Prog. Ther. Phys. **106**, 873 (2001)
Phys. Rev. C**66**, 045202 (2002)

Chiral Symmetry for Baryons

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Contents

International Symposium on
Hadron Spectroscopy, Chiral Symmetry and
Relativistic Description of Bound Systems
Nihon Daigaku Kaikan, Ichigaya, Tokyo, February 24-26, 2003

Introduction

Chiral symmetry is important in hadron physics.

- Its dynamical breaking gives rich physics in low energy

  e.g. Low energy theorem,

  Numbu-Goldstone bosons,

  perturbative expansion.... etc.

- symmetry for ***quark fields*** in QCD
- ***nontrivial for hadron fields***

Non-linear representation

- most general Lagrangian in the chirally broken phase
- NG bosons : special degrees of freedom
- perturbative expansion in terms of meson momenta
- dynamics of mesons and baryons

Linear representation

- at the chiral symmetric limit, hadrons can be classified into representations of chiral group
- group theoretical approach
- embody mechanism of spontaneous breaking

In this talk, we discuss baryon properties in the linear representations.

Chiral symmetry for Baryons in the linear realization

$\mathrm{SU(N_f)}_L \times \mathrm{SU(N_f)}_\mathrm{R}$

$$N_r \to RN_r \qquad N_l \to LN_l$$

$\mathrm{SU(N_f)}_L \in L, \qquad \mathrm{SU(N_f)}_\mathrm{R} \in R$

- Mass term breaks chiral symmetry

$$m_N \bar{N} N = m_N (\bar{N}_l N_r + \bar{N}_r N_l)$$

Generation of the baryon mass by scalar meson

$$m_N \bar{N} N \longrightarrow \sigma \bar{N} N \longrightarrow \bar{N}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) N$$

N_1 : positive parity $\quad N_2$: negative parity

Naive case

positive parity

$N_{1r} \to RN_{1r} \qquad N_{1l} \to LN_{1l}$

negative parity

$N_{2r} \to RN_{2r} \qquad N_{2l} \to LN_{2l}$

Chiral symmetry cannot mix N_1 and N_2.

Introduction of chiral partners

$$N \leftrightarrow N^*$$

$$m_N \bar{N} N \longrightarrow m_0(\bar{N} N + \bar{N}^* N^*)$$

$$N = \frac{1}{\sqrt{2}}(N_1 + \gamma_5 N_2) \qquad N^* = \frac{1}{\sqrt{2}}(\gamma_5 N_1 - N_2)$$

Mirror case

positive parity

$N_{1r} \to RN_{1r} \qquad N_{1l} \to LN_{1l}$

negative parity

$N_{2r} \to LN_{2r} \qquad N_{2l} \to RN_{2l}$

chiral invariant mass term

$$m_0(\bar{N}_2 \gamma_5 N_1 - \bar{N}_1 \gamma_5 N_2)$$

SU(2) x SU(2)

Jido *et. al.* hep-ph/9805306
NPA 671, 471 (2000)
PTP 106, 873 (2001)

the naive case

$$N_{1r} \to RN_{1r} \qquad N_{1l} \to LN_{1l}$$
$$N_{2r} \to RN_{2r} \qquad N_{2l} \to LN_{2l}$$

Linear σ model

no mass term at first

$$\begin{aligned}\mathcal{L}_{\text{nai}} &= \bar{N}_1 i\partial\!\!\!/ N_1 + \bar{N}_2 i\partial\!\!\!/ N_2 \\ &+a\bar{N}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})N_1 + b\bar{N}_2(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})N_2 \\ &+c\{\bar{N}_2(\gamma_5\sigma + i\vec{\tau}\cdot\vec{\pi})N_1 - \bar{N}_1(\gamma_5\sigma + i\vec{\tau}\cdot\vec{\pi})N_2\} + \mathcal{L}_M\end{aligned}$$

mass matrix

$$M \sim \sigma_0 \begin{pmatrix} a & -\gamma_5 c \\ \gamma_5 c & b \end{pmatrix}$$

π coupling matrix

$$C \sim \begin{pmatrix} a & -\gamma_5 c \\ \gamma_5 c & b \end{pmatrix} i\gamma_5\vec{\pi}\cdot\vec{\tau}\,.$$

Masses of N+ and N-

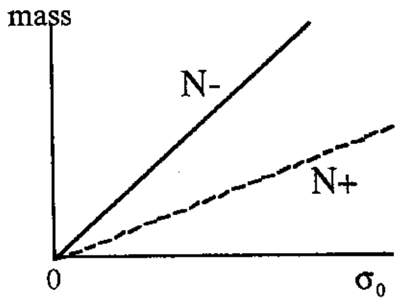
$$m_{\pm} = +\underbrace{\sigma_0}_{f_\pi} \left(\sqrt{\frac{(a+b)^2}{4} + c^2} \mp \frac{(a-b)}{2}\right)$$

$\sigma_0 = +f_\pi$

$$\begin{pmatrix} N_+ \\ N_- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

mixing angle : δ

$$\sinh\delta = -\frac{(a+b)}{2c}$$



π coupling

$M \propto C \;\Rightarrow$

$$g_{\pi N_+ N_-} = 0$$

$$g_{\pi N_+ N_+} = \frac{m_{N_+}}{f_\pi} \qquad g_{\pi N_- N_-} = \frac{m_{N_-}}{f_\pi}$$

$g_{\pi NN} \sim 13 \gg g_{\pi NN^*(1535)} \sim 1$

$g_A = 1$ a GT relation

This σ model is just the sum of two σ models for two independent nucleons.

$$\begin{aligned}\mathcal{L}_{\text{nai}} &= \bar{N}_+ i\partial\!\!\!/ N_+ - m_{N_+}\bar{N}_+N_+ + g_{\pi N_+N_+}\bar{N}_+ i\gamma_5\vec{\tau}\cdot\vec{\pi}N_+ \\ &+\bar{N}_- i\partial\!\!\!/ N_- - m_{N_-}\bar{N}_-N_- + g_{\pi N_-N_-}\bar{N}_- i\gamma_5\vec{\tau}\cdot\vec{\pi}N_-\end{aligned}$$

symmetry $SU(2)_V$ is an unbroken symmetry in the NG phase. Δ_M and N^*_m are obtained by the following isospin decomposition:

$$\psi^{A,a} = \sum_{M=-\frac{3}{2}}^{\frac{3}{2}} (T^A_{3/2})_{aM}\Delta_M + \sum_{m=-\frac{1}{2}}^{\frac{1}{2}} (T^A_{1/2})_{am}N^*_m \quad (1)$$

where the isospin projection matrix $T^A_{3/2}$ and $T^A_{1/2}$ are defined through the Clebsh-Gordon coefficients:

$$(T^A_{3/2})_{aM} = \sum_{r=1,0,-1} (1r\frac{1}{2}a|\frac{3}{2}M)U^{rA} \quad (2)$$

$$(T^A_{1/2})_{am} = \sum_{r=1,0,-1} (1r\frac{1}{2}a|\frac{1}{2}m)U^{rA} \quad (3)$$

and U is an unitary matrix transposing from $A = (1,2,3)$ basis to $r = (+1,0,-1)$ basis for the triplet index:

$$U^{rA} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & -i & 0 \end{pmatrix}. \quad (4)$$

In order to take into account of the chiral partners of Δ and N^*, we introduce two field, $\psi_1^{A,a}$ and $\psi_2^{A,a}$ which belong to $(1,\frac{1}{2})\oplus(\frac{1}{2},1)$ multiplet. Now in the ψ_1 and ψ_2 we have four particles Δ_+, Δ_-, N^*_+ and N^*_-, where the subscripts on Δ and N^* denote their parity. We take the mirror assignment for ψ_1 and ψ_2, that is, ψ_{1R} and ψ_{2L} belong to $(1,\frac{1}{2})$ and ψ_{1L} and ψ_{2R} do to $(\frac{1}{2},1)$. The corresponding infinitesimal transformations are generated by the chiral charges Q^A_R and Q^A_L of the $SU(2)_R \times SU(2)_L$ group with $(A = 1,2,3)$:

$$[Q^A_{R,L}, \psi^{B,a}_{1R,1L}] = i\epsilon^{ABC}\psi^{C,a}_{1R,1L} \quad , \quad (5a)$$

$$[Q^A_{L,R}, \psi^{B,a}_{1R,1L}] = -\frac{1}{2}(\tau^A)^a{}_b\psi^{B,b}_{1R,1L} \quad , \quad (5b)$$

$$[Q^A_{R,L}, \psi^{B,a}_{2R,2L}] = -\frac{1}{2}(\tau^A)^a{}_b\psi^{B,b}_{2R,2L} \quad , \quad (5c)$$

$$[Q^A_{L,R}, \psi^{B,a}_{2R,2L}] = i\epsilon^{ABC}\psi^{C,a}_{2R,2L} \quad , \quad (5d)$$

where τ^A $(A = 1,2,3)$ is Pauli matrix. The particles we are going to descrive here as a mixture of ψ_1 and ψ_2 $J = \frac{3}{2}$ resonances, $\Delta_+(P_{33})$, $\Delta_-(D_{33})$, $N^*_+(P_{13})$ and $N^*_-(D_{13})$. However, the same argument holds also to other multiplets with different spin.

The meson field $M \equiv \sigma + i\vec{\pi}\cdot\vec{\tau}$ belongs to the $(\frac{1}{2},\frac{1}{2})$ multiplet. Then the transformation rules are given by

$$[Q^A_{R,L}, \sigma] = \pm\frac{i}{2}\pi^A \quad , \quad (6a)$$

$$[Q^A_{R,L}, \pi^B] = \frac{i}{2}\left[\mp\delta^{AB} + \frac{i}{2}\epsilon^{ABC}\pi^C\right] \quad , \quad (6b)$$

Now let us consider the case The chiral transformations (5) and (6) as well as parity and time-reversal invariance makes severe constraints on the structure of the Lagrangian written by the fields ψ_1, ψ_2 and M:

$$\begin{aligned}\mathcal{L} = &(\text{Kinetic Terms}) + m_0(\bar\psi_1^A\psi_2^A + \bar\psi_2^A\psi_1^A) \\ &+a\bar\psi_1^A\tau^B(\sigma - i\vec{\pi}\cdot\vec{\tau}\gamma_5)\tau^A\psi_1^B \\ &+b\bar\psi_2^A\tau^B(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5)\tau^A\psi_2^B + \mathcal{L}_M \ ,\end{aligned} \quad (7)$$

where a, b and m_0 are free parameters independent of chiral symmetry. $\mathcal{L}_M$ is an invariant Lagrangian for mesons and its explicit form is irrelevant in the present argument as far as it causes the spontaneous breaking of chiral symmetry.

****** *More should be discussed here on the derivation of this lagrangian. What kind of conditions were used etc etc.* ******

This Lagrangian is an extension of the parity-doublet model for the nucleons proposed by DeTar and Kunihiro [7] to the $(1,\frac{1}{2})\oplus(\frac{1}{2},1)$ fields. The higher terms do not contribute to the tree level masses [12] and we will first study the phenomenological consequence of the tree masses. Also, at this stage, we assume that ψ_1 and ψ_2 have the same parity (we change it later???).

***** *Since we change parity later, why not change it from the beginning???* ******

In general, there are infinite numbers of invariant terms with many M's and derivatives.

With the invariant Lagrangian (7), we shall next consider the masses of Δs and N^*s. A finite condensation of sigma with a certain condition on $\mathcal{L}_m$ breaks spontaneously the $SU(2)_R \times SU(2)_L$ chiral symmetry to the isospin $SU(2)_V$ symmetry. Redefining the sigma field as $\sigma = \langle\sigma\rangle + \tilde\sigma$, we obtain the mass terms for Δs and N^*s:

$$\begin{aligned}\mathcal{L}_m = &(\bar\Delta_1, \bar\Delta_2)\begin{pmatrix} 2a\sigma_0 & m_0 \\ m_0 & 2b\sigma_0 \end{pmatrix}\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \\ &+(\bar N_1^*, \bar N_2^*)\begin{pmatrix} -a\sigma_0 & m_0 \\ m_0 & -b\sigma_0 \end{pmatrix}\begin{pmatrix} N_1^* \\ N_2^* \end{pmatrix} \ ,\end{aligned} \quad (8)$$

where $\sigma_0 \equiv \langle\sigma\rangle$. Since the mass terms for Δs and N^*s have off-diagonal components, we have to diagonalize these mass matrices to get the physical basis. Then the masses of Δs and N^*s are given by the eigenvalues of the mass matrices, which are $-\sigma_0(a+b) \pm \sqrt{(a-b)^2\sigma_0^2 + m_0^2}$ for Δs and $\frac{\sigma_0}{2}(a+b) \pm \frac{1}{2}\sqrt{(a-b)^2\sigma_0^2 + 4m_0^2}$ for N^*s. These eigenvalues could be negative, for example, $\pm m_0$ with $\sigma_0 = 0$. However, at the restoration limit of chiral symmetry, i.e. $\sigma_0 = 0$, the chiral partners should be degenerate with the same mass and one of the partners should have odd parity. Therefore the negative masses are interpreted as *positive* masses for the particles with odd parity. Then we redefine the fields correspond to the lower eigenvalue to fields with the odd parity multiplying γ_5. Finally the physical basis (Δ_+, Δ_-) and (N^*_+, N^*_-) are given by

$$\begin{pmatrix} \Delta_+ \\ \Delta_- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\xi}}\begin{pmatrix} e^{-\xi/2} & -e^{\xi/2} \\ -\gamma_5 e^{\xi/2} & -\gamma_5 e^{-\xi/2} \end{pmatrix}\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix}$$

$$\begin{pmatrix} N^*_+ \\ N^*_- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\eta}}\begin{pmatrix} e^{-\eta/2} & -e^{\eta/2} \\ -\gamma_5 e^{\eta/2} & -\gamma_5 e^{-\eta/2} \end{pmatrix}\begin{pmatrix} N^*_1 \\ N^*_2 \end{pmatrix}$$

the mirror case

$$N_{1r} \to RN_{1r} \qquad N_{1l} \to LN_{1l}$$
$$N_{2r} \to LN_{2r} \qquad N_{2l} \to RN_{2l}$$

Linear σ model (DeTar - Kunihiro)

$$\begin{aligned}\mathcal{L}_{\rm DK} &= \bar{N}_1 i\partial\!\!\!/ N_1 + \bar{N}_2 i\partial\!\!\!/ N_2 + m_0(\bar{N}_2\gamma_5 N_1 - \bar{N}_1\gamma_5 N_2) \\ &\quad + a\bar{N}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})N_1 + b\bar{N}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})N_2 + \mathcal{L}_M\end{aligned}$$

mass matrix

$$M \sim \begin{pmatrix} a\sigma_0 & -\gamma_5 m_0 \\ \gamma_5 m_0 & b\sigma_0 \end{pmatrix}$$

π coupling matrix

$$C \sim \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} i\gamma_5\vec{\pi}\cdot\vec{\tau}.$$

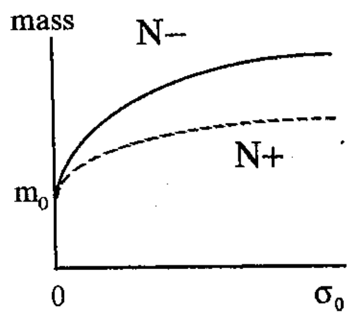
Masses of $N+$ and $N-$

$$m_\pm = \sqrt{\frac{(a+b)^2}{4}\sigma_0^2 + m_0^2} \mp \frac{(a-b)}{2}\sigma_0$$

$$\begin{pmatrix} N_+ \\ N_- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}}\begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

mixing angle : δ

$$\sinh\delta = -\frac{(a+b)\sigma_0}{2m_0}$$



π coupling

$$g_{\pi N_+ N_+} = +\frac{m_{N_+}}{f_\pi}\tanh\delta$$

$$g_{\pi N_- N_-} = -\frac{m_{N_-}}{f_\pi}\tanh\delta$$

$$g_A^{N_\pm} = \pm\tanh\delta$$

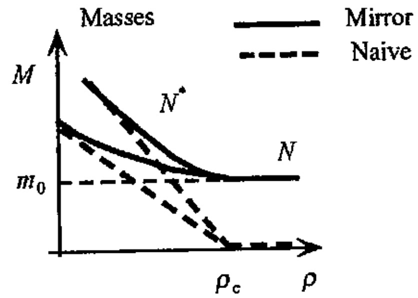
$N_- \Leftarrow N(1535)\ S_{11}\ \frac{1}{2}^-$

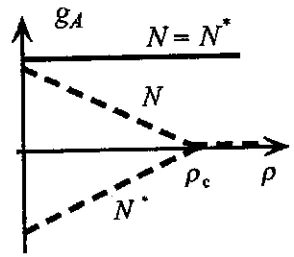
[$\Gamma_{N_- \to \pi N_+} \sim 70$ MeV

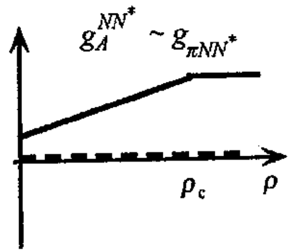
$m_{N_+} = 940$ MeV , $m_{N_-} = 1535$ MeV , $g_{\pi NN^*} = 0.7$

$m_0 = 270$ MeV

	Naive case	Mirror case
Definition	positive parity $N_{1r} \to RN_{1r}$ $N_{1l} \to LN_{1l}$ negative parity $N_{2r} \to RN_{2r}$ $N_{2l} \to LN_{2l}$	positive parity $N_{1r} \to RN_{1r}$ $N_{1l} \to LN_{1l}$ negative parity $N_{2r} \to LN_{2r}$ $N_{2l} \to RN_{2l}$
Mass term	Generated by scalar meson	Introduced with chiral partner
Mass in Wigner phase	massless	massive
Role of σ_0	mass generation	mass splitting
Chiral partner	$N \longleftrightarrow \gamma_5 N$, $N^* \longleftrightarrow \gamma_5 N^*$	$N \longleftrightarrow N^*$
Sign of pN*N* coupling	positive	negative
πNN* coupling in medium	suppressed	enhanced







Observation of sign of πN^*N^* coupling

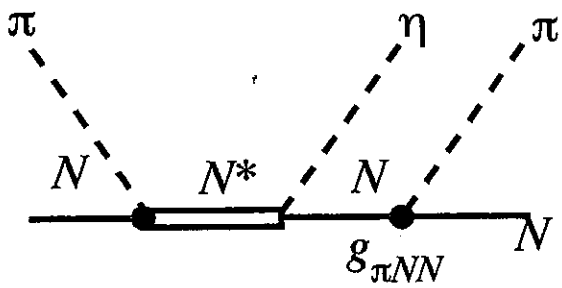
$$\pi N \xrightarrow{N^*} \eta\pi N$$

N^*: S_{11} N(1535) I=1/2 $J^p = 1/2^-$

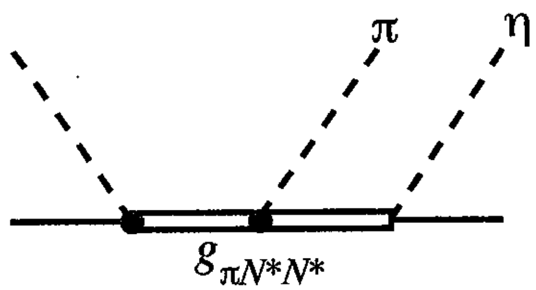
$$N^* \longrightarrow \begin{array}{ll} \pi N & \sim 50\% \\ \eta N & \sim 50\% \end{array}$$

- η meson as a probe of N(1535)
- see just above the threshold

(1)

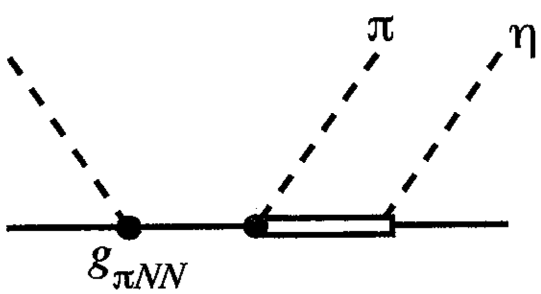


(2)



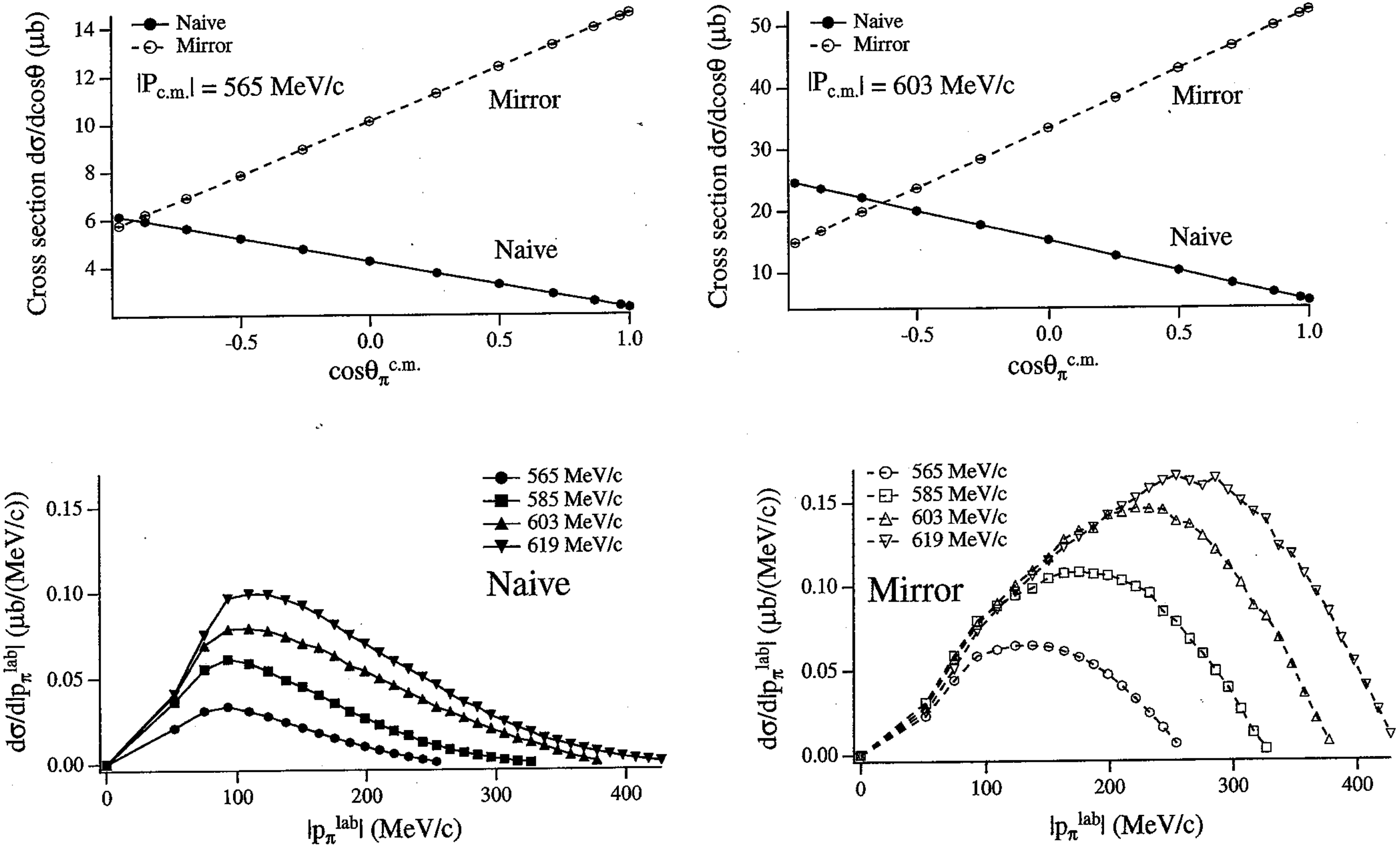
S wave P wave

(3)

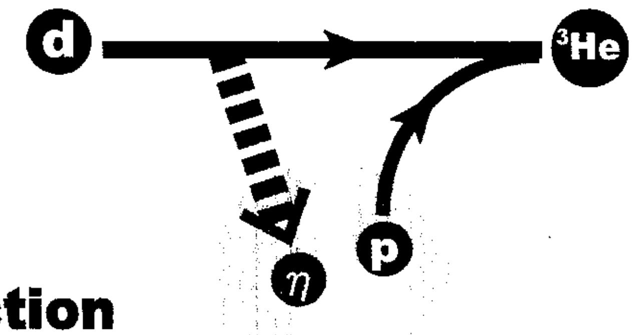


P wave S wave

$\pi^- p \rightarrow \eta \pi^- p$



η mesic nuclei



GSI

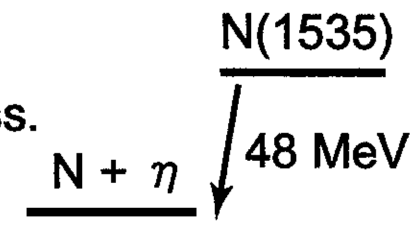
- **recoilless (d,^{3}He) reaction**
- **proton pick-up process**
- **small momentum transfer**

$q_\eta \sim 0$ MeV/c at $T_d = 3.5$ GeV

Special features of the η mesic nuclei

1) The η-N system dominantly couples to N(1535) at the threshold region.
2) The isoscalar particle, η, filters out contaminations of the isospin-3/2 excitations.
3) There is no threshold suppression like the p-wave coupling, as a result of the s-wave nature of the η NN* coupling,

The η optical potential may be sensitive to the medium modification of the N* mass.



Therefore,

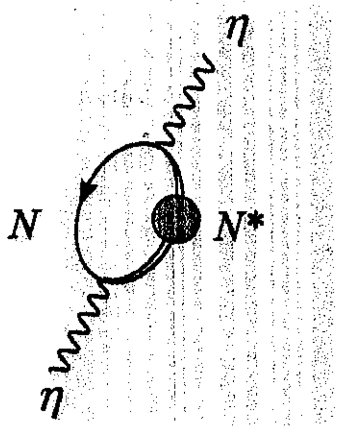
N(1535) plays an important role in the η mesic nuclei.

→ Medium effects to the N(1535)

Optical potential of η in nucleus

N* dominance

s-wave coupling for η NN*



$$g_\eta \sim 2.0 \quad \Longleftarrow \quad \Gamma_{N^* \to \eta N} \simeq 75 \text{ MeV}$$

$$V_\eta(\omega) = \frac{g_\eta^2}{2\mu} \frac{\rho(r)}{\omega + m_N^*(\rho) - m_{N^*}^*(\rho) + i\Gamma_{N^*}(s;\rho)/2}$$

$a_{\eta N} = 0.24 + i\,0.38$ fm

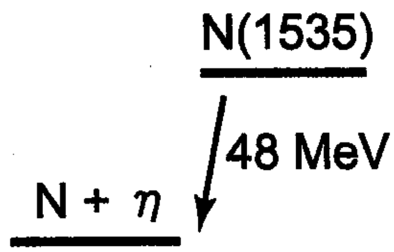
weakly bound : $\omega \simeq m_\eta$

- no medium modification for the masses of N and N*

$$V_\eta \sim \rho T_{\eta N}$$

$$m_\eta + m_N - m_{N^*} < 0$$

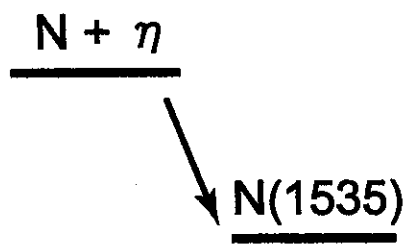
Attractive

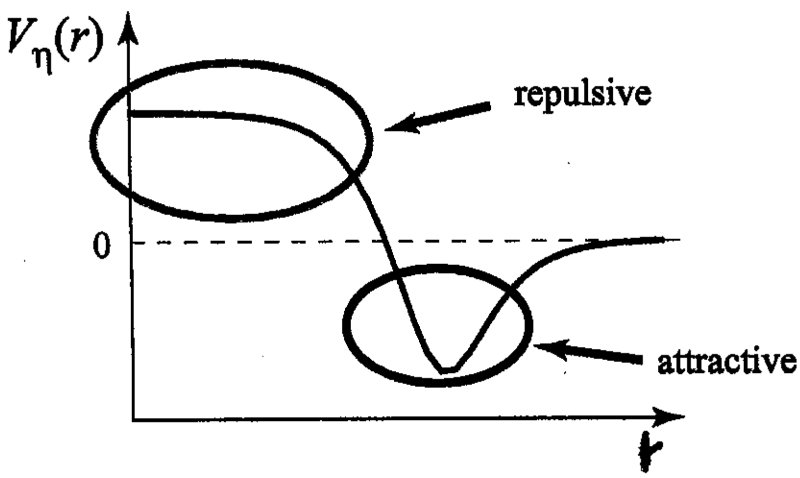


- reduction of the mass difference of N and N*

$$m_\eta + m_N^*(\rho) - m_{N^*}^*(\rho) > 0$$

Repulsive





Optical potential of η in nucleus

Chiral Double model
N* dominance

C: strength of chiral restoration in nucleus

$$\langle\sigma\rangle = \Phi(\rho)\langle\sigma\rangle_0$$

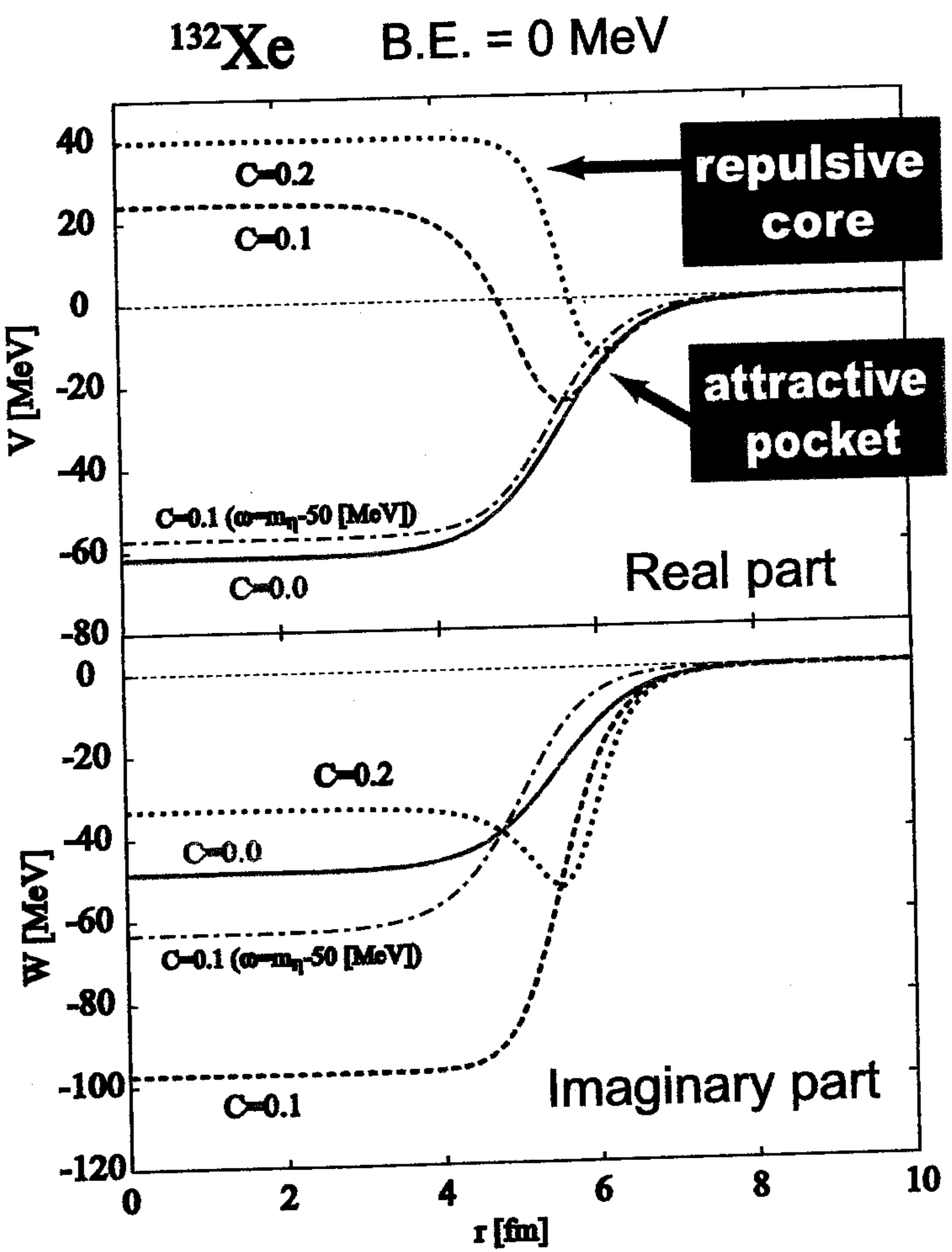
$$\Phi(\rho) = 1 - C\rho/\rho_0 \quad (C = 0.1 \sim 0.3)$$

Local density approximation
Fermi distribution of nuclear density

$$\rho(r) = \frac{\rho_0}{1+\exp[(r-R)/a]}$$

$$R = 1.18A^{1/3} - 0.48\ \mathrm{fm}, \quad a = 0.5\mathrm{fm}$$

'pocket potential'
energy dependence



Summary

Investigate baryon properties in the aspect of chiral symmetry

Two ways of chiral assignment (Naive and Mirror)
- different consequences
  mass at sym. limit,
  sign of πN^*N^* coupling, etc.

Observation of the sign of πN^*N^* coupling

$$\pi N \text{ (or } \gamma N) \to \pi\eta N$$

As a general conclusion,
a sufficient reduction of the mass difference of N and N* makes the potential turn to be repulsive.

of η
Pocket potential

Spectra of the recoilless (d, ^{3}He) reactions are useful to investigate the medium effect of N*.