{3}$	61.8 (45~120)	~ 50	((50~250) × (10~60)%)
	$\Sigma^*\pi$	$-\frac{8}{3}$	11.4 (3~54)		
	NK	$-\frac{4}{\sqrt{3}}$	10.3 (7~23)	~ 30	(× (15~30)%)
$\Sigma(1660)$ (1630 ~ 1690)	$\Lambda\pi$	$\frac{4\sqrt{2}}{3\sqrt{3}}$	48.1 (41~56)		
	$\Sigma\pi$	$\frac{16}{9}$	83.5 (68~101)		
	$\Sigma^*\pi$	$-\frac{8\sqrt{2}}{9}$	7.3 (5~11)		
	NK	$\frac{4}{9}$	0.6 (0.5~0.8)	~ 20	((40~200) × (10~30)%)
$\Xi(1710)$ (~1810)	$\Xi\pi$	$\frac{2\sqrt{2}}{3\sqrt{3}}$	4.6 (~9)		
	ΛK	$\frac{2\sqrt{2}}{3\sqrt{3}}$	0.3 (~0.8)		
	ΣK	$\frac{10\sqrt{2}}{3\sqrt{3}}$	0.7 (~9)		
	$\Xi^*\pi$	$-\frac{8}{3\sqrt{3}}$	1.3 (~12)		
$\Delta(1600)$ (1550 ~1700)	$N\pi$	$-\frac{8}{3\sqrt{3}}$	163 (130~244)	~ 60	((250~450) × (10~25)%)
	$\Delta\pi$	$\frac{10\sqrt{2}}{3\sqrt{3}}$	92.5(57~196)	~190	(× (40~70)%)
$\Sigma^*(1760)$ (~1860)	$\Sigma\pi$	$\frac{8}{9}$	36.9 (~59)		
	$\Lambda\pi$	$\frac{4\sqrt{2}}{3\sqrt{3}}$	78.2 (~118)		
	NK	$-\frac{8}{9}$	4.8 (~7.9)		
	$\Sigma^*\pi$	$\frac{8\sqrt{5}}{9}$	53.5 (~112)		
	ΔK	$\frac{8\sqrt{5}}{9}$	0.6 (~5.5)		
$\Xi^*(1880)$ (~1980)	$\Xi\pi$	$\frac{4\sqrt{2}}{3\sqrt{3}}$	54.6 (~87)		
	ΛK	$\frac{4\sqrt{2}}{3\sqrt{3}}$	5.3 (~9.2)		
	ΣK	$\frac{4\sqrt{2}}{3\sqrt{3}}$	3.1 (~6.2)		
	$\Xi^*\pi$	$\frac{2\sqrt{10}}{3\sqrt{3}}$	15.9 (~36)		
$\Omega(2000)$ (~2100)	ΞK	$\frac{8\sqrt{2}}{3\sqrt{3}}$	11.7 (~24)		
	Ξ^*K	$\frac{4\sqrt{10}}{3\sqrt{3}}$	—	(~3.3)	

$56_E - 56_F$ Mixing)

$$|c_1|^2 + |s_1|^2 = 1$$

$$56 = c_1\, 56_E + s_1\, 56_F \supset N(939),\ \Delta(1232)$$

$$56' = -s_1^*\, 56_E + c_1^*\, 56_F \supset N(1440),\ \Delta(1600)$$

$$\begin{cases} g_{\pi NN} = 13.45 & \longrightarrow g\left(|c_1|^2 - \frac{1}{3}|s_1|^2\right) = 1.462 \\ \Gamma_{N(1440)\to N\pi} = \sim 227.5\,\text{MeV} & \longrightarrow \left| g\ c_1^* s_1^* \right| = 0.4383 \end{cases}$$

(Two Solutions)

(A1) $N \sim N_E$ $\quad N(1440) \sim N_F$

$|c_1|^2 = 0.922 \gg |s_1|^2 = 0.0782 \quad g = 1.63$

(A2) $N \sim N_F$ $\quad N(1440) \sim N_E$

$|c_1|^2 = 0.00934 \ll |s_1|^2 = 0.9907 \quad g = 4.56$

$\Delta(1600) \longrightarrow N(1440)\pi$

$$\Gamma = \left(\sqrt{\tfrac{4}{3}}\right)^2 \frac{|\mathbf{p}|\, M_{1440}}{2\pi M_{1600}} \left| g\left(|s_1|^2 - \tfrac{1}{3}|c_1|^2\right)\right|^2 \left(ch^2\theta_2\, sh(\theta_1+\theta_2)\, ch\theta_3\right)^2$$

B'	$B\pi$	f	Γ_{theor} /MeV	$\Gamma_{exp.}$ /MeV
$\Delta(1600)$ (1550~1700)	$N\pi$	$-\frac{8}{3\sqrt{3}}$	163 (130~244)	~ 60 ((250~450) × (10~25)%)
	$\Delta\pi$	$\frac{10\sqrt{2}}{3\sqrt{3}}$	92.5 (57~196)	~ 190 (× (40~70)%)
	$N(1440)\pi$	$\sqrt{\frac{4}{3}}$	(A1) 0.19 (– ~ 3.6)	~ 80 (× (10~35%))
			(A2) 27 (– ~ 521)	

- **Small mixing of $56_E \leftrightarrow 56_F$**
- **Sizable $\Delta(1600) \to N(1440)\pi$ seems to suggest (A2).**

$$\begin{cases} N(939),\ \Delta(1232) \sim |\rho\ {-\tfrac{1}{2}}\rangle_S & \text{chiral states} \\ N(1440),\ \Delta(1600) \sim |\rho\ \tfrac{3}{2}\rangle_S & \text{NR states} \end{cases}$$

IV. Concluding Remarks

- **Covariant Level Scheme based on $\tilde{U}(12)_{SF} \otimes O(3,1)_L$**

  Ground-State B and $\bar{B}$ are classified as 364.

  $56^{\oplus}$

$56'^{\oplus}$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	; $\Delta(1600)$
$70^{\ominus}$	$\Lambda^{\ominus}(1405)$			

  Chiral States

- Mass formula

  $$M_{N(1440)}^2 - M_{N(939)}^2 \approx M_{\Lambda(1600)}^2 - M_{\Lambda(1116)}^2 \approx M_{\Sigma(1660)}^2 - M_{\Sigma(1191)}^2 \approx M_{\Delta(1600)}^2 - M_{\Delta(1232)}^2$$

- **Magnetic M. of $56'^{\oplus}$ is the same as $56^{\oplus}$**

- **Decay properties of $56'^{\oplus}$ predicted by $SU(6)_{SF}$ seems to be consistent with exp. data.**

⇒ Covariant Level Scheme : Promising !

More accurate exp. data Necessary for confirmation.

- Property of $70_G^{\ominus}$:

  Mixing with ordinary P-wave states $70_1^{\ominus}$

- Excited States of Chiral States.

  candidates $\Lambda^{\ominus}_{\frac{5}{2}}(1830)$?, $N^{\oplus}_{\frac{5}{2}}(1680)$, ⋯

(Assignment of Baryons) in $U(12)_{SF} \times O(3,1)_L$

N=0

$56_0^{\oplus}$

$N_{\frac{1}{2}}^{\oplus}$ $N(939)$ $\Lambda(1116)$ $\Sigma(1192)$ $\Xi(1318)$

$\Delta_{\frac{3}{2}}^{\oplus}$ $\Delta(1232)$ $\Sigma^*(1385)$ $\Sigma^*(1385)$ $\Omega(1672)$

$56_0'^{\oplus}$

$N_{\frac{1}{2}}'^{\oplus}$ $N(1440)$ $\Lambda(1600)$ $\Sigma(1660)$

$\Delta_{\frac{3}{2}}'^{\oplus}$ $\Delta(1600)$

$70_G^{\ominus}$

$\Lambda_{\frac{1}{2}}^{\ominus}$ $\Lambda^{\ominus}(1405)$?

$N_{\frac{1}{2}}^{\ominus}$

$N_{\frac{3}{2}}^{\ominus}$ ($\Sigma_{\frac{3}{2}}^{\ominus}(1580)$)

$\Delta_{\frac{1}{2}}^{\ominus}$ ($\Sigma_{\frac{1}{2}}^{\ominus}(1620)$)

N=1

$70_1^{\ominus}$

$\Lambda_{\frac{1}{2}}^{\ominus}$

$\Lambda_{\frac{3}{2}}^{\ominus}$ $\Lambda_{\frac{3}{2}}^{\ominus}(1520)$

$N_{\frac{1}{2}}^{\ominus}$ $N_{\frac{1}{2}}^{\ominus}(1535)$ $\Lambda_{\frac{1}{2}}^{\ominus}(1670)$ $\Sigma_{\frac{1}{2}}^{\ominus}(1750)$

$N_{\frac{3}{2}}^{\ominus}$ $N_{\frac{3}{2}}^{\ominus}(1520)$ $\Lambda_{\frac{3}{2}}^{\ominus}(1690)$ $\Sigma_{\frac{3}{2}}^{\ominus}(1670)$ $\Xi_{\frac{3}{2}}^{\ominus}(1820)$

$N_{\frac{1}{2}}^{\ominus}$ $N_{\frac{1}{2}}^{\ominus}(1650)$ $\Lambda_{\frac{1}{2}}^{\ominus}(1800)$?

$N_{\frac{3}{2}}^{\ominus}$ $N_{\frac{3}{2}}^{\ominus}(1700)$ $\Sigma_{\frac{3}{2}}^{\ominus}(1940)$

$N_{\frac{5}{2}}^{\ominus}$ $N_{\frac{5}{2}}^{\ominus}(1675)$ $\Lambda_{\frac{5}{2}}^{\ominus}(1830)$? $\Sigma_{\frac{5}{2}}^{\ominus}(1775)$

$\Delta_{\frac{1}{2}}^{\ominus}$ $\Delta_{\frac{1}{2}}^{\ominus}(1620)$

$\Delta_{\frac{3}{2}}^{\ominus}$ $\Delta_{\frac{3}{2}}^{\ominus}(1700)$

$70_1'^{\ominus}$ $\Lambda_{\frac{5}{2}}^{\ominus}(1830)$?

$70_{G1}^{\oplus}$ $N_{\frac{1}{2}}^{\oplus}(1710)$ $N_{\frac{3}{2}}^{\oplus}(1720)$ $N_{\frac{5}{2}}^{\oplus}(1680)$ $\Delta_{\frac{1}{2}}^{\oplus}(1750)$ $\Lambda_{\frac{1}{2}}^{\oplus}(1810)$

$56_{G1}^{\oplus}$

$20_{G1}^{\oplus}$

Chiral States

N=2

$56_2^{\oplus}$

$N_{\frac{3}{2}}^{\oplus}$ $N_{\frac{3}{2}}^{\oplus}(1810)$ $\Lambda_{\frac{3}{2}}^{\oplus}(1890)$

$N_{\frac{5}{2}}^{\oplus}$ $\Lambda_{\frac{5}{2}}^{\oplus}(1820)$ $\Sigma_{\frac{5}{2}}^{\oplus}(1915)$ $\Xi_{\frac{5}{2}}(2030)$

$\Delta_{\frac{1}{2}}^{\oplus}$ $\Delta_{\frac{1}{2}}^{\oplus}(1910)$

$\Delta_{\frac{3}{2}}^{\oplus}$ $\Delta_{\frac{3}{2}}^{\oplus}(1920)$

$\Delta_{\frac{5}{2}}^{\oplus}$ $\Delta_{\frac{5}{2}}^{\oplus}(1905)$

$\Delta_{\frac{7}{2}}^{\oplus}$ $\Delta_{\frac{7}{2}}^{\oplus}(1950)$ $\Sigma_{\frac{5}{2}}^{\oplus}(2030)$

$56_0^{\oplus}$

$70_2^{\oplus}$ ($\Lambda_{\frac{7}{2}}^{\oplus}(2020)$)

$70_0^{\oplus}$

$20_1^{\oplus}$

N=3

$\Delta_{\frac{5}{2}}^{\ominus}(1930)$ $\Lambda_{\frac{7}{2}}^{\ominus}(2100)$

($\Delta_{\frac{3}{2}}^{\ominus}(1940)$)

($\Delta_{\frac{1}{2}}^{\ominus}(1900)$)