
**LIGHT SCALAR MESONS
AND
CHARM DECAYS**

**I. BEDIAGA, CARLA GÖBEL
J. M. DE MIRANDA, A. C. DOS REIS**

**Centro Brasileiro de Pesquisas Físicas
(CBPF)**

Rio de Janeiro, Brazil

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The Amplitude Analysis in Charm Decays

- **3-body decays: Dalitz plot fit deals directly with the physical amplitudes on the (only) two independent dynamical variables (e.g. s_{12} , s_{13})**
- **very sensitive to the interfering processes** PHASES
- **Isobar Model for individual amplitudes, with Zemach Formalism**

$$\mathcal{A}_i = F_D \times F_{R_i} \times \mathcal{M}_i^J \times BW_i$$

- **All contributions added coherently for the total amplitude:**

$$\mathcal{A} = a_{nr} e^{i\delta_{nr}} + \sum_j a_j e^{i\delta_j} \mathcal{A}_j$$

- **The phases δ_j accommodate Final State Interactions (FSI) allowed in 3-body D decays.**

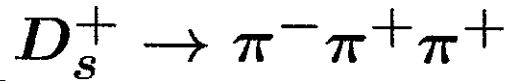
E791 DALITZ PLOT ANALYSES

MEASUREMENTS OF MASSES AND WIDTHS OF SCALAR STATES

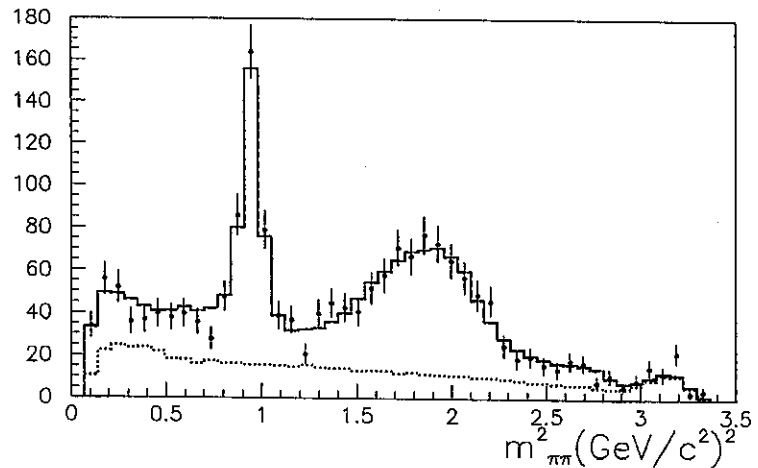
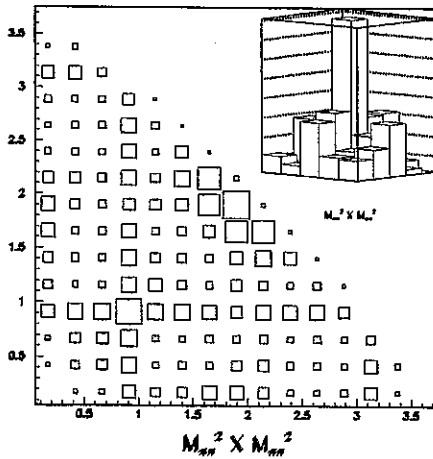
$\Rightarrow D_s^+ \rightarrow \pi^- \pi^+ \pi^+$: masses and widths for
 $f_0(980)$ and $f_0(1370)$
PRL 86, 765 (2001)

$\Rightarrow D^+ \rightarrow \pi^- \pi^+ \pi^+$: evidence for the σ
mass and width measured
PRL 86, 770 (2001)

$\Rightarrow D^+ \rightarrow K^- \pi^+ \pi^+$: evidence for the κ
mass and width measured
PRL 89, 121801 (2002)



Measurement of f_0 masses and widths



- **Coupled channel Breit-Wigner for $f_0(980)$:**

$$m_0 = (977 \pm 3 \pm 2) \text{ MeV}$$

$$g_\pi = 0.09 \pm 0.01 \pm 0.01$$

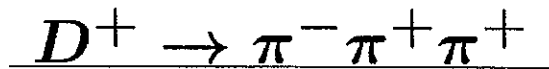
$$g_K = 0.02 \pm 0.04 \pm 0.03 \Rightarrow \text{small coupling to } KK$$

- $f_0(980)$ appears narrow: $\Gamma_0 \sim 45 \text{ MeV}$

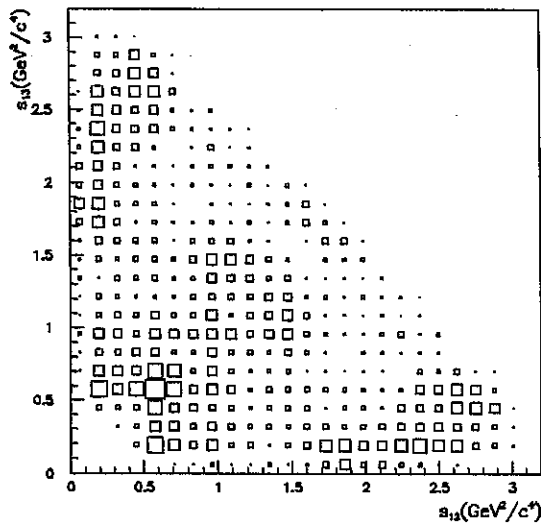
- $f_0(1370)$: $m_0 = (1434 \pm 18 \pm 9) \text{ MeV}$

$$\Gamma_0 = (172 \pm 32 \pm 6) \text{ MeV}$$

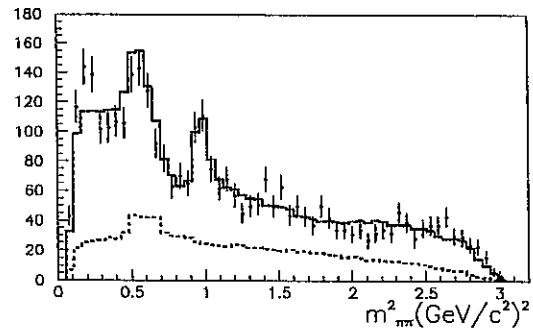
- **scalars give the main contribution to the decay (over 90%)**



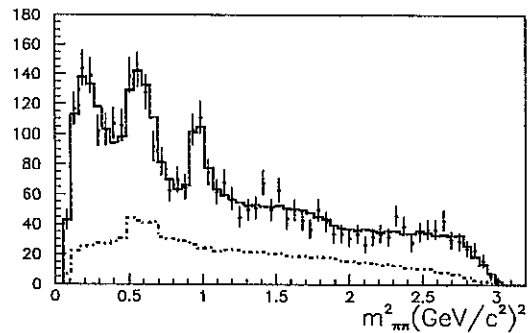
Evidence for a light, broad σ



Without σ



With σ



- In the fit without σ the non-resonant contribution is dominant ($\sim 40\%$), with a bad fit quality
- Including σ the fit quality is good, and $\sigma\pi^+$ is responsible for 50 % of the decay rate

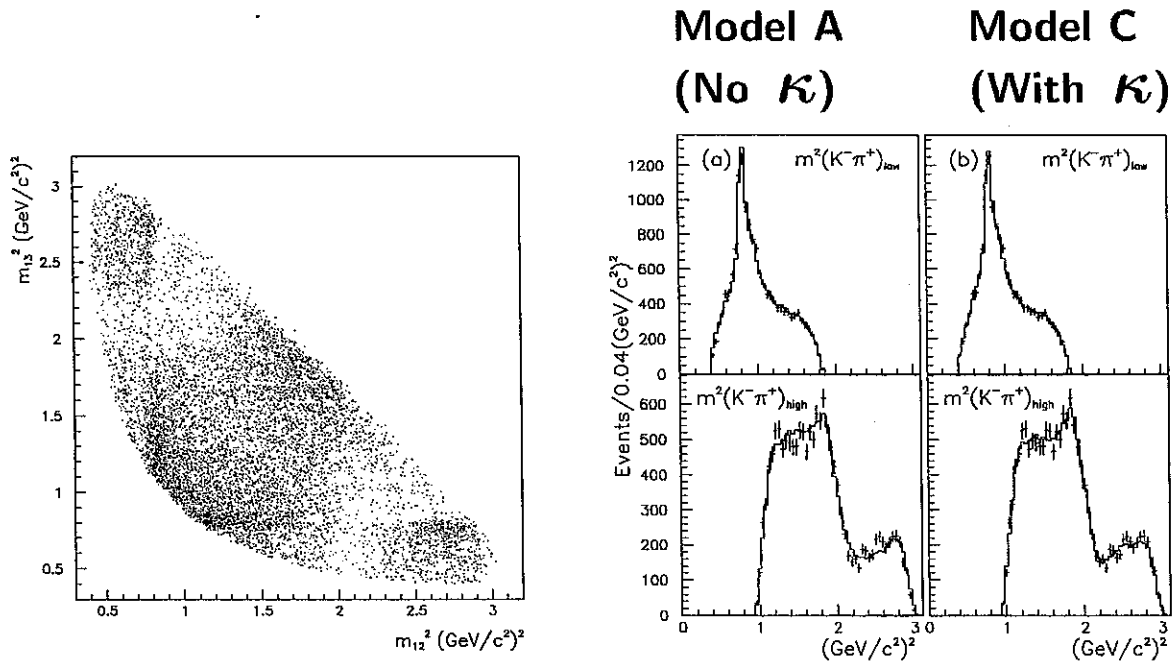
Standard Breit-Wigner parametrization:

$$M_\sigma = 478_{-23}^{+24} \pm 17 \text{ MeV}/c^2$$

$$\Gamma_\sigma = 324_{-40}^{+42} \pm 21 \text{ MeV}/c^2$$



Evidence for a light, broad κ



- In Model A the NR contribution is dominant ($\sim 90\%$, unusual in D), with a bad fit quality
- Including κ in Model C, the fit quality is much superior, $\kappa\pi^+$ has 50 % of the decay rate

Breit-Wigner parametrizations:

$$M_{\kappa} = 797 \pm 19 \pm 43 \text{ MeV}$$

$$\Gamma_{\kappa} = 410 \pm 43 \pm 87 \text{ MeV}$$

$$M_{K_0^*(1430)} = 1459 \pm 7 \pm 6 \text{ MeV}$$

$$\Gamma_{K_0^*(1430)} = 175 \pm 12 \pm 12 \text{ MeV}$$

Main Conclusions on σ and κ from E791

- ★ Their inclusion improves very much the description of the decays considered
 - ★ The NR level becomes consistent with other 3-body D decays
- ★ Vector, tensor or “toy model” hypotheses do not describe the data as well as the scalar hypothesis
- ★ For $D^+ \rightarrow K^- \pi^+ \pi^+$ a number of studies were done to check whether different shapes for the NR decay could explain the data

Can the data from these decays give extra information on the phase behavior for the scalar amplitudes?

⇒ Phase Motion in $D^+ \rightarrow \pi^- \pi^+ \pi^+$

⇒ Additional studies from $D^+ \rightarrow K^- \pi^+ \pi^+$

Preliminary, unpublished studies with E791 Data

We acknowledge the E791 Collaboration
for the use of the data

Phase Motion in $D^+ \rightarrow \pi^- \pi^+ \pi^+$

E791 finds evidence for a σ at low $\pi\pi$ mass using a standard Breit-Wigner parametrization

Can the phase motion of such state be measured without assuming the Breit-Wigner shape?

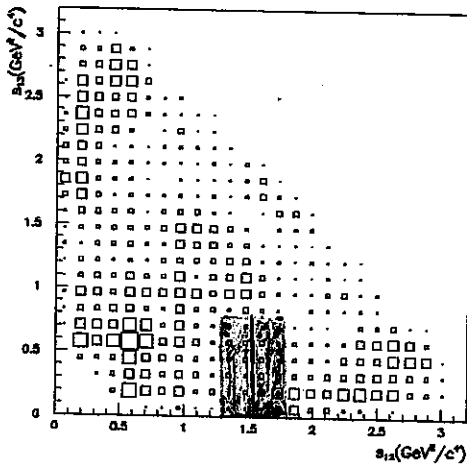
**Amplitude Difference
Method (AD) :**

**Bediaga & Miranda
Phys. Lett. B 550, 135 (2002)**

Main Idea:

Measure the phase motion of an under study state in s_{13} through its interference with a well established resonance in the crossed channel s_{12}

CASE UNDER STUDY: Effect of σ on $f_2(1270)$



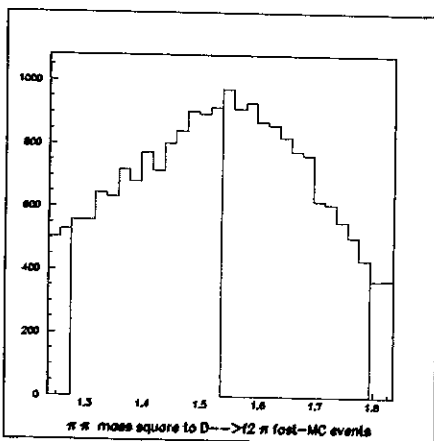
- $f_2(1270)\pi^+$ has large decay fraction ($\sim 20\%$)
- f_2 mass is at node of ρ in the crossed channel
- f_2 is broad \rightarrow wide interference range

$\Rightarrow f_2(1270)$ appearing at s_{12}

\Rightarrow complex amplitude to be studied at s_{13}

$$\mathcal{A}(s_{12}, s_{13}) = a_{f_2} BW_{f_2}(s_{12}) \mathcal{M}_{f_2}(s_{12}, s_{13}) + a_{\sigma} \sin \delta(s_{13}) e^{i(\delta(s_{13}) + \gamma)}$$

The s_{12} projection for $f_2(1270)$ (by fast MC)



find m_{eff} for which the number of events are the same in both windows
 $m_{\text{eff}}^2 = 1.535 \text{ GeV}^2$, $\epsilon = 0.24 \text{ GeV}^2$

$$\int_{m_{\text{eff}} - \epsilon}^{m_{\text{eff}}} |\mathcal{A}_{f_2}|^2 ds_{12} = \int_{m_{\text{eff}}}^{m_{\text{eff}} + \epsilon} |\mathcal{A}_{f_2}|^2 ds_{12}$$

Study the difference:

$$\Delta \int |A|^2 \equiv \int_{m_{\text{eff}} - \epsilon}^{m_{\text{eff}}} |\mathcal{A}|^2 ds_{12} - \int_{m_{\text{eff}}}^{m_{\text{eff}} + \epsilon} |\mathcal{A}|^2 ds_{12}$$

in slices of s_{13}

After some algebra one gets:

$$\Delta \int |A|^2 \sim C [\sin(2\delta(s_{13}) + \gamma) - \sin \gamma] \bar{\mathcal{M}}_{f_2}(s_{13})$$

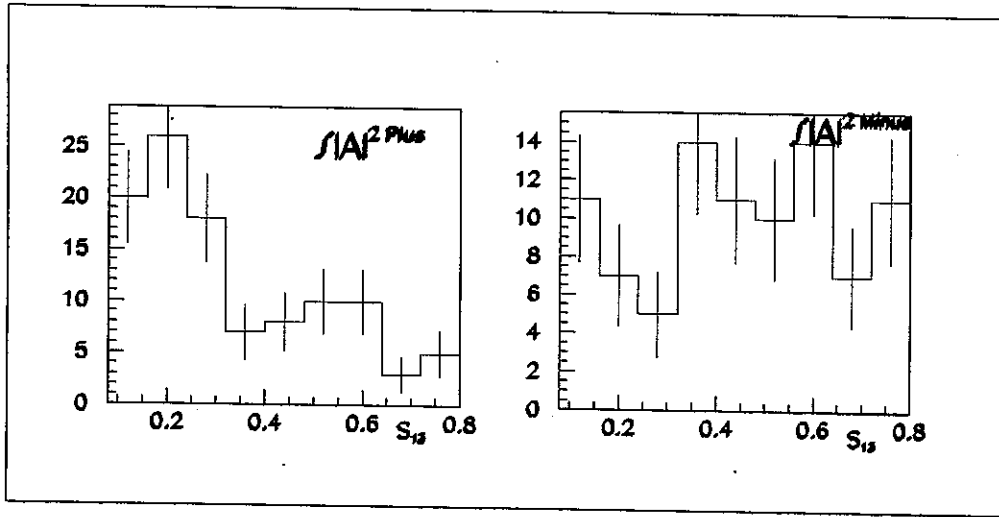
▷ $\bar{\mathcal{M}}_{f_2}(s_{13})$ is a known function and can be deconvoluted. Define :

$$\mathcal{F}(s_{13}) = \Delta \int |A|^2 / \bar{\mathcal{M}}_{f_2}(s_{13})$$

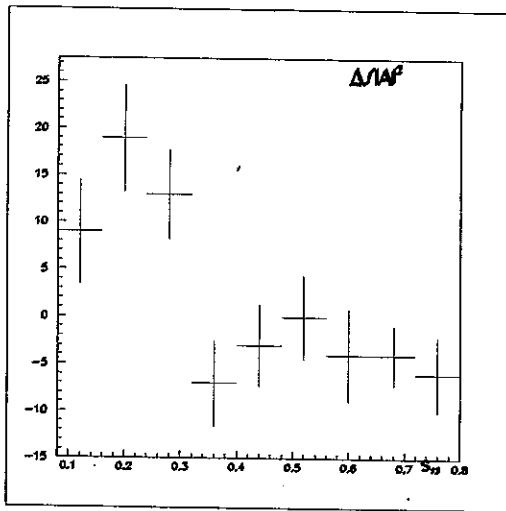
▷ Any departure of $\mathcal{F}(s_{13})$ from a constant is indication of phase variation (γ is constant)

- **slow phase variation** → slowly varying $\mathcal{F}(s_{13})$
- **resonance behavior** should produce zero, maximum, minimum values of $\mathcal{F}(s_{13})$

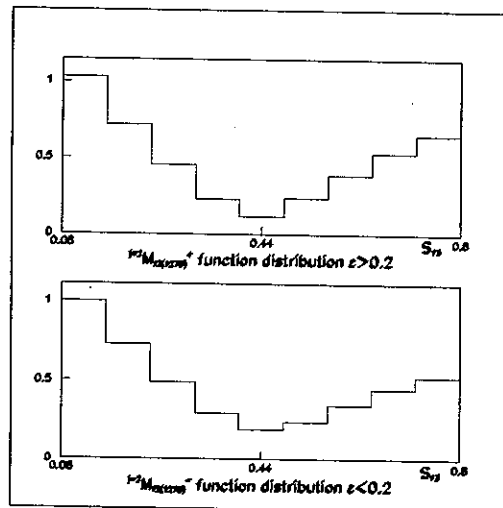
FROM E791 DATA



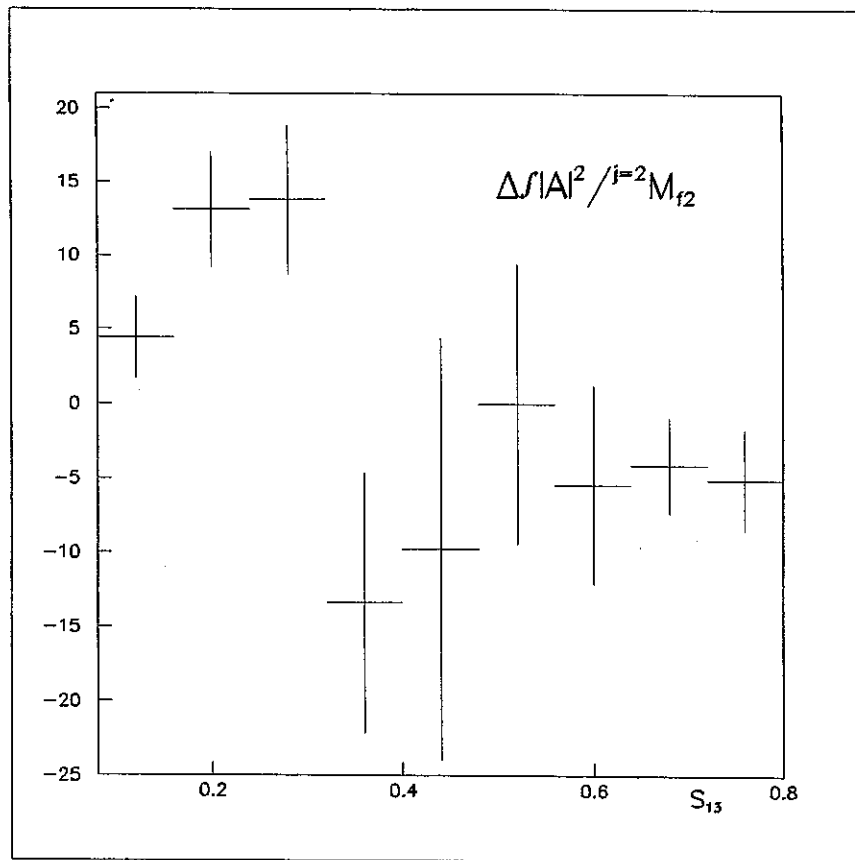
Subtracting and dividing...



\div



$\mathcal{F}(s_{13})$ Plot



★ \mathcal{F} starts at ~ 0 , ends at ~ 0

★ Significant difference between maximum and minimum values at $s_{13} \sim 0.3$

Consistency considerations : $\delta(s_{13})$ crescent, continuous starting from $\sim 0^\circ$

Thus, between maximum and minimum:

$$\Delta(2\delta + \gamma) \sim 180^\circ \rightarrow \Delta\delta \sim 90^\circ$$

How to get the Phase Motion

Conditions of maximum and minimum

$$\mathcal{F}_{\max} \leftrightarrow \sin(2\delta(s_{13}) + \gamma) = 1$$

$$\mathcal{F}_{\min} \leftrightarrow \sin(2\delta(s_{13}) + \gamma) = -1$$

By replacing in the equation for \mathcal{F}

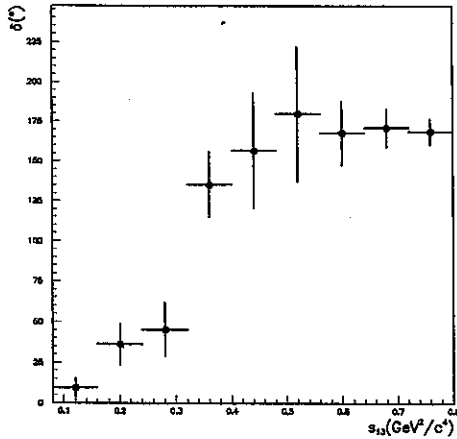
$$C = (\mathcal{F}_{\max} - \mathcal{F}_{\min})/2$$

$$\gamma = \arcsin\left(\frac{\mathcal{F}_{\min} + \mathcal{F}_{\max}}{\mathcal{F}_{\min} - \mathcal{F}_{\max}}\right)$$

And

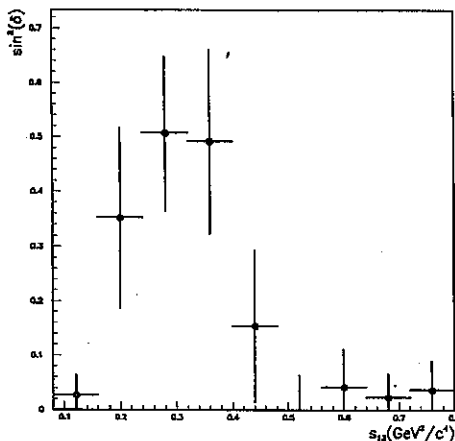
$$\delta(s_{13}) = \frac{1}{2} \left[\arcsin\left(\frac{1}{C} \mathcal{F}(s_{13}) + \sin \gamma\right) - \gamma \right]$$

$\delta(s_{13})$ Plot



shows phase variation $\sim 180^\circ$ from threshold to about $m_{\pi\pi} = 0.9\text{GeV}$

$\sin^2 \delta(s_{13})$ Plot



$m_{\pi\pi} \sim 0.55\text{GeV}$
 $\Gamma_{\pi\pi} \sim 0.4\text{GeV}$

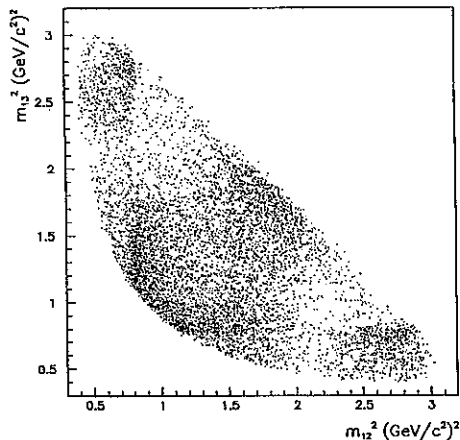
Model-independent measurement of the PHASE MOTION at low $\pi\pi$ mass is totally compatible with E791 published results on a light, broad σ state

Further Studies with $D^+ \rightarrow K^- \pi^+ \pi^+$

Is it possible to get a hint of the phase motion due to the κ state in $D^+ \rightarrow K^- \pi^+ \pi^+$?

Ochs & Minkowski
hep-ph/0209225

$\cos \theta_v$ distribution around $K^*(890)$ to provide indication of interference with the crossed channel



strong asymmetry of $K^*(890)$ is evident

From E791 published results: Asymmetry is due to interference with

→ a dominant NR component in “Model A”

→ a broad, light κ in “Model C”

Can this interference indicate the phase motion of the κ near the $K^*(890)$?

An Illustration: κ pure Breit-Wigner phase

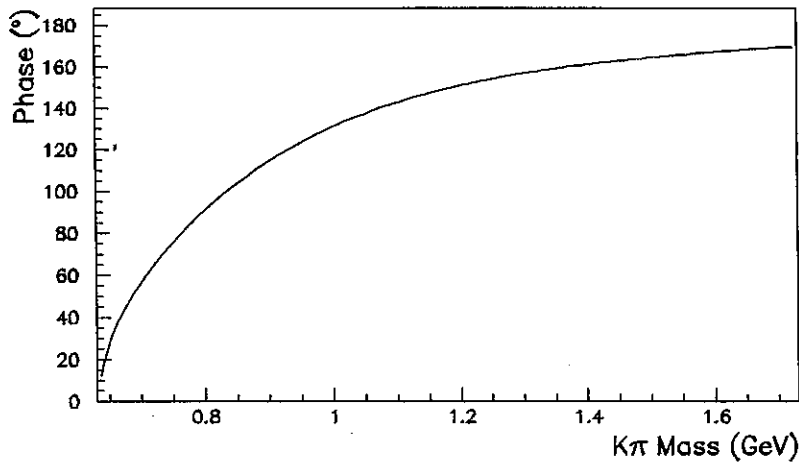
$$BW_{\kappa} = \frac{1}{m_{\kappa}^2 - m_{12}^2 - im_{\kappa}\Gamma(m_{12})}$$

$$\text{with } \Gamma(m_{12}) = \frac{F_R^0(p^*)^2 p^* m_{\kappa} \Gamma_{\kappa}}{F_R^0(p^{*0})^2 p^{*0} m_{12}}$$

E791 central values:

$$m_{\kappa} = 797 \text{ MeV}$$

$$\Gamma_{\kappa} = 410 \text{ MeV}$$



**main phase variation
occurs at low mass**



Difficulties:

- K^* and κ both present at low mass

A phase variation due to κ appears in both s_{12} and s_{13}
(symmetrization)

- the region $0.9\text{GeV} < \sqrt{s_{12}} < 1.0\text{GeV}$ (K^*)
excludes $\sqrt{s_{13}} < 0.95\text{GeV}$

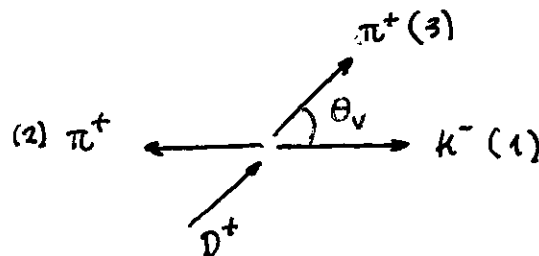
→ Above κ mass:

misses its main phase variation in the crossed channel

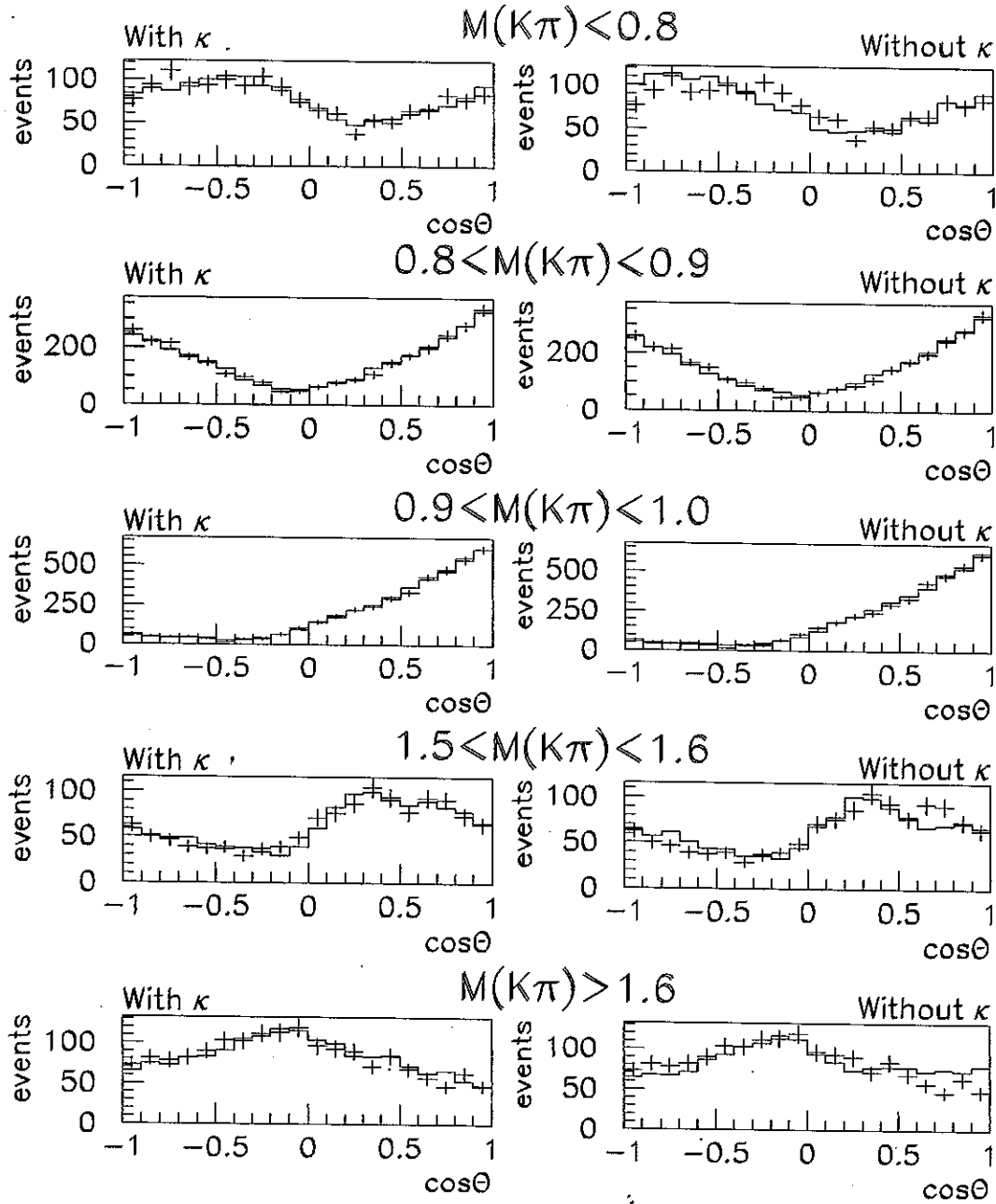
Exercise Plots:

★ $\cos \theta_V$ as a function of m_{12}

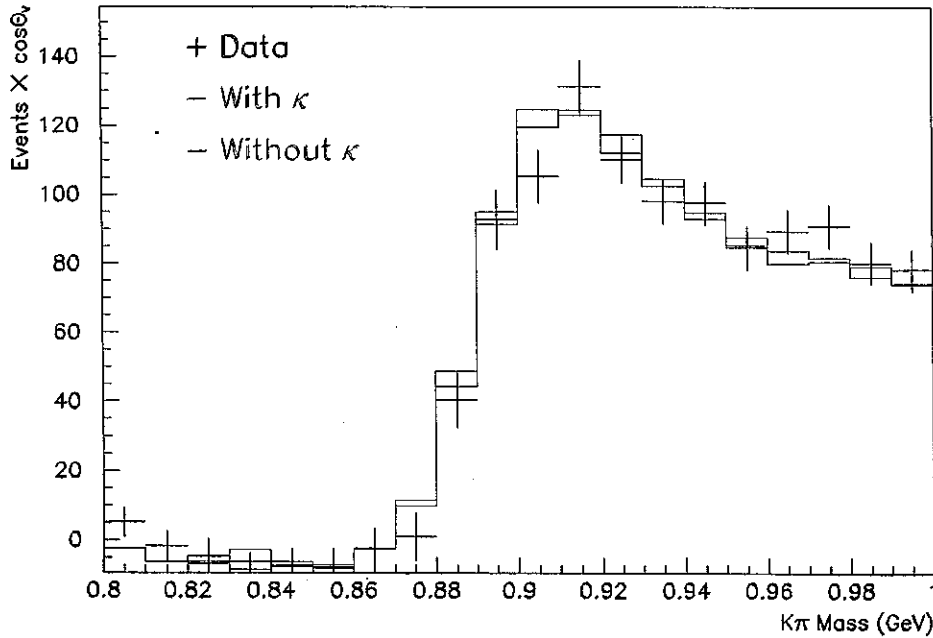
★ Asymmetry of $\langle \cos \theta_V \rangle$ in m_{12}



cos θ_V Plots



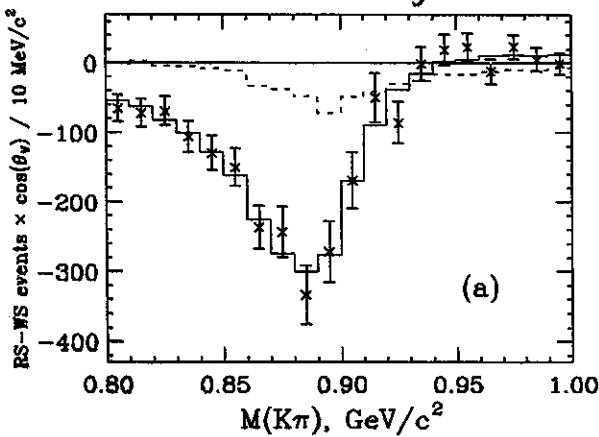
Asymmetry Plot



⇒ In the region 0.8 – 1.0 GeV it is not possible to distinguish between the two models

Reasonable, since above 1 GeV (in the crossed channel) the κ phase rises very slowly

FOCUS result on $D^+ \rightarrow K^- \pi^+ \mu^+ \nu$:
Phys. Lett. B 535, 43 (2002)



Needs an S-wave component (either NR or broad scalar) interfering with K^*

⇒ No contradiction with E791 results

Another Exercise

Can the LASS results for their $K\pi$ S-wave describe our data?

\Rightarrow What do we get by imposing the phase motion measured by LASS?

First Step: Translate scattering to production amplitude

Ochs & Minkowski, private communication

$$A_S = a_{\text{NR}} \frac{1}{p^*(\cot \delta_{\text{NR}} - i)} + a_{K^*(1430)} \mathcal{A}_{K^*(1430)} e^{i\phi}$$

with an effective range form

$$\cot \delta_{\text{NR}} = \frac{1}{a p^*} + \frac{1}{2} b p^*$$

LASS results	$\phi = 2\delta_{\text{NR}}$ $a = 2.19 \text{ GeV}^{-1}, b = 3.74 \text{ GeV}^{-1}$ $M_{K^*(1430)} = 1435 \text{ MeV}$ $\Gamma_{K^*(1430)} = 279 \text{ MeV}$
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1st Fit :

- Fix LASS parameters: $a, b, M_{K^*(1430)}, \Gamma_{K^*(1430)}$
- Fix relative phase $\phi = 2\delta_{NR}$

⇒ Very bad fit quality ($\chi^2/\nu = 4.2$)
even worse than Model A

(no κ , constant NR, free relative phase to $K_0^*(1430)$)

2nd Fit :

- Fix LASS parameters: $a, b, M_{K^*(1430)}, \Gamma_{K^*(1430)}$
- Do not fix relative phase

⇒ Bad fit quality ($\chi^2/\nu = 3.1$)
comparable but still worse than Model A

3rd Fit :

- Release all parameters

$$a = 2.48 \pm 0.29 \text{ GeV}^{-1}$$

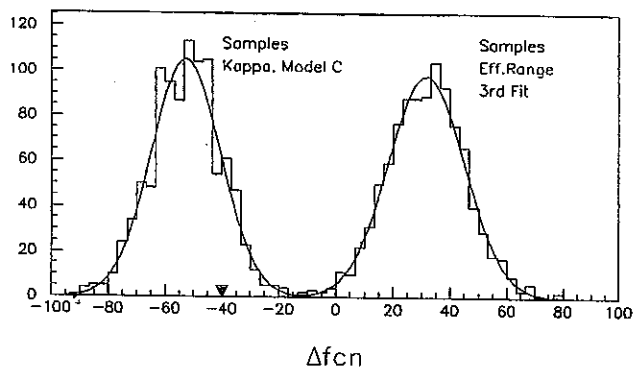
$$b = -2.23 \pm 0.18 \text{ GeV}^{-1}$$

$$M_{K^*(1430)} = 1460 \pm 7 \text{ MeV}$$

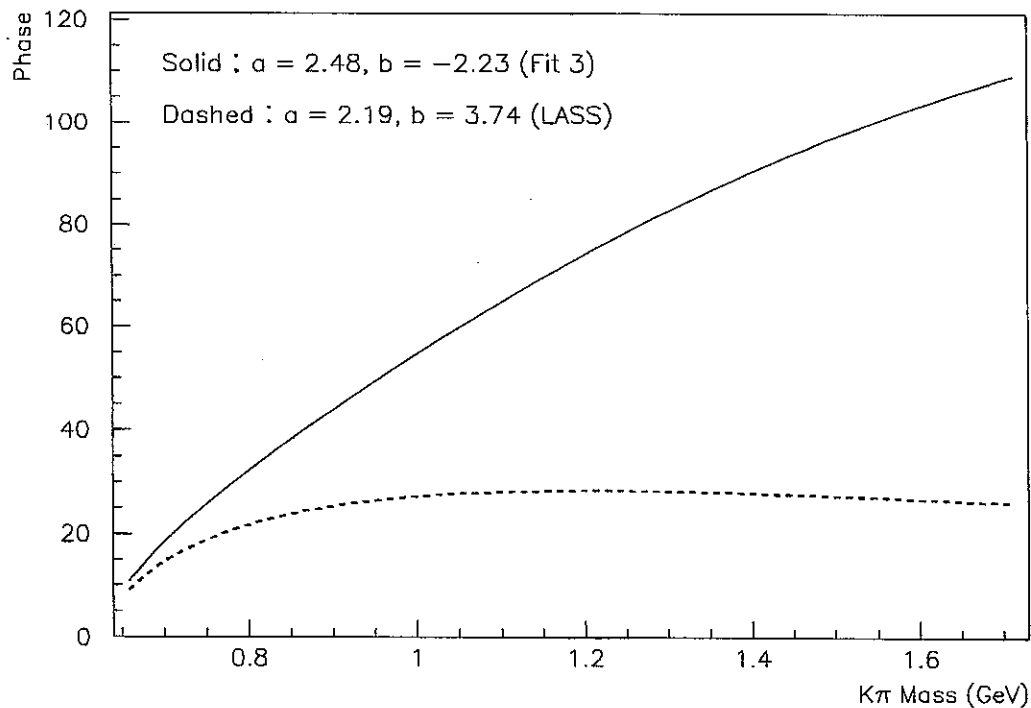
$$\Gamma_{K^*(1430)} = 150 \pm 11 \text{ MeV}$$

Fit is good ($\chi^2/\nu = 1$) but

Model C is better



Comparison of 3rd Fit to LASS



★ We cannot impose LASS parameters to $D^+ \rightarrow K^- \pi^+ \pi^+$ decay, especially the constraint on the relative phase NR/ $K_0^*(1430)$

★ Forcing the effective range parametrization, data asks for a crescent phase variation for the “NR”, much more pronounced than LASS.

★ It is very important to keep the phases of the different scalar contributions (NR, κ , $K_0^*(1430)$) as free parameters to accommodate FSI

Conclusions

★ Fermilab E791 studied the resonant contributions to the decays $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$, $D^+ \rightarrow \pi^- \pi^+ \pi^+$ and $D^+ \rightarrow K^- \pi^+ \pi^+$, focusing on the scalars

The data clearly favors the existence of

$$\sigma(500)$$

$$\kappa(800)$$

parametrized as Breit-Wigners in the fit

★ In $D^+ \rightarrow \pi^- \pi^+ \pi^+$ an Amplitude Difference method was applied to measure the phase motion of the scalar amplitude at low mass through its interference with $f_2(1270)$

Results are consistent with E791 on the existence of a light, broad σ

★ In $D^+ \rightarrow K^- \pi^+ \pi^+$ a number of further exercises was made

→ difficulty for measuring the phase motion of the κ in the $K^*(890)$ mass region. No sensibility there.

→ the $\cos \theta_V$ plots do confirm the effect at low mass, where the phase motion of a κ is much more evident.

→ the phase motion measured by LASS do not explain our data

**$K\pi$ FROM D^+ DECAY AND
FROM ELASTIC SCATTERING DIFFER**

**FURTHER CONFIRMATION/INFORMATION FOR
 σ AND κ ARE NEEDED
preferrably from different sources**

From D Decays:

**Large statitcal samples coming from
FOCUS, BaBar, BELLE, CLEO-C**