1 Two-Body Dirac Equations from Constraint Dynamics

Relativistic two-body bound state wave equations and their connection to quantum field theory.

An old problem:Eddington and Gaunt in 1928.

One without a generally agreed-upon solution.

Steven Weinberg: "It must be said that the theory of relativistic effects and radiative corrections in bound states is not yet in satisfactory shape."

Work done in collaboration with Peter Van Alstine, Bin Liu.

hep-ph/0208186 + nucl-th/0208045

2 The One-Body Dirac Equation

$$(\gamma \cdot p + m)\psi = 0$$ a relativistic version of Newton/s 1st law

With the four-vector substitution

$$p_\mu \longrightarrow p_\mu - A_\mu$$

for electromagnetic interaction and the minimal mass substitution

$$m \longrightarrow m + S$$

for scalar interactions

$$(\gamma \cdot (p - A) + m + S)\psi = 0.$$ a relativistic version of Newton/s 2nd law

Constraint two-body Dirac equations generalize this one to the interacting two-body system.

3 Two-Body Dirac Equations from Constraint Dynamics

1970's, Todorov, Komar, and Van Alstine - Dirac's constraint mechanics

i) Covariantly controls the relative time variable: eliminated negative norm states and cirmcumvented the no-interaction theorem.

Combining constraint dynamics with particle supersymmetries, HC and Van Alstine obtained two-body Dirac equations:

ii) Correct the defects in the Breit equation and in the ladder approximation to the Bethe-Salpeter equation.

iii) Are manifestly covariant

iv) Yield simple three-dimensional Schrödinger-like forms similar to their nonrelativistic counterparts.

v) Their spin dependence is determined naturally by the Dirac-like structure of the equations.

vi) They have well defined strong potential structures that have passed numerous tests, reproducing correct QED perturbative results when solved nonperturbatively.

vii) Dirac forms of the equations make unnecessary the ad hoc introduction of cutoff parameters generally used to avoid singular potentials.

viii) Relativistic potentials can be related directly to the interactions of perturbative quantum field theory or (e.g. for QCD) may be introduced semiphenomenologically.

4 TBDE for World Vector and Scalar Interactions.

For particles interacting through world vector and scalar interactions:

$$\mathcal{S}_1\psi \equiv \gamma_{51}(\gamma_1 \cdot (p_1 - \tilde{A}_1) + m_1 + \tilde{S}_1)\psi = 0$$

$$\mathcal{S}_2\psi \equiv \gamma_{52}(\gamma_2 \cdot (p_2 - \tilde{A}_2) + m_2 + \tilde{S}_2)\psi = 0.$$

i) Provide a non-perturbative or strong potential framework for extrapolating perturbative field theoretic results into the highly relativistic regime of bound light quarks, in a quantum mechanically well defined way.

Framework incorporates two related properties:

ii) Minimal interaction structure

iii) Compatibility of the two-equations

$[\mathcal{S}_1, \mathcal{S}_2]\psi = 0.$ Originates from presence of supersymmetries

a) leads to a relativistic 3rd law

b) Covariantly restricts the relative momentum and energy while correctly structuring spin-dependent recoil.

5 Vector Interactions

i) Compatibility conditions forces "hyperbolic" forms

$$\tilde{A}_1 = [1 - \cosh(\mathcal{G})]p_1 + \sinh(\mathcal{G})p_2 - \frac{i}{2}(\partial \exp \mathcal{G} \cdot \gamma_2)\gamma_2$$

$$\tilde{A}_2 = [1 - \cosh(\mathcal{G})]p_2 + \sinh(\mathcal{G})p_1 + \frac{i}{2}(\partial \exp \mathcal{G} \cdot \gamma_1)\gamma_1$$

$\mathcal{G}(\mathcal{A})$: an invariant interaction function from Wheeler-Feynman CFT or QED from eikonal summation of ladder and cross-ladder diagrams.

$$\mathcal{G}(\mathcal{A}) = -\frac{1}{2}\log(1 - \frac{2\mathcal{A}}{w}) \; ; \quad w \text{ the total c.m. energy}$$

ii) Compatibility condition requires 3rd law and controls relative time

$$\mathcal{A} = \mathcal{A}(x_\perp) - 3RD \;\; LAW - \text{one invariant function}$$

$x_\perp$ is covariant spacelike particle separation

$$x_\perp^\mu = x^\mu + \hat{P}^\mu(\hat{P}\cdot x)$$ covariant spacelike separation perpendicular to P

. ($\hat{P} \equiv \frac{P}{w}$ is a time-like unit vector.)

iii) Dynamics is independent of the relative time in the c.m system

iv) For lowest order electrodynamics,

$$\mathcal{A} = \mathcal{A}(x_\perp) = -\frac{\alpha}{r}$$ From perturbative QFT

$$r \equiv \sqrt{x_\perp^2}.$$

6 Scalar Interactions

i) Scalar potentials $\tilde{S}_i$ given in terms of $\mathcal{G}(\mathcal{A}(x_\perp))$ and invariant functions $M_1(x_\perp)$, $M_2(x_\perp)$, related to each other by

$$\begin{aligned} \tilde{S}_1 &= M_1 - m_1 - \frac{i}{2} \exp \mathcal{G}(\mathcal{A}) \gamma_2 \cdot \frac{\partial M_1}{M_2}, \\ \tilde{S}_2 &= M_2 - m_2 - \frac{i}{2} \exp \mathcal{G}(\mathcal{A}) \gamma_1 \cdot \frac{\partial M_2}{M_1}, \end{aligned}$$

$$M_1^2 - M_2^2 = m_1^2 - m_2^2 \Longrightarrow \begin{array}{l} M_1 = m_1 \ \cosh L \ + m_2 \sinh L \\ M_2 = m_2 \ \cosh L \ + m_1 \ \sinh L \end{array} ; 3RD \ \ LAW\text{—one invariant}$$

ii) Counterpart to invariant $\mathcal{A}$ for scalar interactions is S ; $L = L(S(x_\perp), \mathcal{A}(x_\perp))$ from

$$\begin{aligned} M_1^2 &= m_1^2 + \exp \mathcal{G}(\mathcal{A})(2 m_w S + S^2) \\ M_2^2 &= m_2^2 + \exp \mathcal{G}(\mathcal{A})(2 m_w S + S^2). \end{aligned}$$

iii) Retardative effects are embodied in the c.m. energy dependence.

iv) Two-body Dirac equations a three-dimensional but manifestly covariant rearrangement of BSE

v) Interaction is instantaneous in the c.m. system, a direct consequence of the compatibility of the two equations

vi) Bypasses difficulties of the Bethe-Salpeter equation

7 Manifestly Covariant and Quantum-Mechanically Well-Defined.

i) Covariant Schrödinger-like forms (with p the relative momentum)

$(p^2 + \Phi_w(\sigma_1, \sigma_2, p_\perp, \mathcal{A}(r), S(r)))\psi = b^2(w)\psi$ – From Pauli reduction of CTBDE

ii) Two-body Relativistic kinematics

$$
\begin{aligned}
b^2(w) &= \frac{1}{4w^2}(w^4 - 2(m_1^2 + m_2^2)w^2 + (m_1^2 - m_2^2)^2) \\
&\equiv \varepsilon_w^2 - m_w^2, \\
m_w &= \frac{m_1 m_2}{w}; \quad \varepsilon_w = \frac{w^2 - m_1^2 - m_2^2}{2w}
\end{aligned}
$$

iii) One can solve i) nonperturbatively for both QED and QCD bound state calculations

iv) Every term in $\Phi_w(\sigma_1, \sigma_2, p_\perp, \mathcal{A}(r), S(r))$ is less singular than $-1/4r^2$.

v) Schrödinger-like forms can be transformed into at most 2 coupled wave functions even when non-central tensor forces or spin-difference-orbit interactions are present.

vi) All portions of the 16 component wave function play essential roles in spectral calculations, either directly or through the strong potential structures that they generate when eliminated.

vii) The specific expressions of the spin dependent potentials that appear in the quasipotential Φ_w are dictated by the interaction structure of the Two-Body Dirac Equations and are not put in by hand.

8 Strong Potential Terms

i) Recast Schrödinger-like form of Two-Body Dirac equations into minimal coupling form

$$(p^2 + (m_w + S)^2 - (\varepsilon_w - \mathcal{A})^2 + \Phi_{sp}(\sigma_1, \sigma_2, p_\perp, \mathcal{A}(r), S(r)))\psi = 0$$

in which

$$\begin{aligned}\Phi_{sp} &= \Phi_{D1}\hat{r}\cdot p + \Phi_{D2} + \Phi_{SO}L\cdot(\sigma_1+\sigma_2) + \Phi_{SOD}L\cdot(\sigma_1-\sigma_2) \\ &+\Phi_{SPO}L\cdot(\sigma_1\times\sigma_2) + \Phi_{SS}\sigma_1\cdot\sigma_2 + \Phi_T\sigma_1\cdot\hat{r}\sigma_2\cdot\hat{r} + \Phi_{DT}\hat{r}\cdot p\sigma_1\cdot\hat{r}\sigma_2\cdot\hat{r}\end{aligned}$$

ii)

$$\Phi_i = \Phi_i(\mathcal{A}, S) \;\; ; \;\; \mathcal{A}, S \text{ fix the potential terms}$$

iii) No freedom to parametrize the numerous Φ_i forms independently.

iv) Minimal and Φ_{sp}: "strong potential" terms can be treated nonperturbatively.

v) Contrast main spin-spin term with that of Fermi-Breit.

$$(\text{Two} - \text{Body Dirac}) \quad -\frac{1}{6}\sigma_1 \cdot \sigma_2 \partial^2 ln(1 - \frac{2\mathcal{A}}{w}) \rightarrow \frac{1}{3}\frac{\sigma_1 \cdot \sigma_2 \partial^2 \mathcal{A}}{m_1 + m_2} \quad \text{Breit}$$

9 Nonperturbative Validity of Strong-Potential Forms

i) Solving analytically and numerically to obtain the standard fine and hyperfine spectra of QED.

ii) E.g. for the singlet positronium system with $\mathcal{A} = -\alpha/r$

$$w = m\sqrt{2 + 2/\sqrt{1 + \frac{\alpha^2}{(n + \sqrt{(l+\frac{1}{2})^2 - \alpha^2} - l - \frac{1}{2})^2}}} \doteq m(2 - \frac{\alpha^2}{4} - \frac{21\alpha^4}{64})_{\text{ground state}}$$

of the fully coupled system of 16-component equations

$$\mathcal{S}_1\psi = \mathcal{S}_2\psi = 0.$$

iii) Such validation ought to be required of all candidate equations for nonperturbative quark model calculations and other semiphenomenalogical applications when their quark-model kernals are replaced by ones appropriate for QED.

iv) No other approaches have yet been fully tested in this way.

v) Otherwise two body formalisms may lead to possibly spurious nonperturbative predictions.

vi) The Abelian vector structure of electrodynamics carries over to the short distance structure of QCD.

vii) In order that the equations be appropriate for QCD bound state calculations in that sector they must give correct answers to the appropriate order in the fine structure constant α when applied to QED bound states.

viii) This we have checked numerically for a range of angular momentum and radial states and for equal as well as unequal masses.

ix) Our spin orbit potential is so attractive for the $^3P_0$ state that it actually turns the repulsive angular momentum over.

This behavior may have some importance for the sigma meson.

10 Connection to Quantum Field Theory

i) Invariant forms $\mathcal{A}$ and S in the CTBDE may be perturbatively obtained from the corresponding quantum field theories.

ii) The connection displays explicitly the elimination of the relative energy in the cm system

$$\Phi_w(p, \sigma_1, \sigma_2, \mathcal{A}(r), S(r)) = \pi i \delta(\hat{P} \cdot p) \mathcal{K}(1 + \bar{\mathcal{K}})^{-1}.$$

iii) Gives quasipotential in terms of the Bethe-Salpeter kernel $\mathcal{K}$ and its projection

$$\bar{\mathcal{K}} = \mathbf{G}\mathcal{K}$$

where

$$\mathbf{G} \equiv \Big(\frac{1}{p_1^2 + m_1^2 - i0} \frac{1}{p_2^2 + m_2^2 - i0} - \pi i \delta(P \cdot p) \frac{w}{p_\perp^2 - b^2(w) - i0}\Big) \tag{5}$$

is the difference between Bethe-Salpeter and the constraint propagators (with the relativistic third law delta function.)

iv) Derived by Sazdjian as a "quantum mechanical transform of the Bethe-Salpeter equation",

11 Two-Body Dirac Equations and Meson Spectroscopy

11.1 The Adler-Piran Potential

i) A constraint version of the naive quark model for mesons

ii) Adler and Piran obtained their static quark potential from an effective non-linear field theory derived from QCD.

$$V_{AP}(r) = \Lambda(U(\Lambda r) + U_0) \; (= \mathcal{A} + S).$$

Since their potential is nonrelativistic it cannot distinguish between world scalar and vector potentials, simply representing the effect of their sum in the nonrelativistic limit.

iii) It incorporates a running coupling constant form in coordinate space

$$\Lambda U(\Lambda r << 1) \sim \frac{1}{r ln \Lambda r}$$

iv) It includes linear confinement plus subdominant logarithm terms

$$V_{AP}(r) = \Lambda(c_1\Lambda r + c_2 \log(\Lambda r) + \frac{c_3}{\sqrt{\Lambda r}} + \frac{c_4}{\Lambda r} + c_5), \quad \Lambda r > 2.$$

v) When used with the nonrelativistic Schrödinger equation for light mesons, the Adler-Piran potential or Richardson potential give meson masses that increase with decreasing quark mass.

vi) Such disastrous results are completely turned around using TBDE. Relativistic treatment essential for light mesons with potentials closely tied to QCD.

12 Relativistic Naive Quark Model

i) Incorporating models a static potential model $V = V_{AP}(r)$ for the quark-antiquark interaction in a covariant way into our equations:

a) replacing nonrelativistic r by $\sqrt{x_{\perp}^2}$

b) parcelling out the static potential V_{AP} into the invariant functions $\mathcal{A}(r)$ and $S(r)$.

$$\mathcal{A} = exp(-\beta r)[V_{AP} - \frac{c_4}{r}] + \frac{c_4}{r} + \frac{e_1 e_2}{r},$$

$$S = V_{AP} + \frac{e_1 e_2}{r} - \mathcal{A}.$$

ii) We impose that at short distance the potential is strictly vector while at long distance the vector portion is strictly Coulombic

iii) The confining portion at long distance is (including subdominant portions) strictly scalar.

iv) The relativistic invariance of S and $\mathcal{A}$ follows by reinterpreting the variable r as $r \equiv \sqrt{x_\perp^2}$ in the c.m. system.

v) Step b) is a partially phenomenological one.

vi) But once $\mathcal{A}$ and S are fixed, so are all the accompanying spin dependences.

vii) Our approach is that of a naive quark model since we ignore flavor mixing and the effects of decays on the bound state energies.

13 Meson Spectroscopy

i) We use the same $S, \mathcal{A}$ for all of the mesons

ii) Results are spectrally quite accurate, from the heaviest upsilonium states to the pion.

iii) Notable exceptions are light meson orbital and radially excitations and their spin-orbit splittings.

iv) With just two parametric functions $\mathcal{A}$ and S we are able to obtain a fit about as good as that obtained by Godfrey and Isgur, who use six parametric functions, basically one for each type of spin dependence.

MESON MASSES FROM COVARIANT CONSTRAINT DYNAMICS

NAME	EXP.	THEORY
$\Upsilon : b\bar{b}\ 1^3S_1$	9.460	9.453
$\Upsilon : b\bar{b}\ 1^3P_0$	9.860	9.842
$\Upsilon : b\bar{b}\ 1^3P_1$	9.892	9.889
$\Upsilon : b\bar{b}\ 1^3P_2$	9.913	9.921
$\Upsilon : b\bar{b}\ 2^3S_1$	10.023	10.022
$\Upsilon : b\bar{b}\ 2^3P_0$	10.232	10.227
$\Upsilon : b\bar{b}\ 2^3P_1$	10.255	10.257
$\Upsilon : b\bar{b}\ 2^3P_2$	10.269	10.277
$\Upsilon : b\bar{b}\ 3^3S_1$	10.355	10.359
$\Upsilon : b\bar{b}\ 4^3S_1$	10.580	10.614
$\Upsilon : b\bar{b}\ 5^3S_1$	10.865	10.826
$\Upsilon : b\bar{b}\ 6^3S_1$	11.019	11.013
$B : b\bar{u}\ 1^1S_0$	5.279	5.273
$B : b\bar{d}\ 1^1S_0$	5.279	5.274
$B^* : b\bar{u}\ 1^3S_1$	5.325	5.321
$B_s : b\bar{s}\ 1^1S_0$	5.369	5.368

$B_s : b\bar{s}\ 1^3S_1$	5.416	5.427
$\eta_c : c\bar{c}\ 1^1S_0$	2.980	2.978
$\psi : c\bar{c}\ 1^3S_1$	3.097	3.129
$\chi_0 : c\bar{c}\ 1^1P_1$	3.526	3.520
$\chi_0 : c\bar{c}\ 1^3P_0$	3.415	3.407
$\chi_1 : c\bar{c}\ 1^3P_1$	3.510	3.507
$\chi_2 : c\bar{c}\ 1^3P_2$	3.556	3.549
$\eta_c : c\bar{c}\ 2^1S_0$	3.594	3.610
$\psi : c\bar{c}\ 2^3S_1$	3.686	3.688
$\psi : c\bar{c}\ 1^3D_1$	3.770	3.808
$\psi : c\bar{c}\ 3^3S_1$	4.040	4.081
$\psi : c\bar{c}\ 2^3D_1$	4.159	4.157
$\psi : c\bar{c}\ 3^3D_1$	4.415	4.454
$D : c\bar{u}\ 1^1S_0$	1.865	1.866
$D : c\bar{d}\ 1^1S_0$	1.869	1.873
$D^* : c\bar{u}\ 1^3S_1$	2.007	2.000
$D^* : c\bar{d}\ 1^3S_1$	2.010	2.005
$D^* : c\bar{u}\ 1^3P_1$	2.422	2.407
$D^* : c\bar{d}\ 1^3P_1$	2.428	2.411

$D^* : c\bar{u}\ 1^3P_2$	2.459	2.382
$D^* : c\bar{d}\ 1^3P_2$	2.459	2.386
$D_s : c\bar{s}\ 1^1S_0$	1.968	1.976
$D_s^* : c\bar{s}\ 1^3S_1$	2.112	2.123
$D_s^* : c\bar{s}\ 1^3P_1$	2.535	2.511
$D_s^* : c\bar{s}\ 1^3P_2$	2.574	2.514
$K : s\bar{u}\ 1^1S_0$	0.494	0.492
$K : s\bar{d}\ 1^1S_0$	0.498	0.492
$K^* : s\bar{u}\ 1^3S_1$	0.892	0.910
$K^* : s\bar{d}\ 1^3S_1$	0.896	0.910
$K_1 : s\bar{u}\ 1^1P_1$	1.273	1.408
$K_0^* : s\bar{u}\ 1^3P_0$	1.429	1.314
$K_1 : s\bar{u}\ 1^3P_1$	1.402	1.506
$K_2^* : s\bar{u}\ 1^3P_2$	1.425	1.394
$K_2^* : s\bar{d}\ 1^3P_2$	1.432	1.394
$K^* : s\bar{u}\ 2^1S_0$	1.460	1.591
$K^* : s\bar{u}\ 2^3S_1$	1.412	1.800
$K_2 : s\bar{u}\ 1^1D_2$	1.773	1.877
$K^* : s\bar{u}\ 1^3D_1$	1.714	1.985

$K_2 : s\overline{u}\ 1^3D_2$	1.816	1.945
$K_3 : s\overline{u}\ 1^3D_3$	1.770	1.768
$K^* : s\overline{u}\ 3^1S_0$	1.830	2.183
$K_2^* : s\overline{u}\ 2^3P_2$	1.975	2.098
$K_4^* : s\overline{u}\ 1^3F_4$	2.045	2.078
$K_2 : s\overline{u}\ 2^3D_2$	2.247	2.373
$K_5^* : s\overline{u}\ 1^3G_5$	2.382	2.344
$K_3^* : s\overline{u}\ 2^3F_3$	2.324	2.636
$K_4^* : s\overline{u}\ 2^3F_4$	2.490	2.757
$\phi : s\overline{s}\ 1^3S_1$	1.019	1.033
$f_0 : s\overline{s}\ 1^3P_0$	1.370	1.319
$f_1 : s\overline{s}\ 1^3P_1$	1.512	1.533
$f_2 : s\overline{s}\ 1^3P_2$	1.525	1.493
$\phi : s\overline{s}\ 2^3S_1$	1.680	1.850
$\phi : s\overline{s}\ 1^3D_3$	1.854	1.848
$f_2 : s\overline{s}\ 2^3P_2$	2.011	2.160
$f_2 : s\overline{s}\ 3^3P_2$	2.297	2.629
$\pi : u\overline{d}\ 1^1S_0$	0.140	0.144
$\rho : u\overline{d}\ 1^3S_1$	0.767	0.792

$b_1 : u\bar{d}\ 1^1P_1$	1.231	1.392
$a_0 : u\bar{d}\ 1^3P_0$	1.450	1.491
$a_1 : u\bar{d}\ 1^3P_1$	1.230	1.568
$a_2 : u\bar{d}\ 1^3P_2$	1.318	1.310
$\pi : u\bar{d}\ 2^1S_0$	1.300	1.536
$\rho : u\bar{d}\ 2^3S_1$	1.465	1.775
$\pi_2 : u\bar{d}\ 1^1D_2$	1.670	1.870
$\rho : u\bar{d}\ 1^3D_1$	1.700	1.986
$\rho_3 : u\bar{d}\ 1^3D_3$	1.691	1.710
$\pi : u\bar{d}\ 3^1S_0$	1.795	2.166
$\rho : u\bar{d}\ 3^3S_1$	2.149	2.333
$\rho_4 : u\bar{d}\ 1^3F_4$	2.037	2.033
$\pi_2 : u\bar{d}\ 2^1D_2$	2.090	2.367
$\rho_3 : u\bar{d}\ 2^3D_3$	2.250	2.305
$\rho_5 : u\bar{d}\ 1^3G_5$	2.330	2.307
$\rho_6 : u\bar{d}\ 1^3H_6$	2.450	2.547
χ^2	0.0	101.0

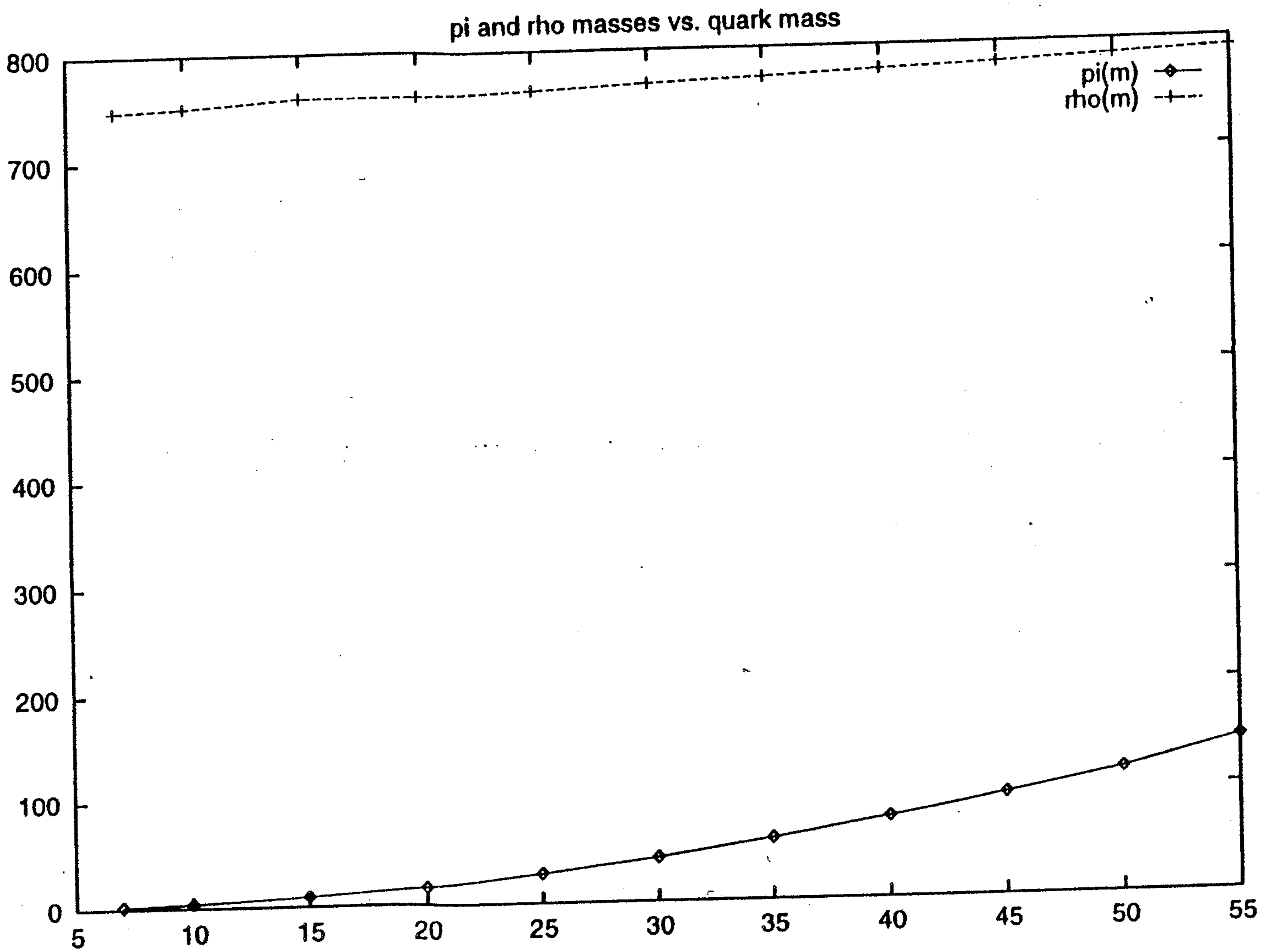
14 Pion as Goldstone Boson

i) Pion is a Goldstone boson in that $m_\pi(m_q \rightarrow 0) \rightarrow 0$.

ii) The ρ and excited pion have finite mass in this limit

iii) The curve does not follow the Goldberger-Trieman relation

$$m_\pi^2 F_\pi = m_q.$$



pi and rho masses vs. quark mass
pi(m)
rho(m)
800
700
600
500
400
300
200
100
0
5
10
15
20
25
30
35
40
45
50
55

15 Two-Body Dirac Equations for Nucleon-Nucleon Scattering

i) The two-body Dirac equations of constraint dynamics provide a natural (Dirac) formulation to extend the early phenomenological work of Reid to the relativistic domain but with all aspects of the effective potentials fixed by connections by way of meson exchanges.

ii) The mesons we include are the pseudoscalar mesons $\pi(135), \eta(548), \eta'(952)$ the vector mesons $\rho(770), \omega(776), \phi(1020)$ and the scalar mesons $\sigma(600), a_0(980), f_0(983)$. The π, ρ, and the a_0 are isovector mesons while the rest are isoscalar mesons.

iii) To incorporate these nine mesons in the context the constraint two-body Dirac equations we need

16 Two-Body Dirac Equations for General Covariant Interactions: The role of supersymmetry

i)Theta matrices.

$$\theta^{\mu} \equiv i\sqrt{\frac{1}{2}}\gamma_5\gamma^{\mu}_{,} \ \mu = 0,1,2,3, \quad \theta_5 \equiv i\sqrt{\frac{1}{2}}\gamma_5,$$

ii) In the "correspondence-principle" limit the they become Grassmann variables

iii) The Dirac equation becomes a constraint imposed on both bosonic (p) and fermionic (θ, θ_5) variables:

$$\mathcal{S}_0\psi \equiv (p\cdot\theta + m\theta_5)\psi = 0; \Longrightarrow \mathcal{S}_0 \equiv (p\cdot\theta + m\theta_5) \approx 0$$

17 Supersymmetry

i) The supersymmetry (generator is $p \cdot \theta + \sqrt{-p^2}\theta_5$) of the free Dirac constraint does not leave invariant the position four-vector x. It displays pseudoclassical zitterbewegegung.

iii) However, the "zitterbewegungless" position variable is supersymmetric:

$$\tilde{x}^\mu = x^\mu + \frac{i\theta^\mu\theta_5}{m}$$

iv) This variable must be modified in the presence of scalar interaction $M = m + S$ to the self-referent form

$$\tilde{x}^\mu = x^\mu + \frac{i\theta^\mu\theta_5}{M(\tilde{x})}.$$

v) The supersymmetric constraints (both fermionic and bosonic) then become

$$\mathcal{S} = p \cdot \theta + M(\tilde{x})\theta_5 \approx 0, \ \frac{1}{i}\{\mathcal{S}, \mathcal{S}\} \equiv \mathcal{H} = p^2 + M^2(\tilde{x}) \approx 0.$$

vi) Since $\theta_5^2 = 0$, the expansion of the self-referent form truncates

$$M(\tilde{x}) = M(x) + \frac{i\partial M(x) \cdot \theta\theta_5}{M(x)}.$$

vi) One arrives back at the Dirac equation by replacing Grassmann variables with theta matrices and dynamical variables x and p with their operator forms.

$$\mathcal{S}\psi = [p \cdot \theta + M(x)\theta_5]\psi = 0$$

$$\mathcal{H}\psi = [p^2 + M^2(x) + 2i\partial M(x) \cdot \theta\theta_5]\psi = 0.$$

viii) The supersymmetry realized through the presence of $\tilde{x}$ is a natural feature of both the free Dirac equation and its standard form for external scalar interaction.

18 Supersymmetric Two-Body Dirac Equations

i) In the presence of interaction, we require the preservation of supersymmetry for each spinning particle.

$$m_i \to M_i(x_1 - x_2) \to M_i(\tilde{x}_1 - \tilde{x}_2) \equiv \tilde{M}_i,\ \ i = 1, 2$$

$$\tilde{x}_i^\mu = x_i^\mu + \frac{i\theta_i^\mu \theta_{5i}}{\tilde{M}_i},\ \ i = 1, 2.$$

ii) Grassmann Taylor expansions of the $\tilde{M}_i$ truncate. Carrying out those expansions

$$\mathcal{S}_1\psi = (\theta_1 \cdot p + \epsilon_1\theta_1 \cdot \hat{P} + M_1\theta_{51} - i\partial L \cdot \theta_2\theta_{52}\theta_{51})\psi = 0,$$

$$\mathcal{S}_2\psi = (-\theta_2 \cdot p + \epsilon_2\theta_2 \cdot \hat{P} + M_2\theta_{52} + i\partial L \cdot \theta_1\theta_{52}\theta_{51})\psi = 0,$$

provided that

$$\partial(M_1^2 - M_2^2) = 0,$$

(the relativistic "third law" condition), and

$$M_i = M_i(x_\perp).$$

$$M_1 = m_1 \ \cosh L \ + m_2 \sinh L, \ M_2 = m_2 \ \cosh L \ + m_1 \ \sinh L,$$

in which

$$L = L(x_\perp)$$

with

$$\partial L = \frac{\partial M_1}{M_2} = \frac{\partial M_2}{M_1}.$$

iii) TBDE are same as in the beginning of this talk when restricted to scalar interactions.

iv) In these coupled Dirac equations the remnants of pseudoclassical supersymmetries are the extra spin dependent recoil corrections to the ordinary one-body Dirac equations.

v) Rewrite constraints in a form which we can generalize to interactions other than scalar.

vi) Without those terms (which vanish when one of the particles becomes infinitely heavy) the two equations would not be compatible.

19 Hyperbolic Form of the Two-Body Dirac Equations for General Covariant Interactions

i) How do we introduce general interactions?

ii) We recast the minimal interaction forms of the two-body Dirac equations into one that generalizes hyperbolic forms we encountered above.

iii) Simple identities such as

$$\cosh^2(\Delta) - \sinh^2(\Delta) = 1$$

transform minimal interaction forms to

$$\mathcal{S}_1\psi = [\cosh(\Delta)\mathrm{S}_1 + \sinh(\Delta)\mathrm{S}_2]\psi = 0,$$

$$\mathcal{S}_2\psi = [\cosh(\Delta)\mathrm{S}_2 + \sinh(\Delta)\mathrm{S}_1]\psi = 0,$$

in which appear auxiliary constraints defined by

$$\mathbf{S}_1\psi \equiv (\mathcal{S}_{10}\cosh(\Delta) + \mathcal{S}_{20}\sinh(\Delta))\psi = 0,$$

$$\mathbf{S}_2\psi \equiv (\mathcal{S}_{20}\cosh(\Delta) + \mathcal{S}_{10}\sinh(\Delta))\psi = 0,$$

with

$$\Delta = -\theta_{51}\theta_{52}L(x_\perp); \quad \mathcal{S}_{i0} = \ (p_i \cdot \theta_i + m\theta_{5i})$$

iv)The interaction enters only through an invariant matrix function Δ

v) Both $\mathcal{S}_i$ and $\mathbf{S}_i$ constraints are compatible for general Δ:

$$[\mathbf{S}_1, \mathbf{S}_2]\psi = 0 \text{ and } [\mathcal{S}_1, \mathcal{S}_2]\psi = 0$$

provided only that

$$\Delta = \Delta(x_\perp).$$

20 Four Polar and Four Axial Interactions

i)For the polar interactions we find

$$\Delta(x_\perp) = -L(x_\perp)\theta_{51}\theta_{52} \text{ scalar}$$

$$\Delta(x_\perp) = J(x_\perp)\hat{P}\cdot\theta_1\hat{P}\cdot\theta_2 \text{ time like vector}$$

$$\Delta(x_\perp) = \mathcal{G}(x_\perp)\theta_{1\perp}\cdot\theta_{2\perp} \text{ space like vector}$$

$$\Delta(x_\perp) = \mathcal{F}(x_\perp)\theta_{1\perp}\cdot\theta_{2\perp}\theta_{51}\theta_{52}\hat{P}\cdot\theta_1\hat{P}\cdot\theta_2 \text{ tensor (polar).}$$

ii) Constraint equations for vector and scalar interactions presented at the beginning of talk are generated in this form by taking $\mathcal{F}=0$ with L and the Feynman gauge combination $\mathcal{G}=-J$.

iii) Constraints for the axial counterparts (note the minus sign) have hyperbolic forms

$$\mathcal{S}_1\psi = (\cosh(\Delta)\mathbf{S}_1 - \sinh\Delta)\mathbf{S}_2)\psi = 0$$

$$\mathcal{S}_2\psi = (\cosh(\Delta)\mathbf{S}_2 - \sinh\Delta)\mathbf{S}_1)\psi = 0,$$

in which $\mathbf{S}_1$ and $\mathbf{S}_2$ are defined as before while the interactions appear through

$$\Delta(x_\perp) = C(x_\perp)/2 \quad \text{pseudoscalar}$$

$$\Delta(x_\perp) = H(x_\perp)\hat{P}\cdot\theta_1\hat{P}\cdot\theta_2\theta_{51}\theta_{52} \quad \text{time like pseudovector}$$

$$\Delta(x_\perp) = I(x_\perp)\theta_{1\perp}\cdot\theta_{2\perp}\theta_{51}\theta_{52} \text{ space like pseudovector}$$

$$\Delta(x_\perp) = Y(x_\perp)\theta_{1\perp}\cdot\theta_{2\perp}\hat{P}\cdot\theta_1\hat{P}\cdot\theta_2 \text{ tensor (axial).}$$

iv) Future research will determine the relative importance of the interactions other than scalar and vector in meson spectroscopy.

21 Nucleon-Nucleon Scattering

i) A problem recently tackled requiring pseudoscalar interactions as well as vector and scalar

ii) With scalar, vector, and pseudoscalar interactions,

$$\Delta(x_\perp) = -L(x_\perp)\theta_{51}\theta_{52} + \mathcal{G}(x_\perp)\theta_1 \cdot \theta_2 - \frac{C(x_\perp)}{2}.$$

iii) With electromagnetic four-vector condition $J(x_\perp) = -\mathcal{G}(x_\perp)$ relating time and space-like components:

$$\theta_1 \cdot \theta_2 = \theta_{1\perp} \cdot \theta_{2\perp} - \hat{P} \cdot \theta_1 \hat{P} \cdot \theta_2; \quad \text{Feynman Gauge}$$

iv)Reduction of these two-body Dirac equations to Schrödinger-like form for combined scalar, time- and space-like vector and pseudoscalar interactions. (Liu (2003) and Long (1998))

$$\Phi_w \rightarrow \Phi_{SI}+\Phi_D+\Phi_{SO}L\cdot(\sigma_1+\sigma_2)+\Phi_{SS}\sigma_1\cdot\sigma_2+\Phi_T\sigma_1\cdot\hat{r}\sigma_2\cdot\hat{r}+\Phi_{SOT}\boldsymbol{\sigma}_1\cdot\hat{r}\sigma_2\cdot\hat{r}L\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}$$

in which (with $K = (L + \mathcal{G})/2$)

$$
\begin{aligned}
\Phi_{SI} &= 2m_w S + S^2 + 2\varepsilon_w A - A^2 \\
\Phi_D &= -4\frac{\sinh^2(K)}{r^2} + \frac{2(3\mathcal{G} - L)' \sinh^2(K)}{r} + \frac{2(L + \mathcal{G})' \sinh(K) \cosh(K)}{r} \\
&\quad + \frac{1}{4}L'^2 + \frac{3}{2}\mathcal{G}'^2 - L'\mathcal{G}' + \frac{1}{4}C'^2 - \frac{1}{2}\vec{\nabla}^2 L + \vec{\nabla}^2 \mathcal{G} \\
\Phi_{SO} &= \frac{(3\mathcal{G} - L)'}{2r} + \frac{(3\mathcal{G} - L)' \sinh^2(K)}{r} - \frac{2\sinh^2(K)}{r^2} + (L + \mathcal{G})'\frac{\cosh(K)\sinh(K)}{r} \\
\Phi_{SS} &= \frac{1}{2}\vec{\nabla}^2 \mathcal{G} + \mathcal{G}'^2 - \frac{1}{2}\mathcal{G}'C' + \frac{1}{2}\frac{(C + L)'}{r} - \frac{1}{2}\mathcal{G}'L' + \frac{4\mathcal{G}'\cosh(K)\sinh(K)}{r} \\
&\quad + \frac{4\mathcal{G}'\sinh^2(K)}{r} - 2\frac{\cosh(K)\sinh(K)}{r^2} - 2\frac{\sinh^2(K)}{r^2} \\
\Phi_T &= \frac{1}{2}\vec{\nabla}^2 C - \frac{3}{2}\mathcal{G}'^2 + \frac{3}{2}\mathcal{G}'C' + \frac{1}{2}L'\mathcal{G}' - \frac{1}{2}C'L' - \frac{3}{2}\frac{(C + L)'}{r} - \frac{1}{2}\vec{\nabla}^2 \mathcal{G} \\
&\quad + \frac{(2L - 10\mathcal{G})'\cosh(K)\sinh(K)}{r} - \frac{(2L + 6\mathcal{G})'\sinh^2(K)}{r} + 6\frac{\cosh(K)\sinh(K}{r^2} \\
\Phi_{SOT} &= -(L + \mathcal{G})'\frac{\sinh^2(K)}{r} + 2\frac{\sinh(K)\cosh(K)}{r^2} - \frac{(L + \mathcal{G})'}{2r} - \frac{(3\mathcal{G} - L)'\sinh(K)}{r}
\end{aligned}
$$

v)This Schrödinger-like form (for equal masses) differs by a scale transformation that eliminates the spin-independent and tensor dependent $\hat{r} \cdot p$ terms from earlier equation.

22 The Meson Exchange Potentials

i) Beyond limitation to the above nine mesons the model dependent assumptions involve specifying how the corresponding 9 Yukawa potentials are included in the three invariant functions $C, L, \mathcal{G} = -J$.

ii) Strong potential terms are structured by

a) assuming that with

$$\mathcal{G} = -\frac{1}{2}\log(1 - \frac{2A}{w})$$

we take

$$\begin{aligned} A &= \mathcal{A} \quad , \mathcal{A} < 0 \\ &= \frac{w}{\pi}\arctan(\frac{\mathcal{A}\pi}{2w}) \quad , \mathcal{A} > 0. ** \end{aligned}$$

where the invariant $\mathcal{A}$ is taken as

$$\mathcal{A} = g_\rho^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\exp(-m_\rho \bar{r})}{\bar{r}} + g_w^2 \frac{\exp(-m_\omega \bar{r})}{\bar{r}} + g_\phi^2 \frac{\exp(-m_\phi \bar{r})}{\bar{r}}.$$

b) For the invariant L we take

$$M_1 = m_1 \cosh L + m_2 \sinh L = \sqrt{m_1^2 + G^2(2m_w S + S^2)} \quad ; S > 0$$

$$M_2 = m_2 \cosh L + m_1 \sinh L = \sqrt{m_2^2 + G^2(2m_w S + S^2)} \quad ; S > 0$$

$$L = -\frac{1}{2}\log(1 - \frac{2S}{w - 2A}); \quad S < 0 \quad **$$

and

$$\mathcal{S} = -g_\sigma^2 \frac{\exp(-m_\sigma \bar{r})}{\bar{r}} - g_{f_0}^2 \frac{\exp(-m_{f_0} \bar{r})}{\bar{r}} - g_{a_0}^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\exp(-m_{a_0} \bar{r})}{\bar{r}}.$$

We need these two modifications (**) of our strong potential terms to accommodate large repulsive vector and large attractive scalar interactions. They will be reflected in corresponding changes in the quasipotential portions.

c) For the pseudoscalar invariant function C

$$C = \frac{1}{w}[g_\pi^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\exp(-m_\pi \bar{r})}{\bar{r}} + g_\eta^2 \frac{\exp(-m_\eta \bar{r})}{\bar{r}} + g_{\eta'}^2 \frac{\exp(-m_{\eta'} \bar{r})}{\bar{r}}].$$

iii) We model effects of form factors by replacing invariant radius r $(=\sqrt{x_\perp^2})$by

$$\bar{r} = \sqrt{r^2 + r_0^2}$$

with r_0 a form factor parameter.

iv) In addition we take into account that the vector mesons may have an anomalous "magnetic moment" type of coupling

$$i\frac{e}{2M}\bar{\psi}[\gamma^\mu, \gamma^\nu]\psi F_{\mu\nu}.$$

The net result is to include pairs of additional vector and scalar Yukawa interactions interactions but with opposite signs

23 Phase Shift Analysis

i) We use the variable phase method developed by Calogero. The details are presented in Phys. Rev C...

We present here results for three different angular momentum states in $n - p$ scattering.

Clearly this model needs some more attention, particularly a) the inclusion of world tensor coupling instead of the field theoretic mass shell assumption, b) the inclusion of pseudovector coupling of the pseudoscalar mesons and c) the off mass shell effects of the vector meson couplings.

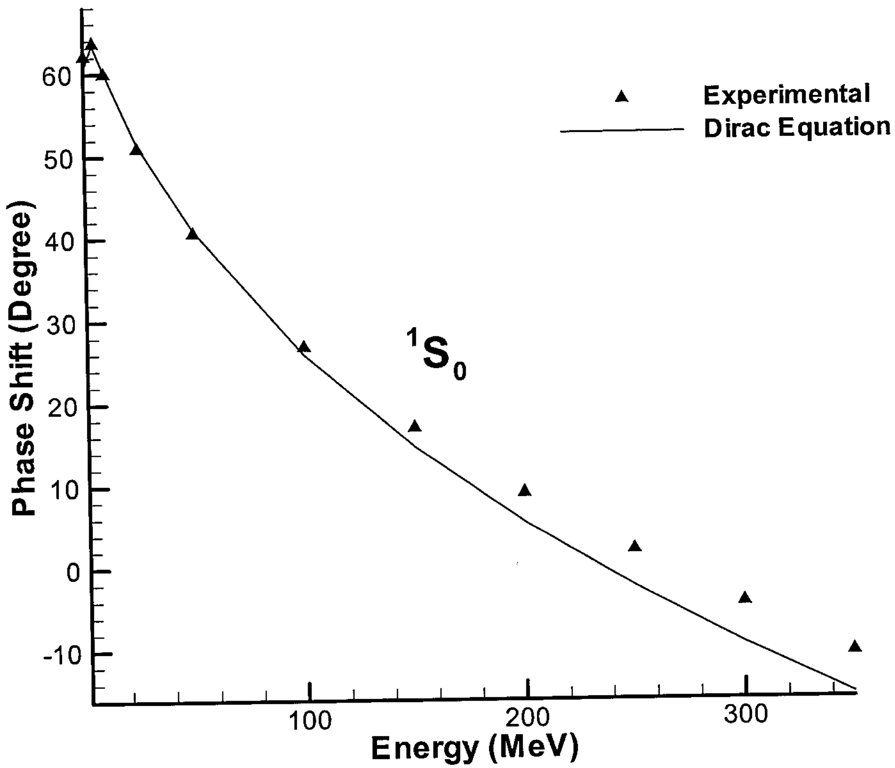


Figure 1: np Scattering Phase Shift of $^1S_0$ State

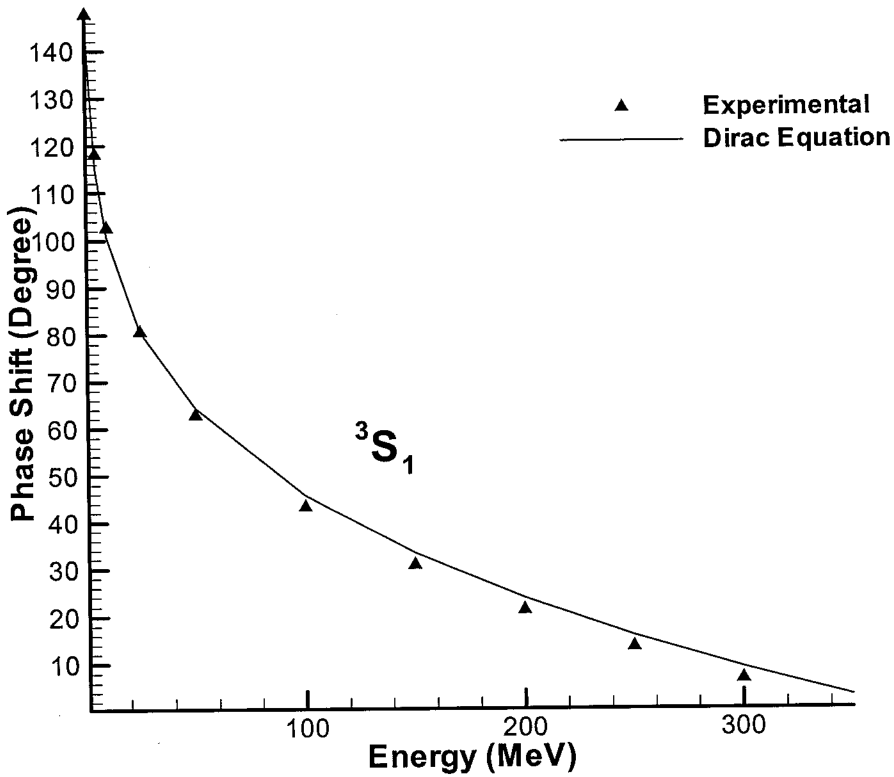


Figure 2: np Scattering Phase Shift of $^3S_1$ State

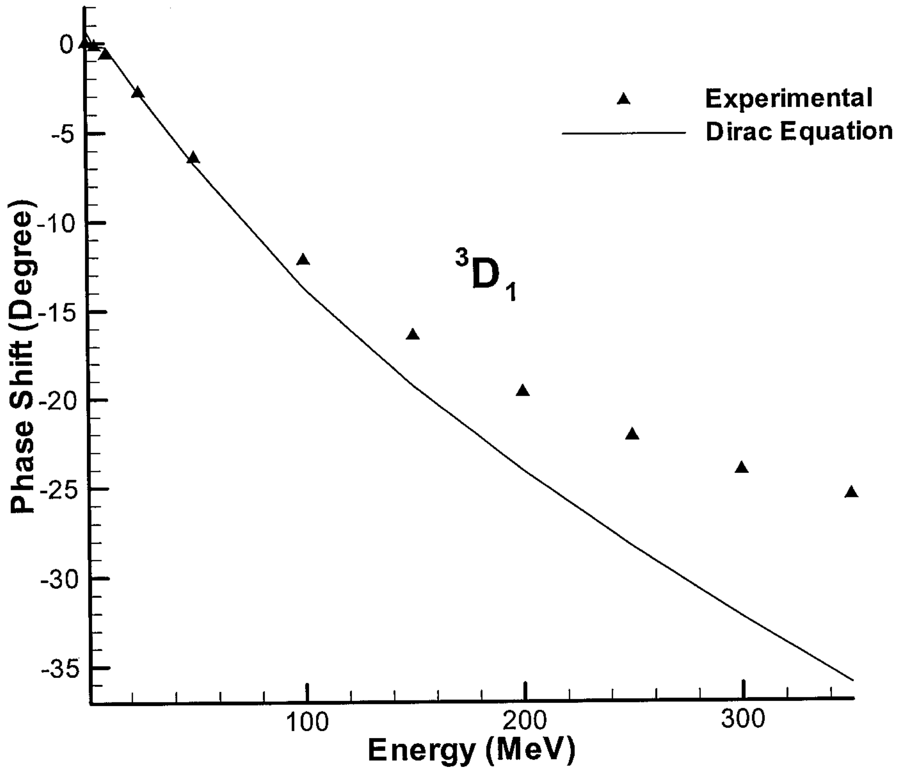


Figure 3: np Scattering Phase Shift of $^3D_1$ State

31 Summary

- Strong Potential form of the two-body Dirac equations tested in QED - spurious results due to form of wave equations unlikely

- Strong Potential form of the two-body Dirac equations tested in QCD -accurate spectral results throughout most of spectrum - Goldstone -boson behavior -two invariant potential functions

- Strong Potential form of two-body Dirac equations tested in NN-scattering. Hyperbolic forms of equation extend to general covariant interactions - Phase shift results promising.

- Covariant and Local Schrödinger-like structure - Simple to implement - able to take advantage of formalisms developed for the nonrelativistic Schrödinger equation -Microscopic theory of meson-meson scattering (Barnes and Swanson) and Unitarized quark model of Törnqvist and Ono

- Direct connection to quantum field theory