Chiral Transition in QCD and Scalar Correlation

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Nihon Univ. Kaikan, Ichigaya, Japan

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§1 Introduction

Tricky points in hadron physics based on QCD:

1. QCD is written solely in terms of quark and gluon fields,
   while, only hadrons are observed in the low-energy regime.
   Why?

   The QCD vacuum$\neq$ the perturbative one.
   Hadrons are elementary excitations on top of the non-perturbative QCD vacuum.

2. Symmetries in (classical) QCD Lagrangian are not manifest;
   Color SU(3) —- Confinenment (Hadrons are white!)
   (Apprximate) chiral symmetry —- Spontaneously broken

   ″

The low-energy hadron physics is a study of the nature of QCD vacuum and hopefully its symmetry properties.

Hadron Physics $\sim$ condensed matter physics +Atomic Physics.

§1 Introduction (cont'd)

Chiral Transition	=	a phase transition of QCD vacuum*)

*): $\langle \bar{q}q \rangle$ being the order parameter.

Eg. Lattice QCD; F. Karsch, Nucl. Phys. Proc. Suppl. **83**, 14 (2000).

The wisdom of many-body theory tells us:

> If a phase transition is of 2nd order or *weak* 1st order,
> $\exists$ soft modes $\sim$ the fluctuations of the order parameter

For chiral transition,

$$\langle (\bar{q}q)^2 \rangle \sim \sigma\text{-meson} \ \ (I = J = 0).$$
$$\Downarrow$$

> The σ meson becomes the soft mode of chiral transition at $T \neq 0$ and/or $\rho_B \neq 0$.

T. Hatsuda and T.K.: Phys. Rev. Lett. **55** (1985), 158; Phys. Rep. **247**(1994) 221.
Phys. Lett. B145 ('84), 7.
$m_\sigma \searrow$. $\Gamma_\sigma \searrow$
($m_\sigma = 2M$: Nambu)

But, what is the σ?

The wisdom of many-body theory tells us:

> If a phase transition is of 2nd order or *weak* 1st order,
> $\exists$ soft modes $\sim$ the fluctuations of the order parameter

Soft modes and phase transitions

phase transition	Softening modes
quadrupole deformation	2^+-phonon
super conductivity	pairing vibration
pion condensation	σ-τ modes*)

*) M. Gyulassy and W. Greiner(1977); M. Ericson and J. Delorme (1978); W.M. Alberico et al (1980); H. Toki and W. Weise (1979); T.K. (1981).

For chiral transition,

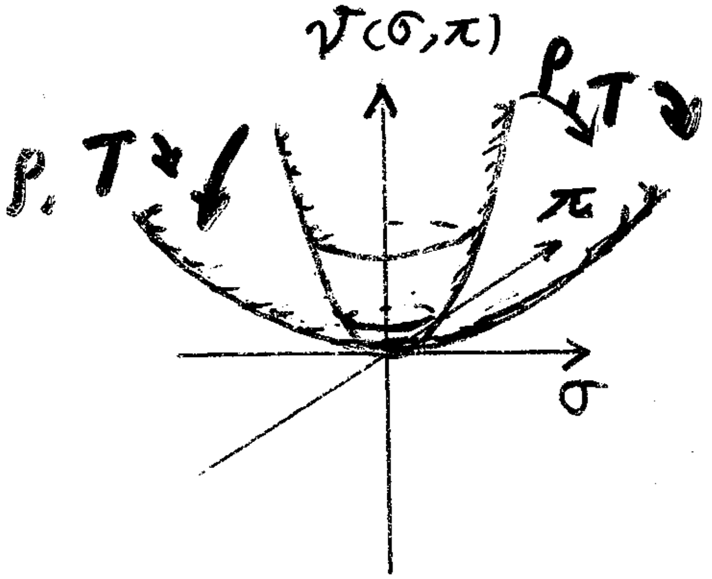
$$\langle(\bar{q}q)^2\rangle \sim \sigma\text{-meson} \quad (I = J = 0).$$

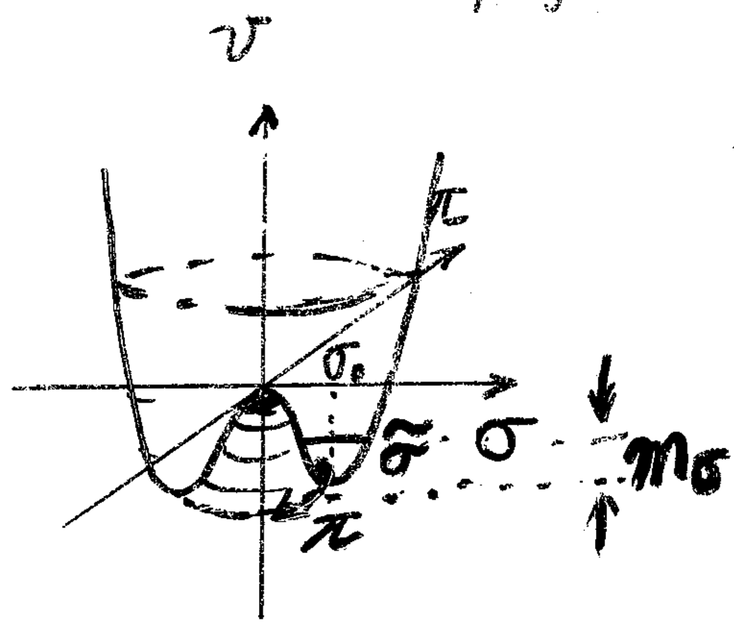
> The σ meson becomes the soft mode of chiral transition at $T \neq 0$ and/or $\rho_B \neq 0$:
> $m_\sigma \to 0$, $\Gamma_\sigma \to 0$

T. Hatsuda and T.K.: Phys. Lett. **B145** (1984),7; Phys. Rev. Lett. **55** (1985), 158.

$m_\sigma = 2M$ (Nambu rel.)

χ-transition $\rightarrow$ $\tilde{\pi}$ ... χ-rotatio..
$\tilde{\sigma}$ ... amp. fluct.





$T > T_c$	$T < T_c$
$\rho > \rho_c$	$\rho < \rho_c$
$g < g_c$	$g > g_c$

W-S model:
cf. Higgs field ϕ
$\downarrow$
$\phi = \langle\phi\rangle + \tilde{\phi}$
Higgs particle

(NG-boson $\downarrow$ W_L^μ)

$\sigma = \sigma_0 + \tilde{\sigma}$
amplitude fluctuation (vibration)
pairing

π : rotational mode
(pairing rotation)

cf. curvature $\rightarrow \infty$, $m_\sigma \nearrow$
$\rightarrow$ non-lin. σ model (Weinberg)

Near critical point, the σ is necessary!

Heuristic evaluation of $\langle\langle\bar{q}q\rangle\rangle_{T,\rho}$ at $T \neq 0$ and/or $\rho \neq 0$ based on Feynman-Hellman theorem

The condensate at $T \neq 0$:

$$\langle\langle\bar{q}_i q_i\rangle\rangle = \frac{1}{Z}\text{Tr}\left[\bar{q}_i q_i \; e^{-(H_{QCD}-\mu N)/T}\right] = \frac{\partial\omega(T)}{\partial m_i} \qquad (1)$$

Z is the QCD partition function, ω is the free energy density.

Free pion gas:

$$\langle\langle\bar{q}_i q_i\rangle\rangle_T = \langle\bar{q}_i q_i\rangle_0 + \delta\langle\langle\bar{q}_i q_i\rangle\rangle_T$$

where,

$$\delta\langle\langle\bar{q}_i q_i\rangle\rangle_T = \sum_p n_\pi(p)\langle\pi(p)|\bar{q}_i q_i|\pi(p)\rangle,$$

$$\langle\pi(p)|\bar{q}_i q_i|\pi(p)\rangle = m_\pi\langle\bar{q}_i q_i\rangle_\pi/E_p^\pi,$$

Gell-Mann-Oakes-Renner relation

$$f_\pi^2 m_\pi^2 = -(m_u + m_d)/2 \cdot \langle\bar{u}u + \bar{d}d\rangle$$

and F-H theorem gives

$\langle\bar{q}_i q_i\rangle_\pi = 6.25 > 0$ $(i = u, d)$.

Thus, $\langle\langle\bar{q}_i q_i\rangle\rangle$ decreases in the absolute value;

i.e. restoration of chiral symmetry at finite temperature.

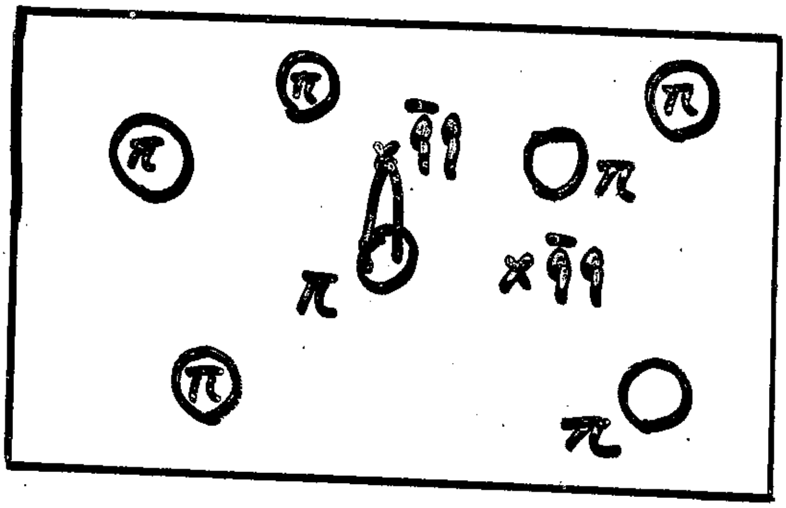
(C) A heuristic approach based on soft pion theorem

T. Hatsuda, Y. Koike and S. Lee ('92)

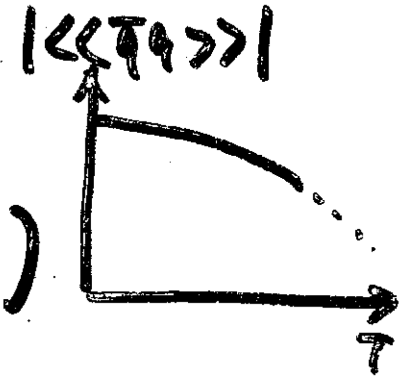
$$\langle\langle \bar{q}q \rangle\rangle = \langle \bar{q}q \rangle + \delta\langle\langle \bar{q}q \rangle\rangle$$

$$\delta\langle\langle \bar{q}q \rangle\rangle = \sum_{a=1}^{3} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_\pi} \langle \pi^a | \bar{q}q | \pi^a \rangle \, n_B(E/T)$$

$$\langle \pi^a | \bar{q}q | \pi^a \rangle \simeq \frac{-1}{2f_\pi^2} \langle 0 | \bar{q}q | 0 \rangle$$



a model for Hadronic matter

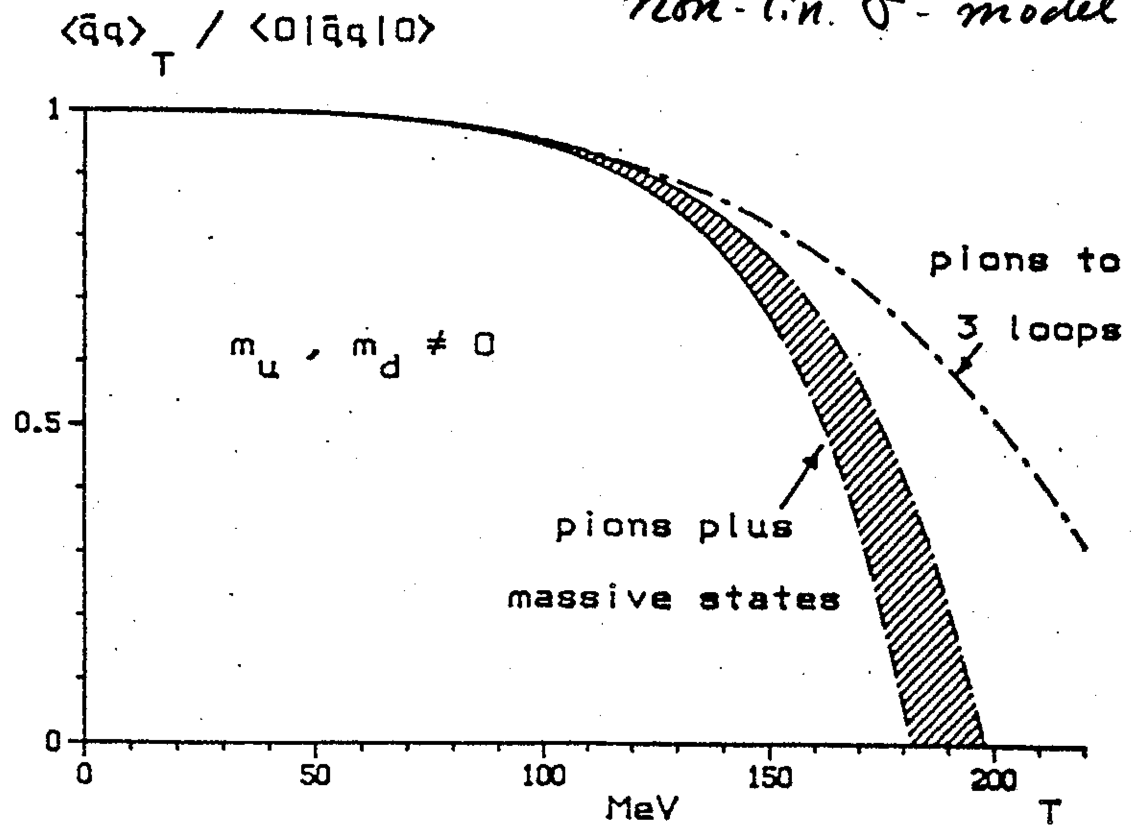


$$\langle\langle \bar{q}q \rangle\rangle \simeq \langle \bar{q}q \rangle \left(1 - \frac{T^2}{8f_\pi^2} B_1\left(\frac{m_\pi}{T}\right) \right)$$

$$B_1(z) \sim z^{1/2} e^{-z}, \quad z \to \infty \quad (T \ll m_\pi)$$

P. Gerber & H. Leutwyler, Nucl. Phys. B321 ('89) 387.

non-lin. σ - model ⊕ χ-pert.

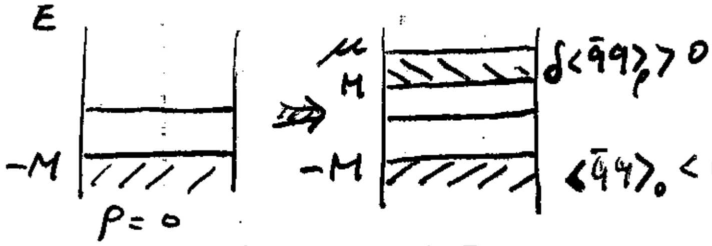


Degenerate nucleon system:

$$\langle N|\bar{q}q|N\rangle = \frac{\partial\langle N|\mathcal{H}_{QCD}|N\rangle}{\partial m_q}$$

$$\langle\Phi|\mathcal{H}_{QCD}|\Phi\rangle = \varepsilon_{vac} + \rho_B[M_N + B].$$

Φ: Nuclear matter,
B: the binding energy.



$$\frac{\langle\bar{q}q\rangle}{\langle\bar{q}q\rangle_0} = 1 - \frac{\rho_B}{f_\pi^2 m_\pi^2}\left[\Sigma_{\pi N} + \widehat{m}\frac{d}{d\widehat{m}}\left(\frac{B(n_B)}{A}\right)\right],$$

$\Sigma_{\pi N} = (m_u + m_d)/2 \cdot \langle N|\bar{u}u + \bar{d}d|N\rangle \sim (40-50)$ MeV; π-N sigma term.

and $B(n_B)/A$ is the nuclear binding energy per particle with $\widehat{m} = (m_u + m_d)/2$. The density-expansion of the r.h.s. in the lowest order gives a reduction of almost 35 % of $\langle\bar{q}q\rangle$ already at the nuclear matter density $n_0 = 0.17\text{fm}^{-3}$. Thus, the quark condensate tends to decreases in the nuclear medium.

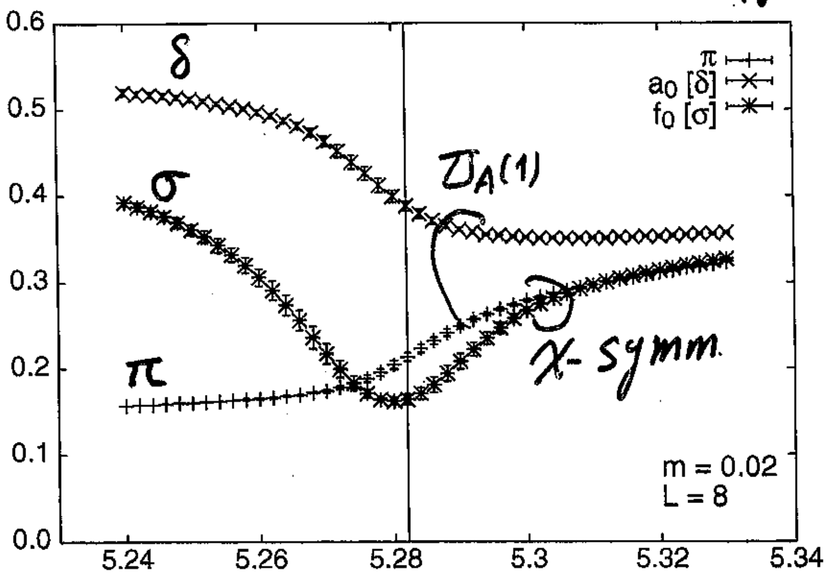
Cf. Lattice Calculation of the ~~screening~~ generalised *masses* F. Karsch, hep-lat/0106019

$N_f = 2$ $\quad 8^3 \times 4$ stagg. fermion

e.g.

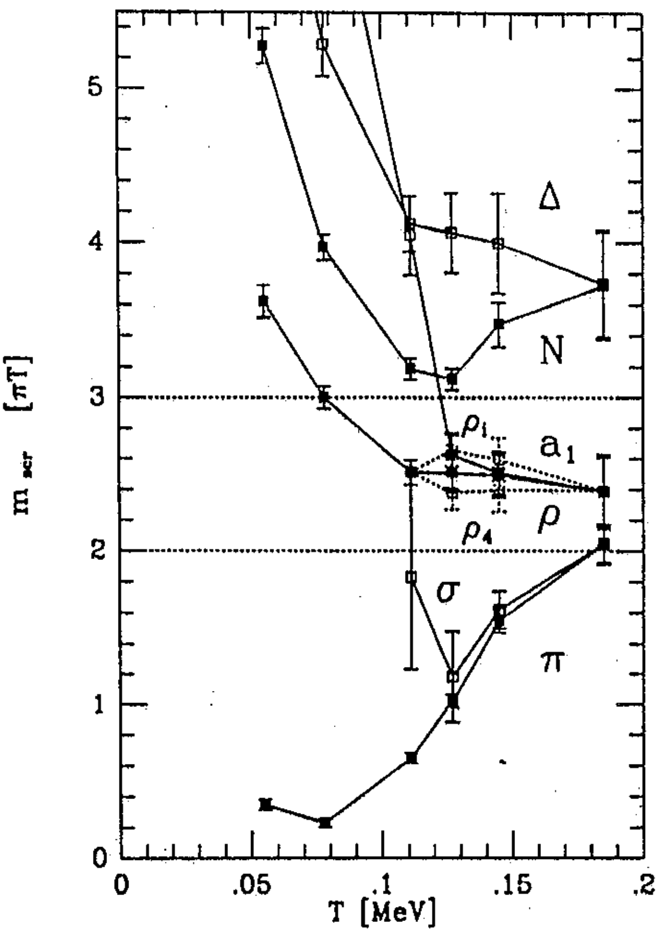
$m_\sigma^2 = \chi_\sigma^{-1}$

$\chi_\sigma = \langle (\bar\psi\psi)^2 \rangle$



1. The softening of σ
2. a degeneracy of the σ and π at high T
3. $U_A(1)$ symmetry not restored even at high T. ($m_\delta \leftrightarrow m_\pi$)

But, what is the siginificance of the σ in hadron physics?



space-like screening masses as a function of temperature, from (Schäfer & Shuryak 1996*b*). ıe lowest fermionic Matsubara frequency πT.

Instanton approach

Fluctuation Effects in Hot Quark Matter: Precursors of Chiral Transition at Finite Temperature

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(Received 2 May 1985)

Fluctuations of the order parameter of chiral transition in a hot and dense quark gas are examined in the random-phase approximation with the use of a QCD-motivated effective Lagrangean. We show that there arise soft modes having a large strength and a narrow width above the critical temperature, which are analogous to the fluctuations of the order parameter in a superconductor above the critical point. It is argued that the modes contribute to the cooling of the quark-gluon plasma.

lowing points: (i) There is a sharp peak with a narrow width ($10 \sim 60$ MeV) above T_χ which implies the ex-

Phase diagram

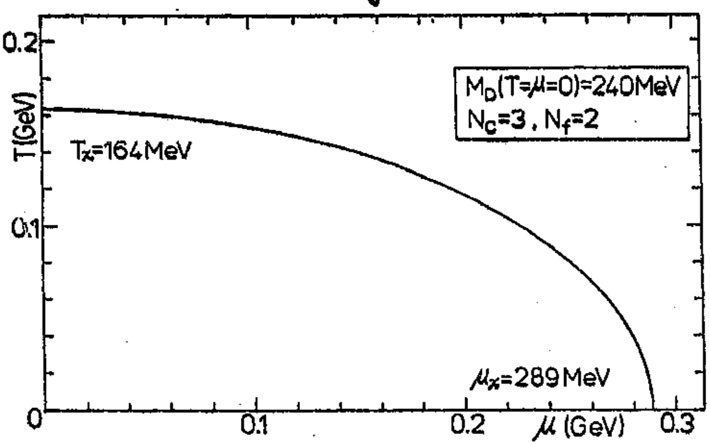


FIG. 1. Critical line calculated by use of the parameters which reproduce f_π ($=93$ MeV) and m_π ($=140$ MeV) in the lowest order of the chiral perturbation.

Spectral function

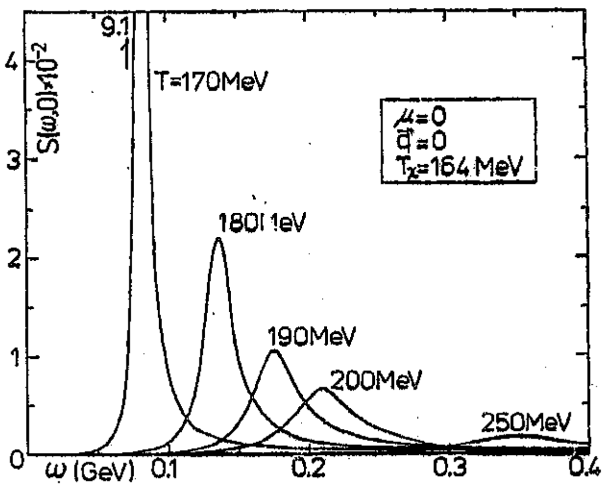
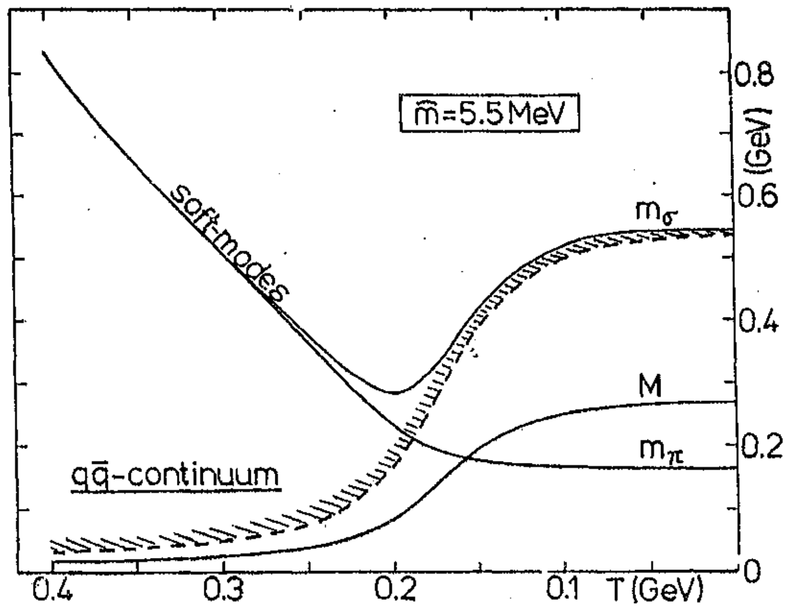


FIG. 2. Strength function at zero momentum transfer ($q=0$) above the critical temperature $T_\chi = 164$ MeV with $\mu = 0$. The shape and the peak position of $S(\omega, q)$ for $q \neq 0$ as a function of $\omega^2 - q^2$ hardly change from those of $S(\omega, 0)$.



The mass shift

FIG. 3. Dynamical quark mass $M = M_D(T, \mu) + \tilde{m}$, and the masses of σ mode (m_σ) and π mode (m_π). The dashed line denotes the $2M$ threshold from which the $q\bar{q}$ continuum starts.

§2 The significance of the σ meson in low energy hadron physics and QCD

1. the σ = the quantum fluctuation of the order parameter$\sim$ the Higgs particle in the WSG model: $\rightarrow m_\sigma = 400$-$800$ MeV, $\Gamma \sim m_\sigma$. (prediction in NJL-like models and mended symmetry of Weinberg.)

2. The pole in this mass range observed in the pi-pi S-matrix. S. Ishida et al.(1997); N. A. Törnqvist and M. Roos, (1996); M. Harada, F. Sannino and J. Schechter,(1996); J. A. Oller, E. Oset and J. R. Peláez, (1998); K. Igi and K. Hikasa, Phys.. Rev. **D59**, 034005 (1999); G. Colangero, J. Gasser and H. Leutwyler, hep-ph/0103088. As a compilation of the pole positions of the σ obatined in the modern analyses, see Z. Xiao and H. Zheng, hep-ph/0011260.

   Significance of respecting chiral symmetry, unitarity and crossing symmetry to reproduce the phase shifts both in the $\sigma(s)$- and $\rho(t)$-channels with a low mass σ pole; (Igi and Hikasa).

3. Responsible for the intermediate range attraction in the nuclear force.

4. Accounts for $\Delta I = 1/2$ enhancement in $K^0 \rightarrow 2\pi$ compared with $K^+ \rightarrow \pi^+\pi^-$. E.P. Shabalin (1988); T. Morozumi, C.S. Lim and I. Sanda (1990).

5. π-N sigma term $\Sigma_{\pi N} \sim 40$-50 MeV.(naively ~ 15 MeV)
   $\leftarrow$ enhanced by the collectiveness of the σ. T.Hatsuda and T.K.(1990) ; see the next slide.

K. Igi & K. Hikasa, Phys. Rev. D59 (00) 034005
with χ-symm & crossing symm.

$\delta^{I=J=0}$

only ρ
no σ

ρ & σ

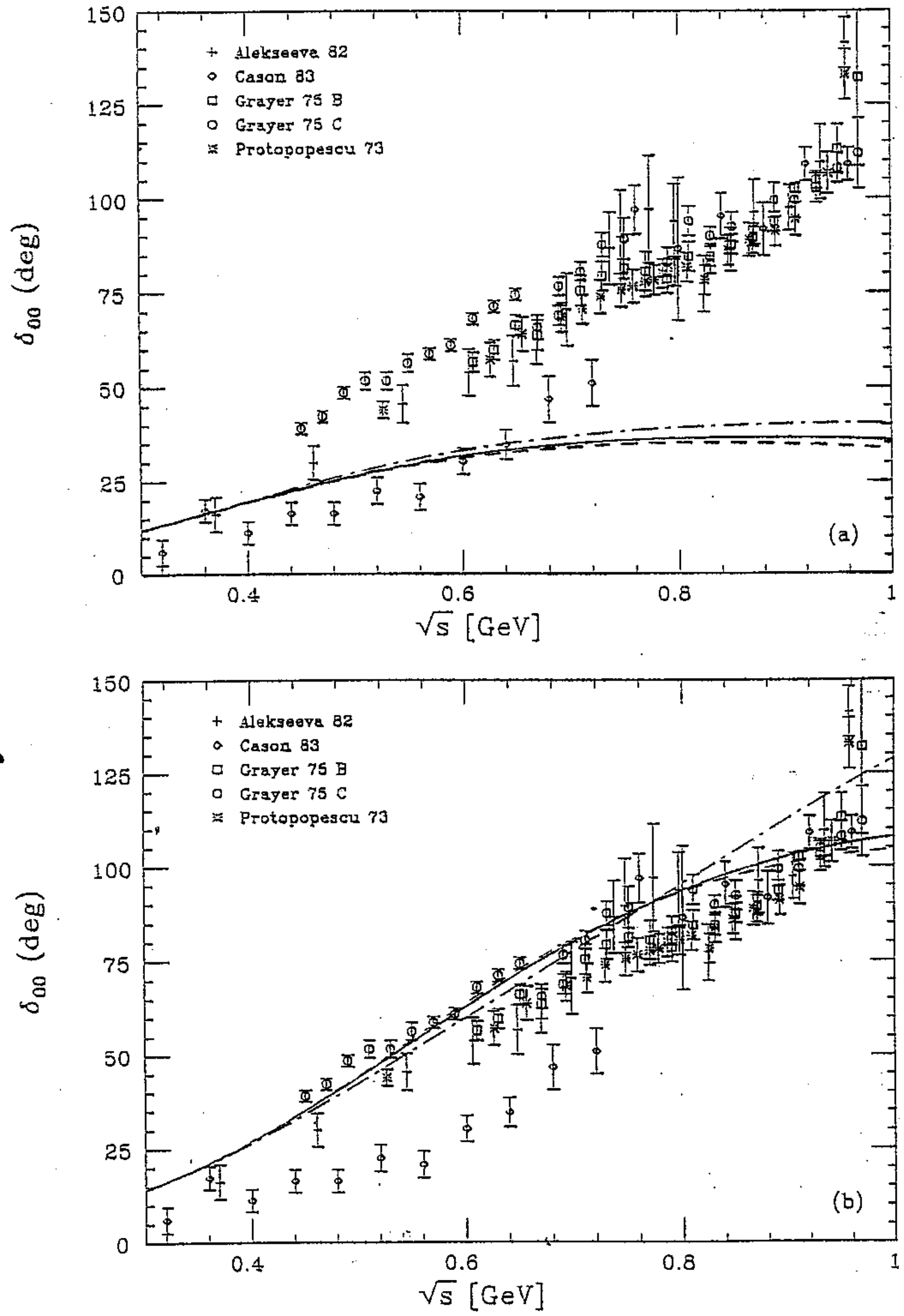


FIG. 2. The $I = J = 0$ $\pi\pi$ phase shift with (a) ρ exchange only; (b) degenerate σ ρ exchanges for the KSRF/Weinberg (dashed), Veneziano (dot-dash), and intermediate $= 0.45 m_\rho^2/2f_\pi^2$ (solid) couplings. Some experimental data are also shown.

J. A. Oller, E. Oset & J. R. Peláez;
(hep-ph/9804209) Phys. Rev. D59, 074001 ('99)
Err. ibid D60, 099906 ('99).

cf. N. Kaiser et al. Nucl. Phys A594 ('95) 325; A612 ('97) 297

Table III. Masses and partial widths in MeV

Channel (I,J)	Resonance	Mass from pole	Width from pole	Mass effective	Width effective	Partial Widths
$(0,0)$	σ	442	454	$\approx$ 600	*very large*	$\pi\pi - 100\%$
$(0,0)$	$f_0(980)$	994	28	≈ 980	≈ 30	$\pi\pi - 65\%$ $K\bar{K} - 35\%$
$(0,1)$	$\phi(1020)$	980	0	980	0	
$(1/2,0)$	κ	770	500	≈ 850	*very large*	$K\pi - 100\%$
$(1/2,1)$	$K^*(890)$	892	42	895	42	$K\pi - 100\%$
$(1,0)$	$a_0(980)$	1055	42	980	40	$\pi\eta - 50\%$ $K\bar{K} - 50\%$
$(1,1)$	$\rho(770)$	759	141	771	147	$\pi\pi - 100\%$

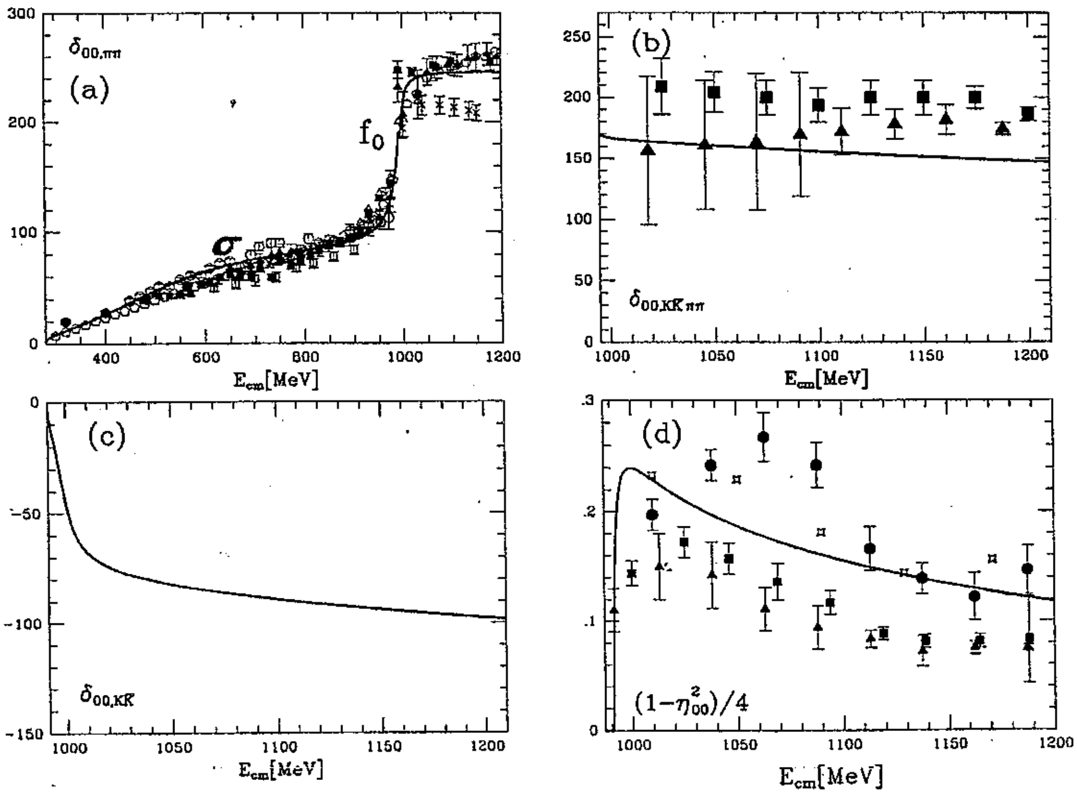


Fig.1: Results in the $I = J = 0$ channel. (a) phase shifts for $\pi\pi \to \pi\pi$ as a fraction of the CM energy of the meson pair: full triangle [19], open circle [20], full square [21], open triangle [22], open square [23] (all these are analysis of the same experiment [18]), cross [24], full circle [25], empty pentagon [26]. (b) phase shifts for $K\bar{K} \to \pi\pi$: full square [27], full triangle [28]. (c) Phase shifts for $K\bar{K} \to K\bar{K}$. (d) Inelasticity: results and data for $(1-\eta^2)/4$: starred square [26], full square [27], full triangle [28], full circle [29].

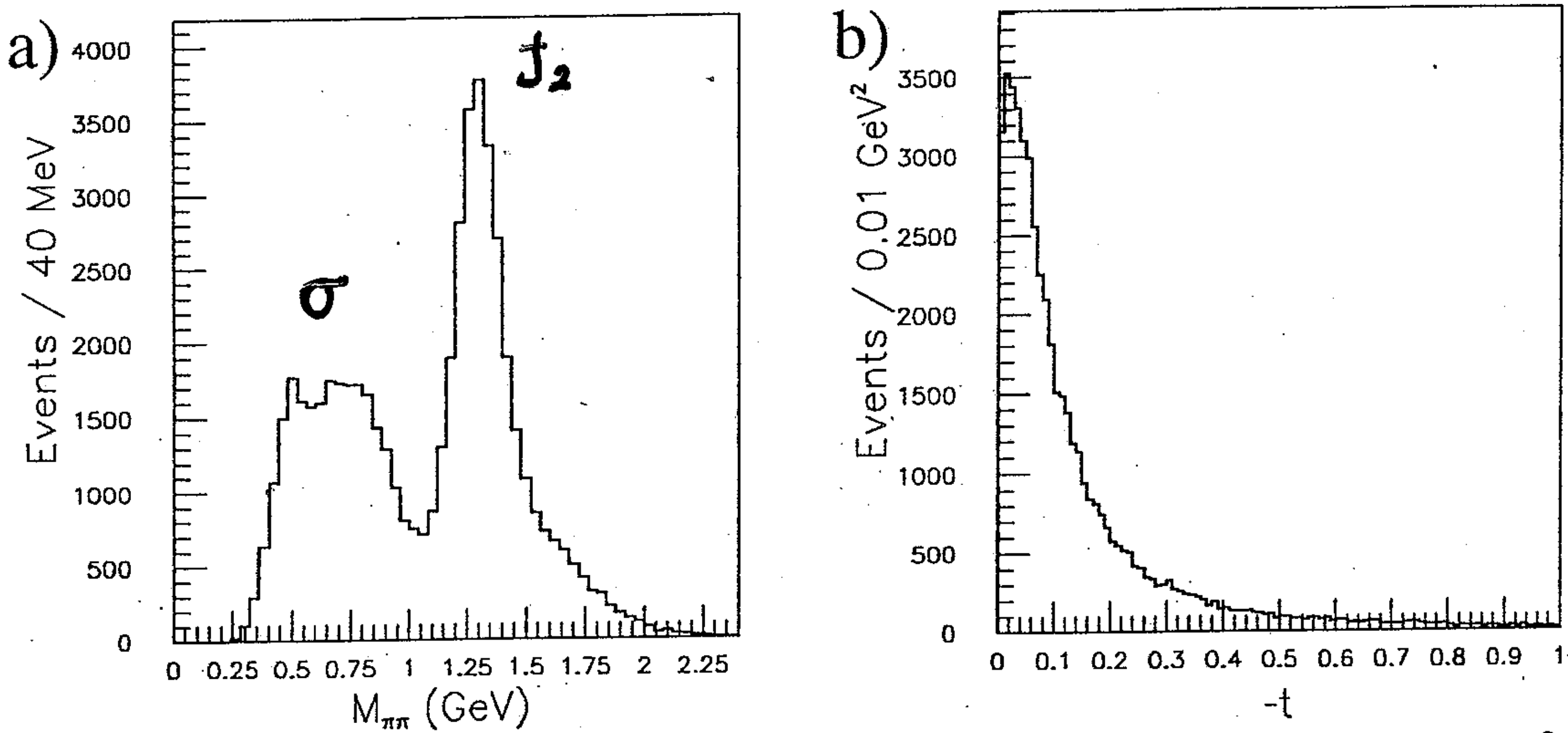


Fig. 1. (a) Effective mass distribution of the $\pi^0\pi^0$ system in $\pi^- p \to \pi^0\pi^0 n$. Data are corrected by the acceptance. (b) t distribution of $\pi^- p \to \pi^0\pi^0 n$.

E135 exp. group at KEK.

Data : E135 at KEK (K. Takamatsu et al)

Analysis : S. Ishida et al ('96, '97)

$m_\sigma = 588 \pm 12$ MeV, $\Gamma_\sigma = 281 \pm 25$ MeV

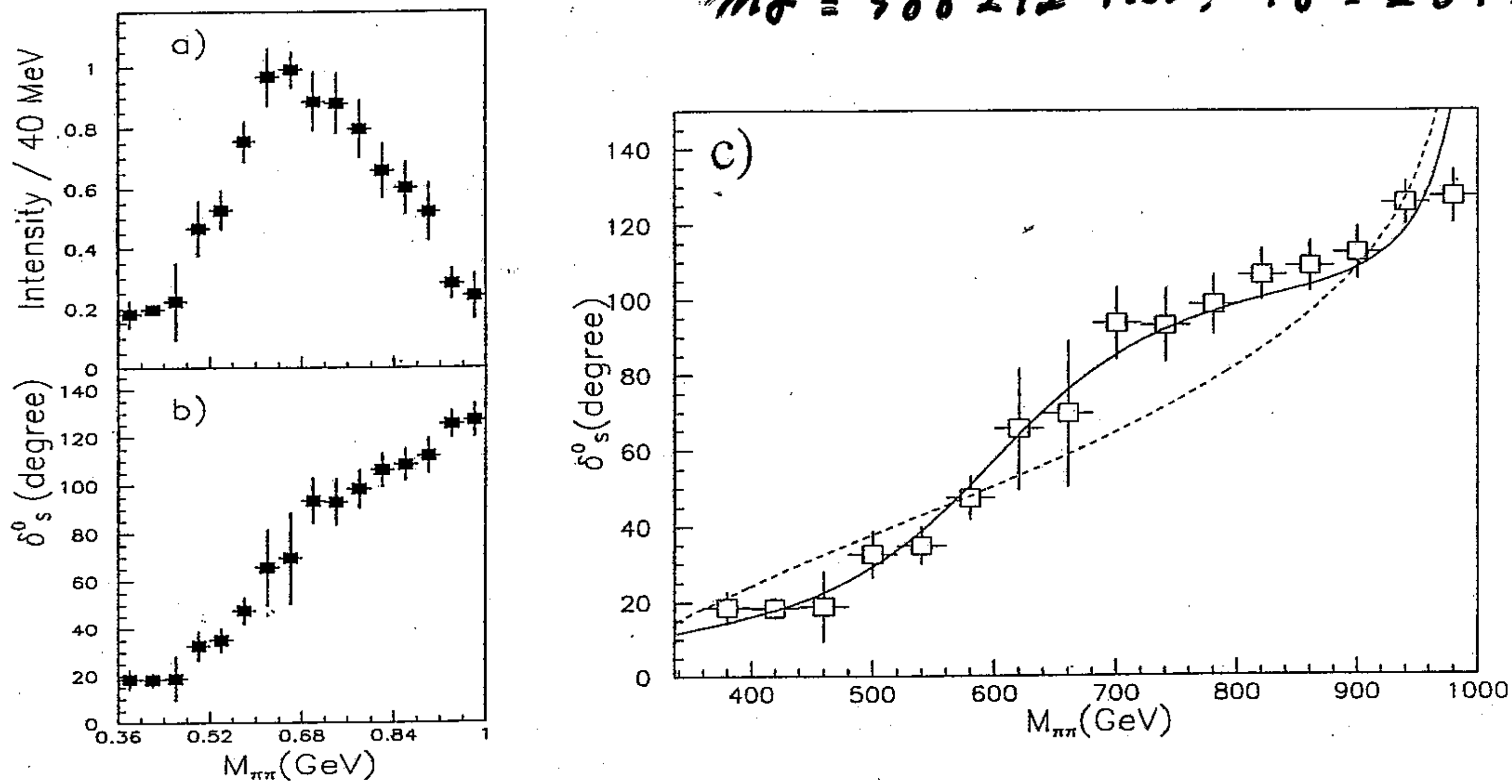
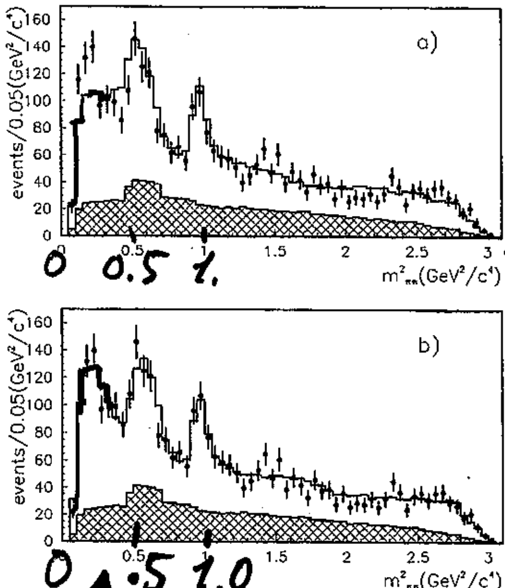


Fig. 3. (a) Normalized intensity distribution of S wave amplitude square below 1.0 GeV. (b) I=0, S wave $\pi^+\pi^- \to \pi^0\pi^0$ scattering phase shift δ^0_S below $K\bar{K}$ threshold. (c) The same for (b). Solid line is the result of fitting with the IA method introducing the negative phase background δ_{BG}. Dotted line shows the result with $r_c = 0$.

E. M. Aitala et al (E791 collab.)
Phys. Rev. Lett. 86, 770 (2001)

σ in $D^+ \to \pi^- \pi^+ \pi^+$



without a σ pole

$0 \quad 0.5 \quad 1.$ $m^2_{\pi\pi}$ (GeV²/c²)

with a σ pole

$0 \quad .5 \quad 1.0$ $m^2_{\pi\pi}$ (GeV²/c²)

σ : $m_\sigma = 478^{+24}_{-23} \pm 17$ MeV/c^2

Figure 2: s_{12} and s_{13} projections for data (error bars) and fast-MC (solid). The shaded area is the background distribution, (a) solution with the Fit 1, (b) solution with Fit 2.

$\Gamma_\sigma = 324^{+42}_{-40} \pm 21$ MeV/c^2

preference for the additional amplitude, as is indicated also by the difference in χ^2/ν for the two models.

We consider the systematic errors associated with the values of the fixed parameters in the fit. The most important ones come from uncertainties in the background model (the background shape, composition, and level), in particular the $D_s^+ \to \eta'\pi^+$ reflection which populates the same region as the $D^+ \to \rho^0\pi^+$ component. We also account for the uncertainties in the parameters describing the acceptance function.

To better understand our data, we also fit it with vector, tensor, and toy models for the sixth (sigma) amplitude, allowing the masses, widths, and relative amplitudes to float freely. The vector and tensor models test the angular distribution of the signal. The toy model tests the phase variation expected of a Breit-Wigner amplitude by substituting a constant relative phase. The vector resonance model converges to poorly defined values of the mass and width: 805 ± 194 MeV/c^2 and 1438 ± 903 MeV/c^2; the tensor model to more poorly defined values: 2350 ± 683 MeV/c^2 and -690 ± 1033; and the toy model to 434 ± 11 MeV/c^2 and 267 ± 37 MeV/c^2. As

T. Morozumi, C.S. Lim & A.I. Sanda,

Phys. Rev. Lett. 65 ('90), 404.

$$\langle \pi^+\pi^- | Q_6 | K^0 \rangle = \langle \pi^0\pi^0 | Q_6 | K^0 \rangle = Y_0 * \frac{F_K}{3F_\pi - 2F_K} ;$$

$$\frac{F_K}{3F_\pi - 2F_K} = \frac{M_\sigma^2 - M_\pi^2}{M_\sigma^2 - M_K^2} \cdot \frac{F_K^2}{F_\pi^2}$$

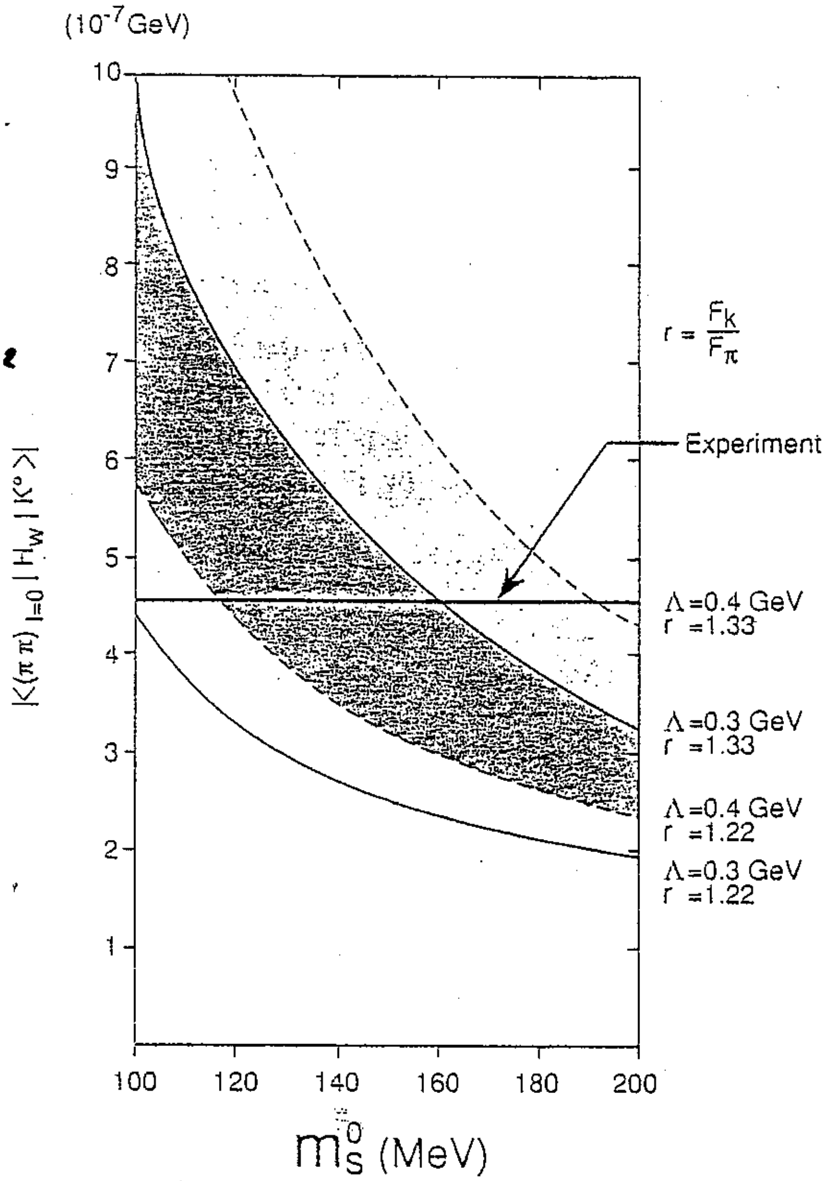


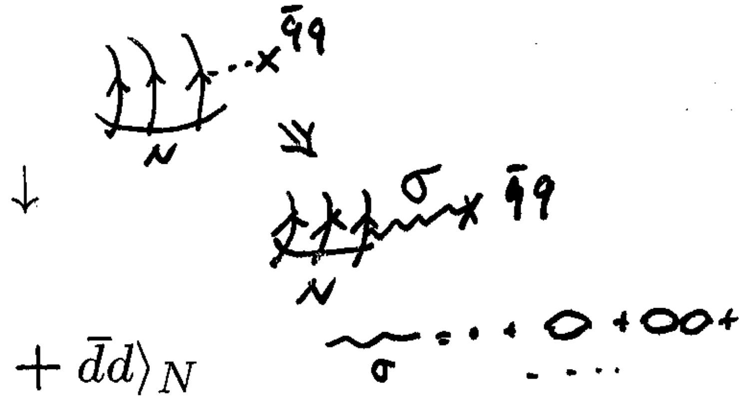
FIG. 1. Comparison of Eq. (21) with the experimental values for the $\Delta I = \frac{1}{2}$ $K \to \pi\pi$ decay amplitude. The two sets of curves correspond to $\Lambda = 0.4$ and 0.3 GeV. These parameters are chosen to be identical to those of Bardeen, Buras, and Gerard (Ref. 9) for ease of comparison. Since F_K/F_π at $\mu = 800$ MeV is not known precisely, we let the ratio vary from 1.22 to 1.33.

Light quark contents of baryons

B	$\langle \bar{u}u \rangle_B$	$\langle \bar{d}d \rangle_B$	$\langle \bar{s}s \rangle_B$
P (938)	4.97 (2)	4.00 (1)	0.53 (0)
Λ^0 (1115)	3.63(1)	3.63(1)	1.74(1)
Δ^{++} (1232)	3.66(2)	0.76 (0)	0.26 (0)
Ω^- (1672)	0.72 (0)	0.72 (0)	3.71 (3)

T.K. and T. Hatsuda, Phys. Lett. **B240** (1990) 209

The numbers in ( ) are those in the naive quark model.



$$\Sigma_{\pi N} = \hat{m}\langle \bar{u}u + \bar{d}d \rangle_N$$
$$= 5.5\text{MeV} \times (4.97 + 4)$$
$$\simeq 50\,\text{MeV}$$
$$>> 5.5 \times (2 + 1) \simeq 17\,\text{MeV}$$ naive case

with

$$y \equiv 2\langle \bar{s}s \rangle_N / \langle \bar{u}u + \bar{d}d \rangle_N = 0.12$$
$$= 0$$ naive

The empirical value of π-N Sigma term is reproduced due to the enhancement of the scalar charge due to the σ-mesonic collective mode! (The small but finite value of y reflects the small flavor mixing in the scalar mesons.)

$$T \to T_c,\ \rho \to \rho_c \Rightarrow \mathrm{Re}\, m_\sigma \searrow 2m_\pi,\ \mathrm{Im}\, m_\sigma \searrow 0$$

Nucl. Phys. A695, 273 (2001)

$2m_\pi$

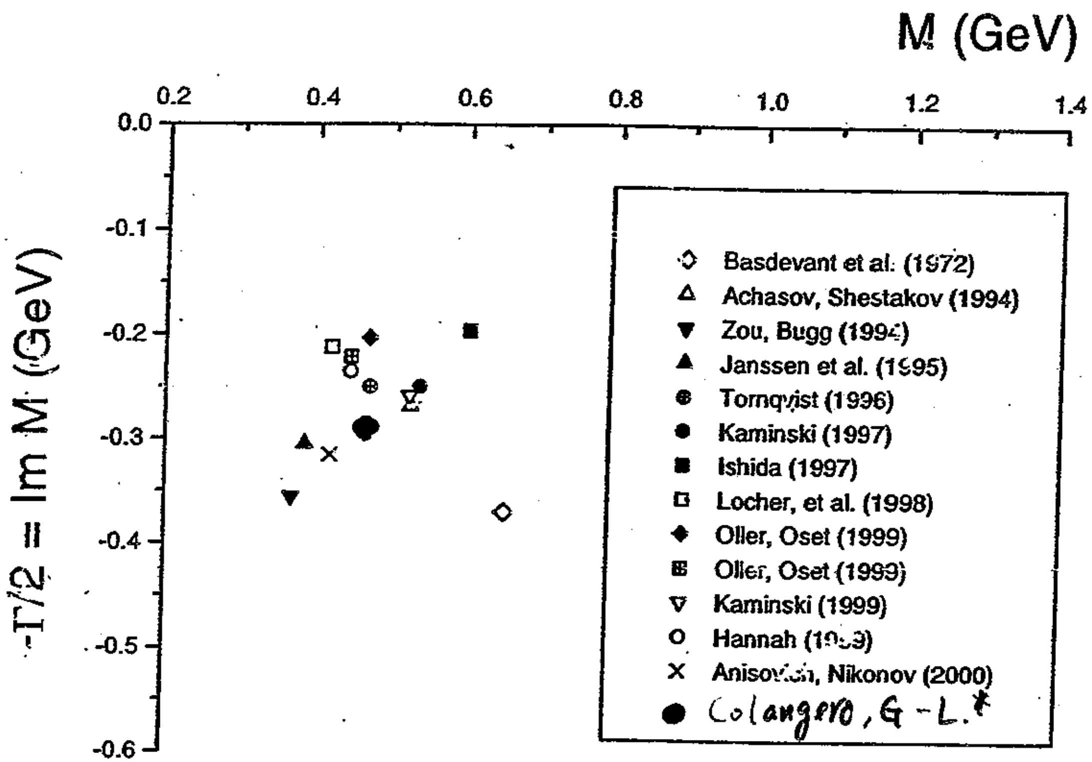
Softening!

Nucl. Phys. B60.

, 125, (2001)

Exp. E791 coll. (Fermi lab)
P.R.L. 86, 770 (2001) : $m_\sigma = 478$ MeV, $\Gamma_\sigma = 324$ MeV

Z. Xiao & H. Zheng:
hep-ph/0011260

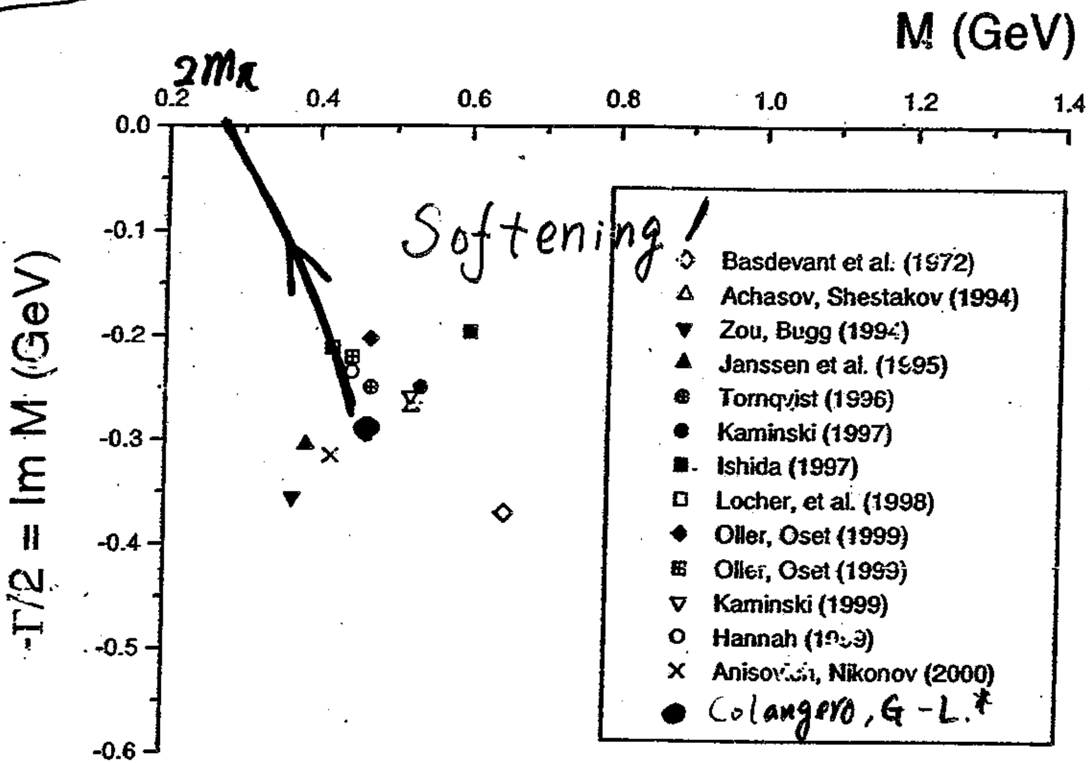


* G. Colangero, J. Gaßer and H. Leutwyler;
hep-ph/0103088

Figure 1: The poles of the S-matrix in the complex mass plane (GeV) corresponding to the lightest scalar resonance according to Refs. [3] – [18].

Z. Xiao & H. Zheng:
hep-ph/0011260
Nucl. Phys. A695, 273 (2001)

$T \to T_c$, $\rho \to \rho_c$ $\Rightarrow$ $\mathrm{Re}\, m_\sigma \searrow 2m_\pi$, $\mathrm{Im}\, m_\sigma \searrow 0$



* G. Colangero, J. Gaßer and H. Leutwyler; Nucl. Phys. B603, 125, (2001)
hep-ph/0103088

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Exp. E791 coll. (Fermi lab)
P.R.L. 86, 770 (2001) : $m_\sigma = 478$ MeV, $\Gamma_\sigma = 324$

Wait!

Is the pole observed in the pi-pi phase shift really the σ as the quantum fluctuation of the order parameter of the chiral transition?

A change of the environment	$\rightarrow$	a change of the mode coupled to the order parameter

$\Downarrow$

§3 Production of the σ-meson in nuclear medium

$\uparrow$

Useful for exploring the existence of the σ and the possible restoration of chiral symmetry at finite density.

T. K., Prog. Theor. Phys. Suppl. **120**(1994), 75

What is a good observables to see the softening in the sigma channel in nuclear medium?

Notice:
A particle might loose it identity when put in a medium.

Eg.
$\sigma \leftrightarrow 2\pi$, $\sigma \leftrightarrow$ p-h, π+p-h,(Δ-h,) $\pi + \Delta$-h ...

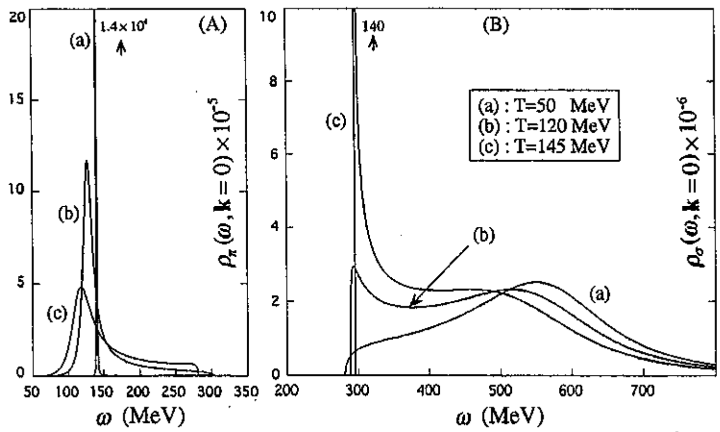
Need of calculation of Strength function

At $T \neq 0$

S. Chiku and T. Hatsuda, Phys. Rev. **D57**(1998)R6

M. K. Volkov et al, Phys. Lett. **424** (1998), 235

Findings:
an enhancement of the spectral function in the sigma channel near the $2m_\pi$ threshold.



S. Chiku and T. Hatsuda, Phys. Rev. **D57**(1998)R6

The surprise was,

> Such an enhancement had been seen by an Experiment by CHAOS*)!, at $T = 0$ but at $\rho_B \neq 0$

*); $A(\pi^+, \pi^+\pi^\pm)A'$ ($A = 2 \to 208$)

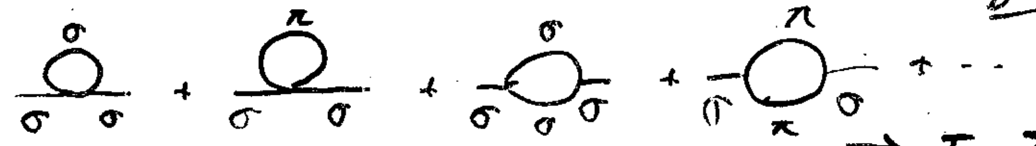
F. Bonutti et al, Phys. Rev. Lett. **77** (1996), 603.;

Nucl. Phys. **A677**, 123(2000)

↑

Motivated to explore possible in-medium π-π correlation; P. Schuck, W. Norönberg and G. Chanfray, Z. Phys. **A330**, 119(1988); G. Chanfray, Z. Aouissat, P. Schuck and W. Nörenberg, Phys. Lett. **B256**, 325 (1991).

S. Chiku and T. Hatsuda, Phys. Rev. D57 ('98) R6.
D58 ('98) 076001



$\rightarrow \mathrm{Im}\,\Sigma_\sigma$

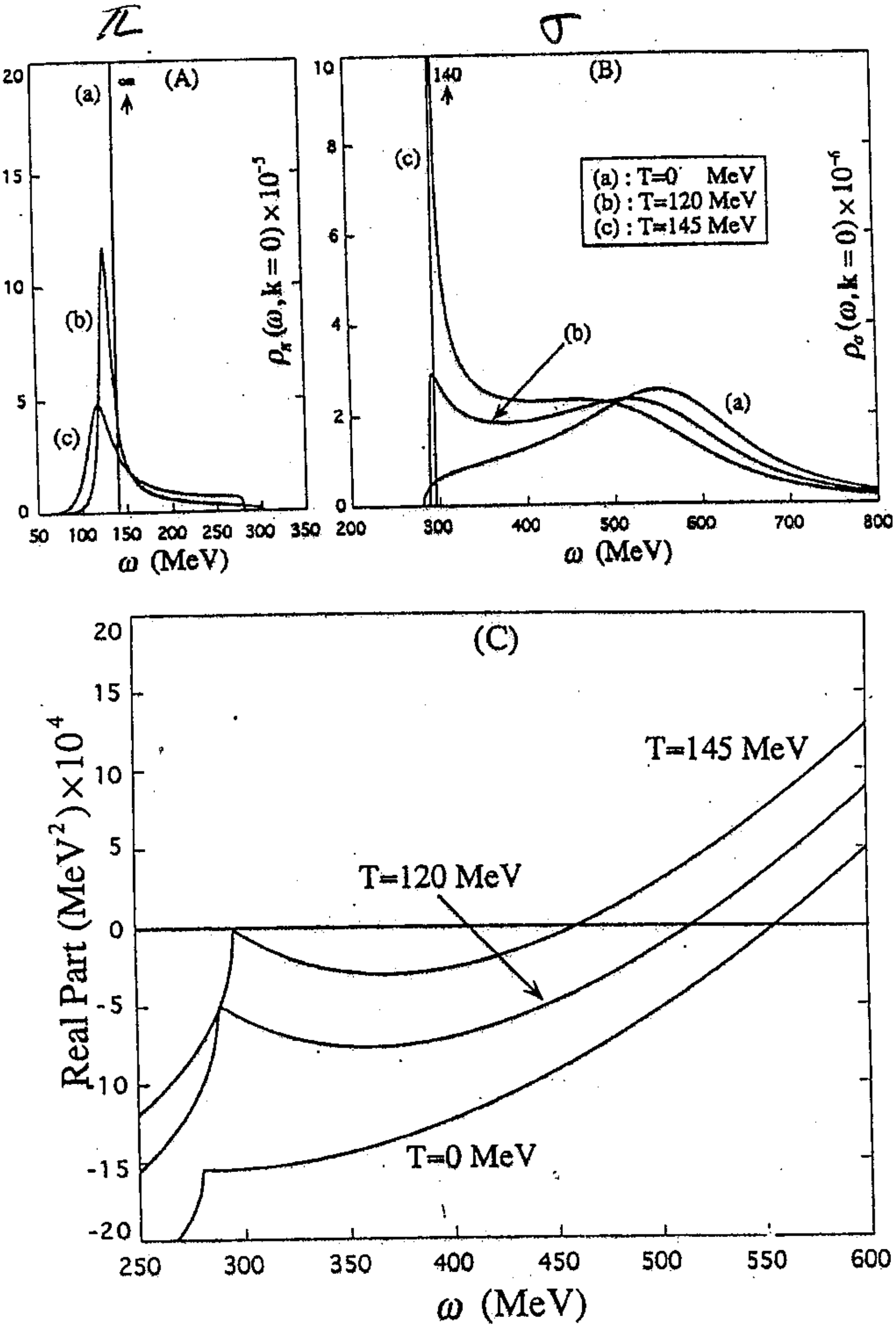
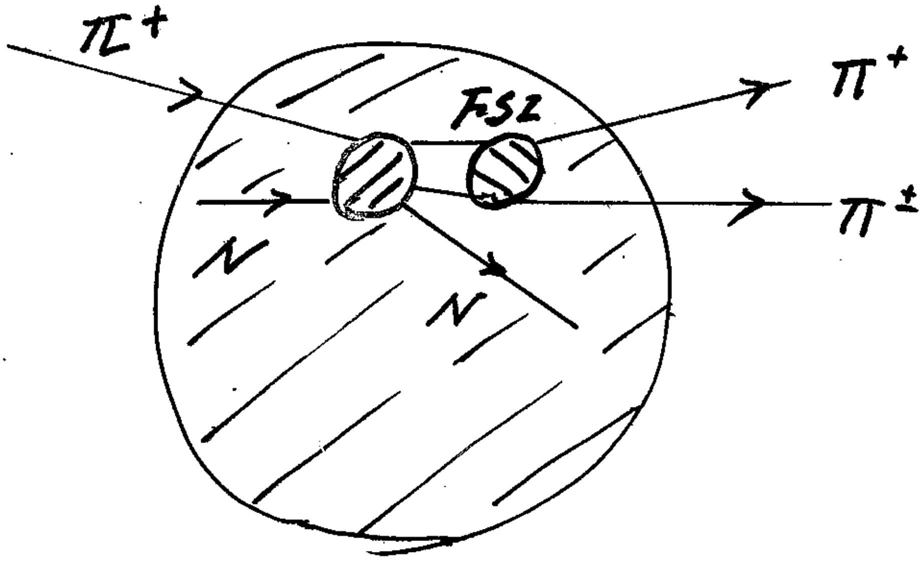


FIG. 3. Spectral function in π channel (A) and in σ channel (B) for $T=0$, 120, 145 MeV. The real part of $(D^R_\sigma(\omega,0;T))^{-1}$ as a function of ω is shown in (C).



$T_\pi^{in} = 283\,MeV$, $P_\pi^{in} = 399\,MeV$

$T_{\pi,av}^{out} = 35\,MeV$ $P_{\pi,av}^{out} = 105\,MeV$

$\Downarrow$

Close to $2m_\pi$ threshold

F. Bonutti et al.
Phys. Rev. Lett. 77 ('96) 603.

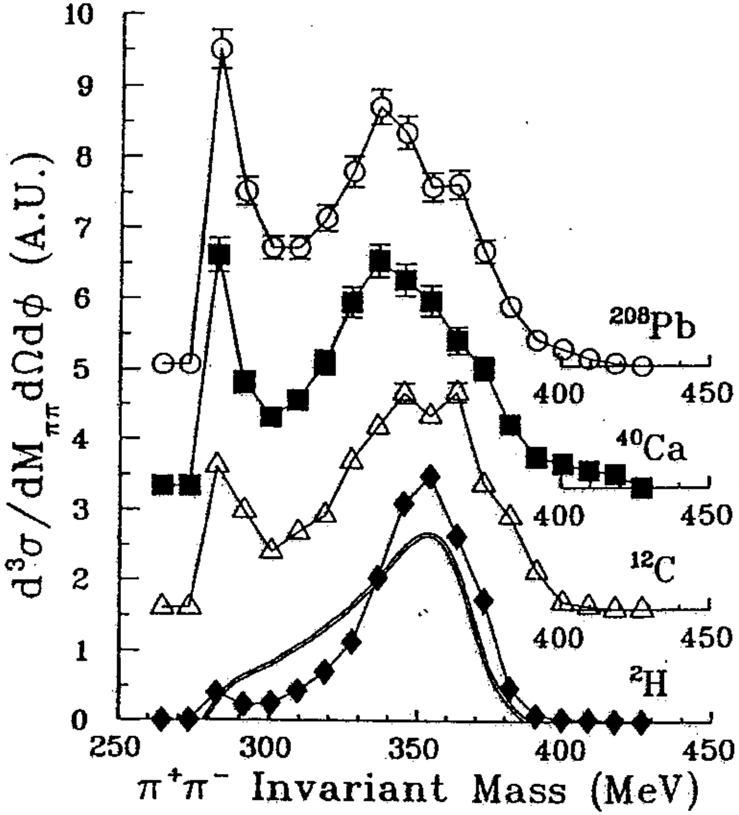
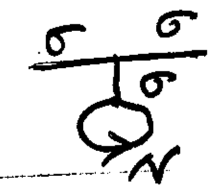
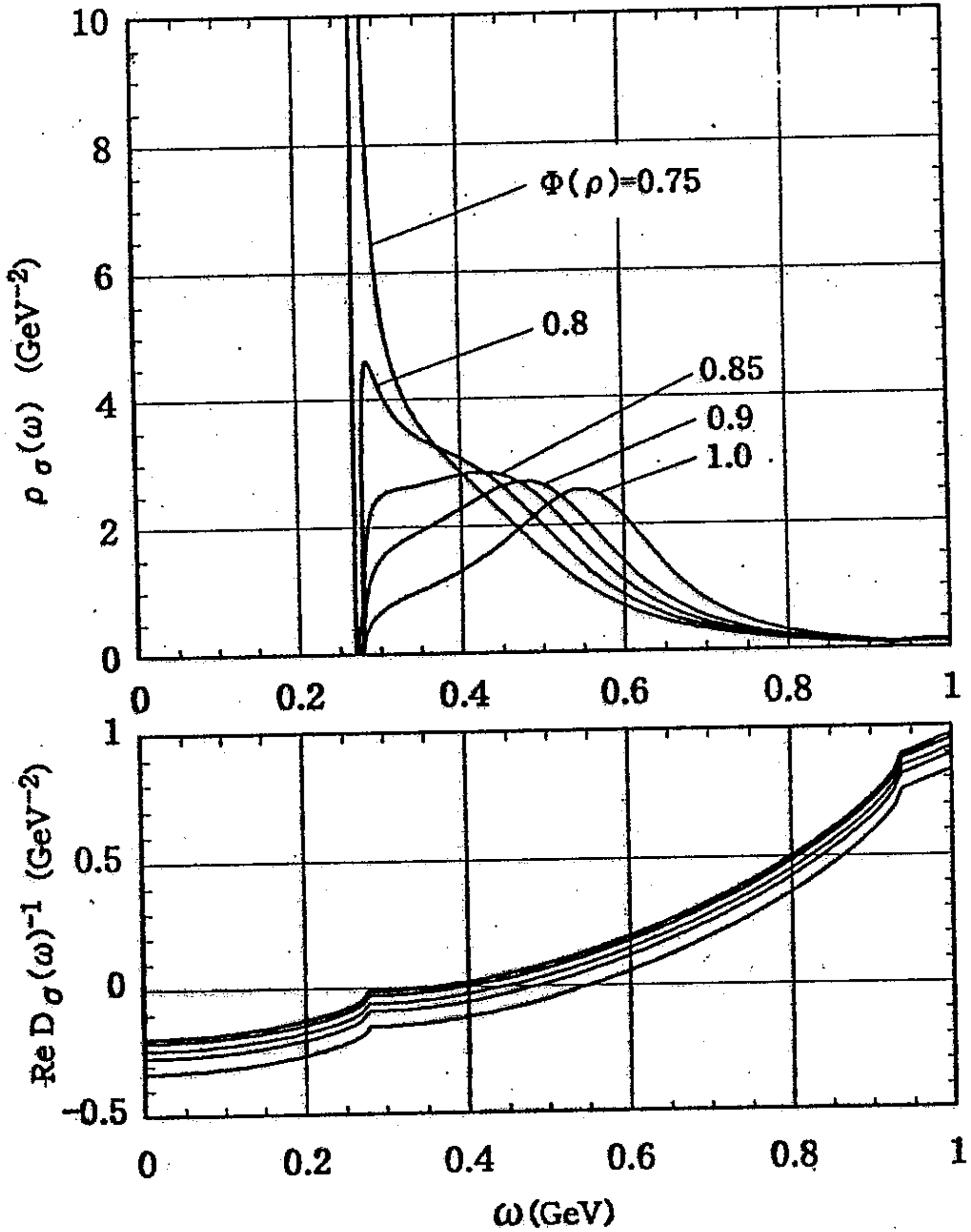


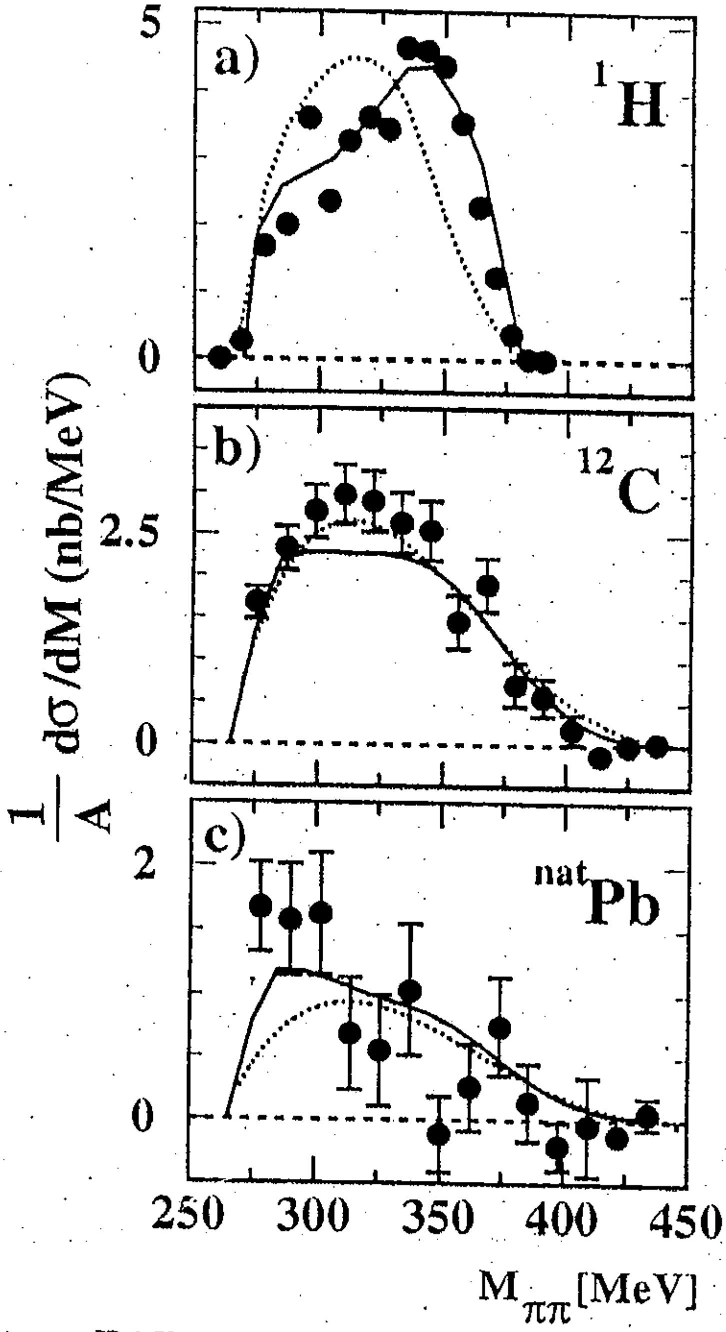
FIG. 1. Invariant mass distributions for the $(\pi^-, \pi^-\pi^-)$ reaction channel. The vertical scale is in arbitrary units. The curve is the prediction of an extended version of Oset's model for ^{2}H [2,9] normalized to the data. The thin lines drawn through the data points are to guide the eye.

T. Hatsuda, T.K. H. Shimizu,
Phys. Rev. Lett. 82 ('99) 2840



$\Downarrow$
m_σ^*





'. 1. Differential cross sections of the reaction $\cdot^{\circ}\pi^{\circ}$) with A=^{1}H,^{12}C,^{nat}Pb for incident photons in the ' range of 400-460 MeV (solid circles). Error bars de- tatistical uncertainties and the curves are explained in xt.

Kienle and T. Yamazaki ('01)

W. Weise, Acta Physica Polonica B31 ('00) 2715.

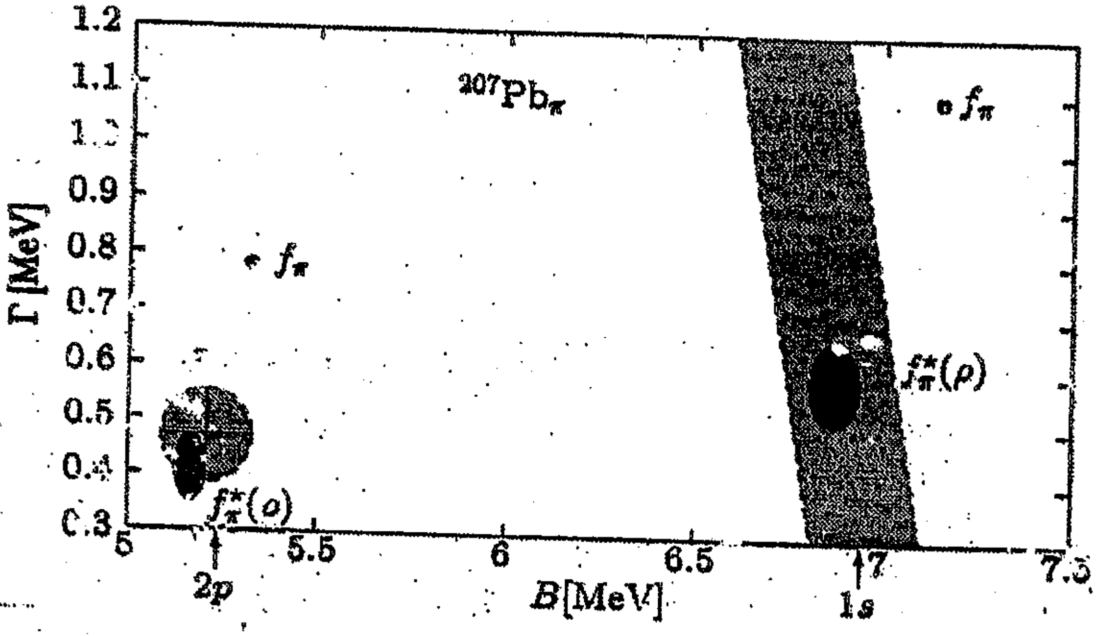


Fig. 2. Binding energy B and width Γ of 1s and 2p pionic atom states in ^{207}Pb. Points (f_π) are obtained [16] using the chiral s-wave optical potential (22, 23) with vacuum pion decay constant (f_π = 92.4 MeV) and ReB_0 = 0. Dark ellipses (f_π^*) are results [16] when replacing f_π by the in-medium decay constant (25) with σ_N = (45 ± 8) MeV. Light shaded areas: empirical range of B, Γ from Ref. [12].

Tomozawa-Weinberg. $T^{(+)}(\omega, q=0) = 0$, $T^{(-)}(\omega, q=0) = \frac{\omega}{2f_\pi^2}$

pion self-energy $\Pi = 2\omega U = -T(\pi^- p)\rho_p - T(\pi^- n)\rho_n$

$T^\pm = \frac{1}{2}\left(T(\pi^- p) \pm T(\pi^- n)\right)$

mass diff. $\Delta m(\pi^\pm) = U(\pi^\pm) = \pm \frac{\rho_p - \rho_n}{4 f_\pi^2}$

$\frac{\langle \bar qq\rangle_\rho}{\langle \bar qq\rangle_0} \simeq 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho$

$m_\pi^{*2} f_\pi^{*2} = -\frac{m_u + m_d}{2}\langle \bar uu + \bar dd\rangle_\rho$

$\to f_\pi^{*2}(\rho) = f_\pi^2 - \frac{\sigma_{\pi N}}{m_\pi^2}\rho$; $f_\pi^*(\rho_0) = .82 f_\pi$

$m_\pi^*(\rho) \simeq m_\pi$

Calculation of the strength function at $\rho_B \neq 0$

T. Hatsuda, T. K. and H. Shimizu,

Phys. Rev. Lett. **82**(1999), 2840.

Chiral restoration in the nuclear medium can lead to the required enhancement near the $2m_\pi$ threshold!†)

$\Updownarrow$

A state of the art calculation in the conventional reaction theory without chiral restoration fail in reproducing the sufficinet enhancement;

R. Rapp *et al.*, Phys. Rev. **C59**, R1237 (1999); M. J. Vicente-Vacas and E. Oset, Phys. Rev. **C60**, 064621 (1999).

†) Later confirmed by other groups; Z. Aouissat, G. Chanfray, P. Schuck and J. Wambach, Phys. Rev. **C61**, 012202 (2000); D. Davesne, Y. J. Zhang and G. Chanfray, Phys. Rev. **C62**, 024604(2000).

§Chiral restoration in nonlinear realization

D. Jido, T. Hatsuda and T. K., Phys. Rev. **D63** (2000), 011901(R).

showed that the nonlinear sigma models also give rise to the near $2m_\pi$ enhancement of the spectral function in nuclear medium:

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}\mathrm{Tr}[\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!}(MM^\dagger)^2 \\ &- h(M + M^\dagger)] + \bar{\psi}(i\partial\!\!\!/ - gM_5)\psi + \cdots, \qquad (1)\end{aligned}$$

where $M = \sigma + i\vec{\tau}\cdot\vec{\pi}$, $M_5 = \sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}$, ψ is the nucleon field, and Tr is for the flavor index.

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}[(\partial S)^2 - m_\sigma^{*2}S^2] - \frac{\lambda\langle\sigma\rangle}{6}S^3 - \frac{\lambda}{4!}S^4 \\ &+ \frac{(\langle\sigma\rangle + S)^2}{4}\mathrm{Tr}[\partial U \partial U^\dagger] + \frac{\langle\sigma\rangle + S}{4}\, h\ \mathrm{Tr}[U^\dagger + U] \\ &+ \mathcal{L}_{\pi N}^{(1)} - gS\bar{N}N\ , \qquad (2)\end{aligned}$$

with

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}(i\partial\!\!\!/ + i v\!\!\!/ + i a\!\!\!/\gamma_5 - m_N^*)N \qquad (3)$$

where $(v_\mu, a_\mu) = (\xi\partial_\mu\xi^\dagger \pm \xi^\dagger\partial_\mu\xi)/2$, and $m_N^* = g\langle\sigma\rangle$.

In heavy S-filed limit

$$\langle\sigma\rangle_0 = f_\pi, g/\lambda = \text{fixed}. \tag{4}$$

Integrating out the S field,

$$\begin{aligned}\mathcal{L} &= (f_\pi^2/4 - g f_\pi/2m_\sigma^2 \cdot \bar{N}N)(\mathrm{Tr}[\partial U \partial U^\dagger] - h/f_\pi \cdot \mathrm{Tr}[U + U^\dagger] \\ &\quad + \mathcal{L}_{\pi N}^{(1)}\end{aligned} \tag{5}$$

In nuclear matter;

$$\bar{N}N \to \langle \bar{N}N \rangle \simeq \rho$$

then,

$$\begin{aligned} f_\pi = \langle\sigma\rangle_0 \to \langle\sigma\rangle &= \langle\sigma\rangle_0 (1 - g\rho/f_\pi m_\sigma^2) \\ &\equiv f_\pi^* \end{aligned} \tag{6}$$

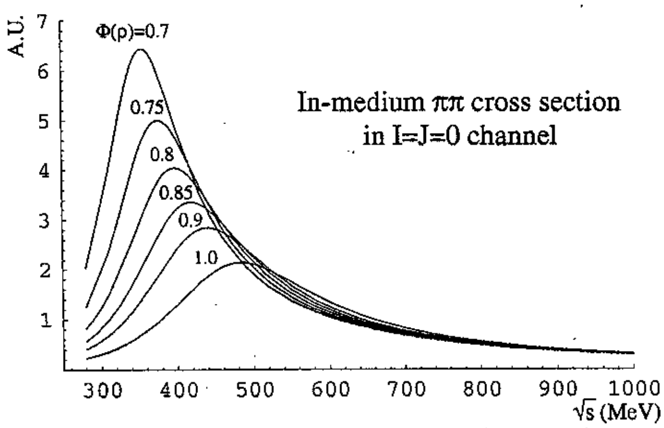
Thus, the properly normalized pion field;

$$\phi^* = (\phi/f_\pi) \cdot f_\pi^* \tag{7}$$

cf.

U.-G. Meissner, J. Oller and A. Wirzba et al, Ann. Phys. **297**(2002), 27

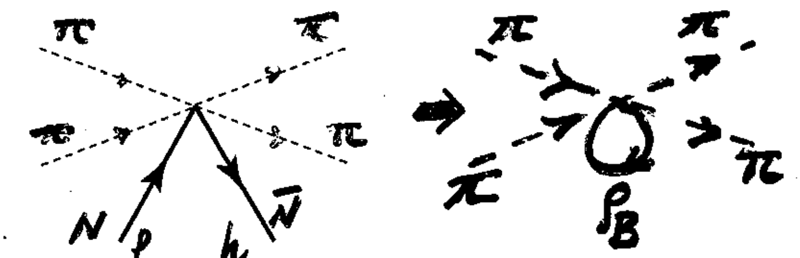
E. Kolomeitsev, N. Kaise and W. Weise, `nucl-th/0207090`.



D. JIdo et al, Phys. Rev. **D63**(2000)011901(R)

1. Although there is no explicit σ-degrees of freedom, there arises a decrease of the pion decay constant f_π^* in nuclear medium.

2. This is due to a new vertex, i.e., 4πN-N vertex absent in the free space and in the previous calculations of the $(\pi, 2\pi)$ reaction. The vertex is responsible for the reduction of f_π^* and hence for the spectral enhancement.



The new vertex in nuclear medium

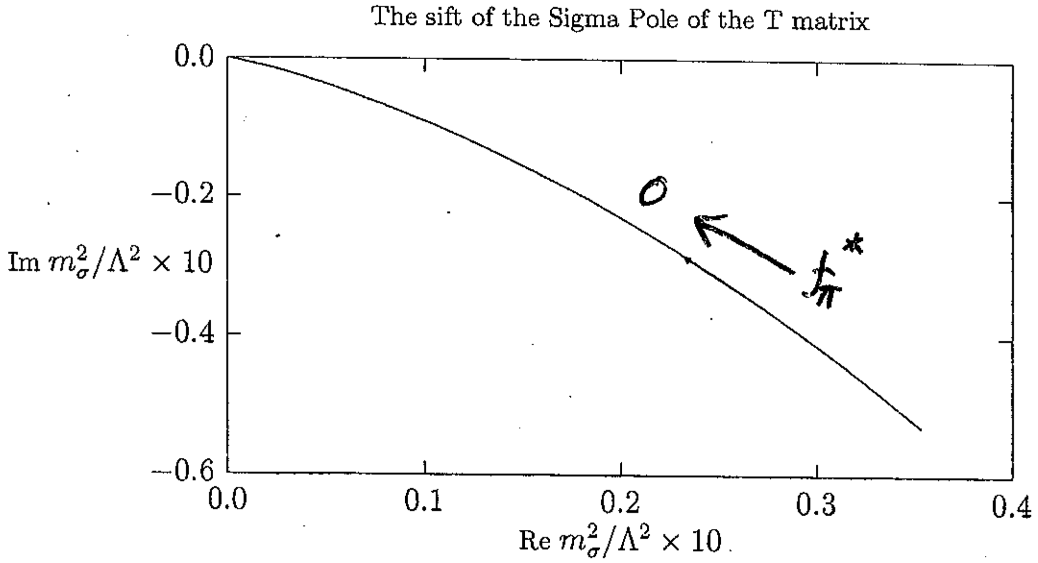


Figure caption:The figure shows the shift of the pole position of the T matrix in the chiral limit as f_π decreases toward 0.

The sigma meson mode is the softening mode with a decreasing width as the chiral restoration occurs!

§Summary

1. The σ meson as the quantum fluctuation of the order parameter of the chiral transition may account for various phenomena in hadron physics which otherwise remain misterious.

2. There have been accumulation of experimental evidence of the σ pole in the pi-pi scattering matrix. $\Leftarrow$ chiral symmetry, analyticity and crossing symmetry.

3. Partial restoration of chiral symmetry in hot and dense medium as represented by the decreasing f_π leads to a softening of the σ meson pole in the 2nd Riemann sheet even in the non-linear realization of chiral symmetry. $\leftarrow$ wave function renormalization $Z^*(P)$

4. Even a slight restoration of chiral symmetry in the hadronic matter leads to a peculiar enhancement in the spectral function in the σ channel near the $2m_\pi$ threshold.

5. Such an enhancement might have been observed in the reaction A(π^+, $(\pi^+\pi^-)_{I=J=0}$)A' by CHAOS group, and also in the reaction A(π^-, $(\pi^0\pi^0)_{I=J=0}$)A' by the CB collaboraion.

6. Further theoretical and experimental works are needed to confirm the above.

7. There are other possible evidences of the partial restoration in chiral symmetry in the hadronic matter.