

# Radiative decays of $\phi$ -meson about nature of light scalar resonances

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## Abstract

We show on gauge invariance grounds that the new threshold phenomenon is discovered in the radiative decays  $\phi \rightarrow \gamma a_0 \rightarrow \gamma \pi^0 \eta$  and  $\phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0$ . This enables to conclude that production of the lightest scalar mesons  $a_0(980)$  and  $f_0(980)$  in these decays is caused by the four-quark transitions, resulting in strong restrictions on the large  $N_C$  expansions of the decay amplitudes. The analysis shows that these constraints give new evidences in favor of the four-quark nature of  $a_0(980)$  and  $f_0(980)$  mesons.

## I. INTRODUCTION

The lightest scalar mesons  $a_0(980)$  and  $f_0(980)$ , discovered more than thirty years ago, became the hard problem for the naive quark-antiquark ( $q\bar{q}$ ) model from the outset. Really, on the one hand the almost exact degeneration of the masses of the isovector  $a_0(980)$  and isoscalar  $f_0(980)$  states revealed seemingly the structure similar to the structure of the vector  $\rho$  and  $\omega$  mesons, and on the other hand the strong coupling of  $f_0(980)$  with the  $K\bar{K}$  channel pointed unambiguously to a considerable part of the strange quark pair  $s\bar{s}$  in the wave function of  $f_0(980)$ .

In 1977 R.L. Jaffe noted that in the MIT bag model, which incorporates confinement phenomenologically, there are light four-quark scalar states [1]. He suggested also that  $a_0(980)$  and  $f_0(980)$  might be these states with symbolic structures  $a_0^0(980) = (us\bar{u}\bar{s} - ds\bar{d}\bar{s})/\sqrt{2}$  and  $f_0(980) = (us\bar{u}\bar{s} + ds\bar{d}\bar{s})/\sqrt{2}$ . From that time  $a_0(980)$  and  $f_0(980)$  resonances came into beloved children of the light quark spectroscopy, see, for example, reviews [2,3].

Ten years later we showed [4] that the study of the radiative decays

$\phi \rightarrow \gamma a_0 \rightarrow \gamma \pi \eta$  and  $\phi \rightarrow \gamma f_0 \rightarrow \gamma \pi \pi$  can shed light on the problem of  $a_0(980)$  and  $f_0(980)$  mesons. Over the next ten years before experiments (1998) the question was considered from different points of view [5–9].

Now these decays have been studied not only theoretically but also experimentally.

The first measurements have been reported by the SND [10,11] and CMD-2 [12] Collaborations which obtain the following branching ratios

$$BR(\phi \rightarrow \gamma \pi^0 \eta) = (0.88 \pm 0.14 \pm 0.09) \cdot 10^{-4} \quad [10] \quad (2000),$$

$$BR(\phi \rightarrow \gamma \pi^0 \pi^0) = (1.221 \pm 0.098 \pm 0.061) \cdot 10^{-4} \quad [11] \quad (2000),$$

$$BR(\phi \rightarrow \gamma \pi^0 \eta) = (0.9 \pm 0.24 \pm 0.1) \cdot 10^{-4},$$

$$BR(\phi \rightarrow \gamma \pi^0 \pi^0) = (0.92 \pm 0.08 \pm 0.06) \cdot 10^{-4} \quad [12].$$

More recently the KLOE Collaboration has measured [13,14]

$$BR(\phi \rightarrow \gamma \pi^0 \eta) = (0.851 \pm 0.051 \pm 0.057) \cdot 10^{-4} \text{ in } \eta \rightarrow \gamma \gamma \quad [13],$$

$$BR(\phi \rightarrow \gamma \pi^0 \eta) = (0.796 \pm 0.060 \pm 0.040) \cdot 10^{-4} \text{ in } \eta \rightarrow \pi^+ \pi^- \pi^0 \quad [13],$$

$$BR(\phi \rightarrow \gamma \pi^0 \pi^0) = (1.09 \pm 0.03 \pm 0.05) \cdot 10^{-4} \quad [14],$$

in agreement with the Novosibirsk data [10–12] but with a considerably smaller error.

Note that the isovector  $a_0(980)$  meson is produced in the radiative  $\phi$  meson decay as intensively as the well-studied  $\eta'(958)$  meson containing  $\approx 66\%$  of  $s\bar{s}$ , responsible for the decay ( $\phi \approx s\bar{s} \rightarrow \gamma s\bar{s} \rightarrow \gamma\eta'(958)$ ). It is a clear qualitative argument for the presence of the  $s\bar{s}$  pair in the isovector  $a_0(980)$  state, i.e., for its four-quark nature.

Since the one-loop model  $\phi \rightarrow K^+K^- \rightarrow \gamma a_0$  and  $\phi \rightarrow K^+K^- \rightarrow \gamma f_0$ , see Fig. 1, suggested at basing the experimental investigations [4], is used in the data treatment [10–14], the question on the mechanism of the scalar meson production in the  $\phi$  radiative decays is put into the shade.

We show below in Section II that the present data give the conclusive arguments in favor of the  $K^+K^-$  loop transition, see Fig. 1, as the principal mechanism of  $a_0(980)$  and  $f_0(980)$  meson production in the  $\phi$  radiative decays.

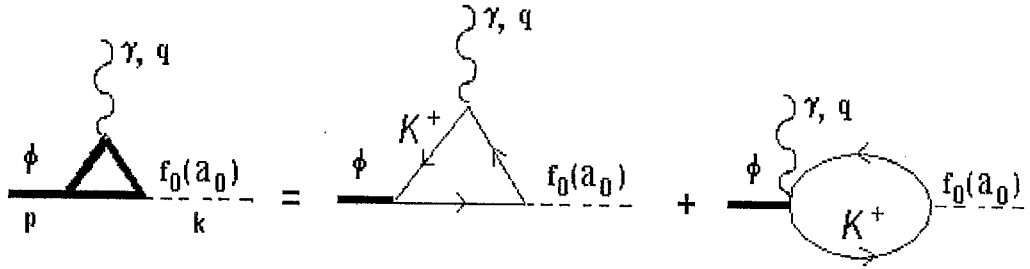


FIG. 1. Diagrams of the  $K^+K^-$  loop model.

In Section III we show that the knowledge of this mechanism allows to conclude that the production of  $a_0(980)$  and  $f_0(980)$  in the  $\phi$  radiative decays is caused by the four-quark transitions. This constrains the large  $N_C$  expansions of the decay amplitudes and gives new impressive evidences in favor of the four-quark nature of  $a_0(980)$  and  $f_0(980)$  [1,3,4,6,15–17].

Section IV contains concluding remarks.

## II. THE MECHANISM OF THE $a_0(980)$ AND $f_0(980)$ PRODUCTION IN THE $\phi$ RADIATIVE DECAYS

In Figs. 2 and 3 are shown the KLOE data on  $\phi \rightarrow \gamma\pi^0\eta$  [13] and the SND data on  $\phi \rightarrow \gamma\pi^0\pi^0$  [11] (2000) respectively. The similar excitations of the  $a_0(980)$  and  $f_0(980)$  resonances are observed also by the SND [10], CMD-2 [12] and KLOE [14] Collaborations, respectively.

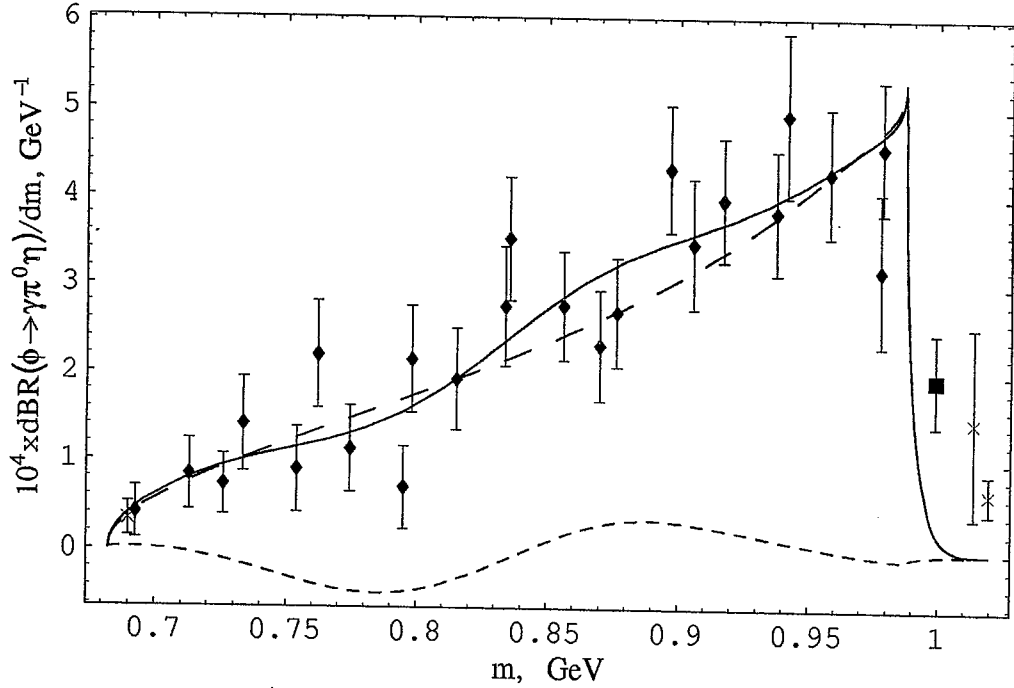


FIG. 2. Fitting of  $10^4 \times dBR(\phi \rightarrow \gamma\pi^0\eta)/dm$  is shown with the solid line, the signal contribution is shown with the dashed line.

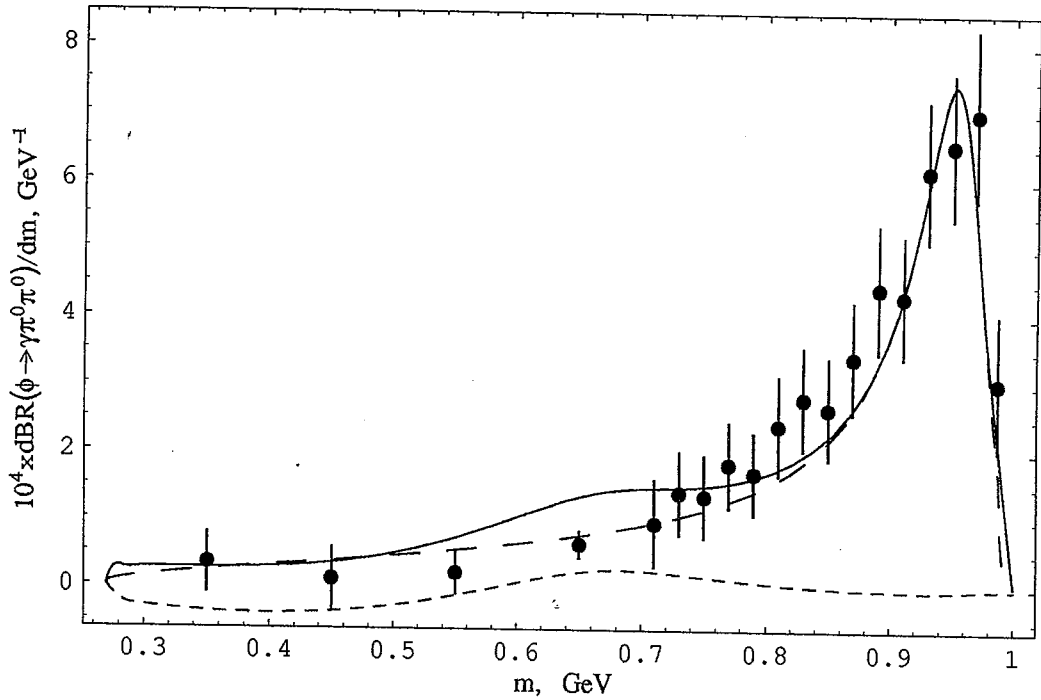


FIG. 3. Fitting of  $10^4 \times dBR(\phi \rightarrow \gamma\pi^0\pi^0)/dm$  with the background is shown with the solid line, the signal contribution is shown with the dashed line. The dotted line is the interference term.

The data are described in the model  $\phi \rightarrow (\gamma a_0 + \pi^0 \rho) \rightarrow \gamma \pi^0 \eta$  and  $\phi \rightarrow (\gamma f_0 + \pi^0 \rho) \rightarrow \gamma \pi^0 \pi^0$ , see details in Ref. [17].

As Figs. 2 and 3 suggest, the  $\phi \rightarrow \gamma a_0 \rightarrow \gamma \pi^0 \eta$  process dominates everywhere over the region of the  $\pi^0 \eta$  invariant mass  $m_{\pi^0 \eta} = m$  and the  $\phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0$  process dominates in the resonance region of the  $\pi^0 \pi^0$  system, the  $\pi^0 \pi^0$  invariant mass  $m_{\pi^0 \pi^0} = m > 780 \text{ MeV}$ <sup>1</sup>.

The resonance mass spectrum is of the form<sup>2</sup>

$$\begin{aligned}
 S_R(m) &= d\Gamma(\phi \rightarrow \gamma R \rightarrow \gamma ab, m)/dm \\
 &= \frac{2 m^2 \Gamma(\phi \rightarrow \gamma R, m) \Gamma(R \rightarrow ab, m)}{\pi |D_R(m)|^2} \\
 &= \frac{4 |g_R(m)|^2 \omega(m) p_{ab}(m)}{3(4\pi)^3 m_\phi^2} \left| \frac{g_{Rab}}{D_R(m)} \right|^2, \tag{1}
 \end{aligned}$$

where  $R = a_0$  or  $f_0$  and  $ab = \pi^0 \eta$  or  $\pi^0 \pi^0$  respectively,  $\omega(m) = (m_\phi^2 - m^2)/2m_\phi$  is the photon energy in the  $\phi$  meson rest frame,  $p_{ab}(m)$  is the modulus of the  $a$  or  $b$  particle momentum in the  $a$

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<sup>1</sup>A.V. Kiselev noted kindly that in Ref. [17] the solid curve at  $m < 780 \text{ MeV}$  is drawn incorrectly. He also kindly prepared the correct figure.

<sup>2</sup>Notice that in Ref. [17] we took into account the mixing of  $f_0(980)$  meson with another scalar isoscalar resonance, see also Ref. [6], but such a complication is not essential for the present investigation.

and  $b$  mass center frame,  $g_{Rab}$  is the coupling constant,  $g_{f_0\pi^0\pi^0} = g_{f_0\pi^+\pi^-}/\sqrt{2}$ ,  $D_R(m)$  is the  $R$  resonance propagator the form of which everywhere over the  $m$  region can be find in [4,18,19],  $g_R(m)$  is the invariant amplitude that describes the vertex of the  $\phi(p) \rightarrow \gamma(k)R(q)$  transition with  $q^2 = m^2$ . This is precisely the function which is the subject of our investigation.

By gauge invariance, the transition amplitude is proportional to the electromagnetic field strength tensor  $F_{\mu\nu}$  (in our case to the electric field in the  $\phi$  meson rest frame):

$$\begin{aligned} A[\phi(p) \rightarrow \gamma(k)R(q)] \\ = G_R(m) [p_\mu e_\nu(\phi) - p_\nu e_\mu(\phi)] [k_\mu e_\nu(\gamma) - k_\nu e_\mu(\gamma)], \end{aligned} \quad (2)$$

where  $e(\phi)$  and  $e(\gamma)$  are the  $\phi$  meson and  $\gamma$  quantum polarization four-vectors,  $G_R(m)$  is the invariant amplitude free from kinematical singularities. Since there are no charge particles or particles with magnetic moments in the process, there is no pole in  $G_R(m)$ .

Consequently, the function

$$g_R(m) = -2(pk)G_R(m) = -2\omega(m)m_\phi G_R(m) \quad (3)$$

is proportional to  $\omega(m)$  (at least!) in the soft photon region.



To describe the experimental spectra similar to the ones in Figs. 2 and 3<sup>3</sup>, the function  $|g_R(m)|^2$  should be smooth (almost constant) in the range  $m \leq 0.99$  GeV, see Eq. (1). Stopping the function  $(\omega(m))^2$  at  $\omega(990 \text{ MeV}) = 29 \text{ MeV}$  with the help of the form-factor  $1/[1 + (R\omega(m))^2]$  requires  $R \approx 100 \text{ GeV}^{-1}$ . It seems to be incredible to explain the formation of such a huge radius in hadron physics. Based on the large, by hadron physics standard,  $R \approx 10 \text{ GeV}^{-1}$ , one can obtain an effective maximum of the mass spectrum under discussion only near 900 MeV. To exemplify this trouble let us consider the contribution of the isolated R resonance:  $g_R(m) = -2\omega(m)m_\phi G_R(m_R)$ . Let also the mass and the width of the R resonance equal 980 MeV and 60 MeV, then  $S_R(920 \text{ MeV}) : S_R(950 \text{ MeV}) : S_R(970 \text{ MeV}) : S_R(980 \text{ MeV}) = 3 : 2.7 : 1.8 : 1$ .

So stopping the  $g_R(m)$  function is the crucial point in understanding the mechanism of the production of  $a_0(980)$  and  $f_0(980)$  resonances in the  $\phi$  radiative decays.

The  $K^+K^-$  loop model  $\phi \rightarrow K^+K^- \rightarrow \gamma R$  [4] solves this prob-

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<sup>3</sup>Note that  $S_R(m) = \Gamma_\phi dBR(\phi \rightarrow \gamma R \rightarrow \gamma ab, m)/dm$ .

lem in the elegant way: the fine threshold phenomenon is discovered, see Fig. 4, where the universal in  $K^+K^-$  loop model function  $|g(m)|^2 = |g_R(m)/g_{RK^+K^-}|^2$  is shown <sup>4</sup>.

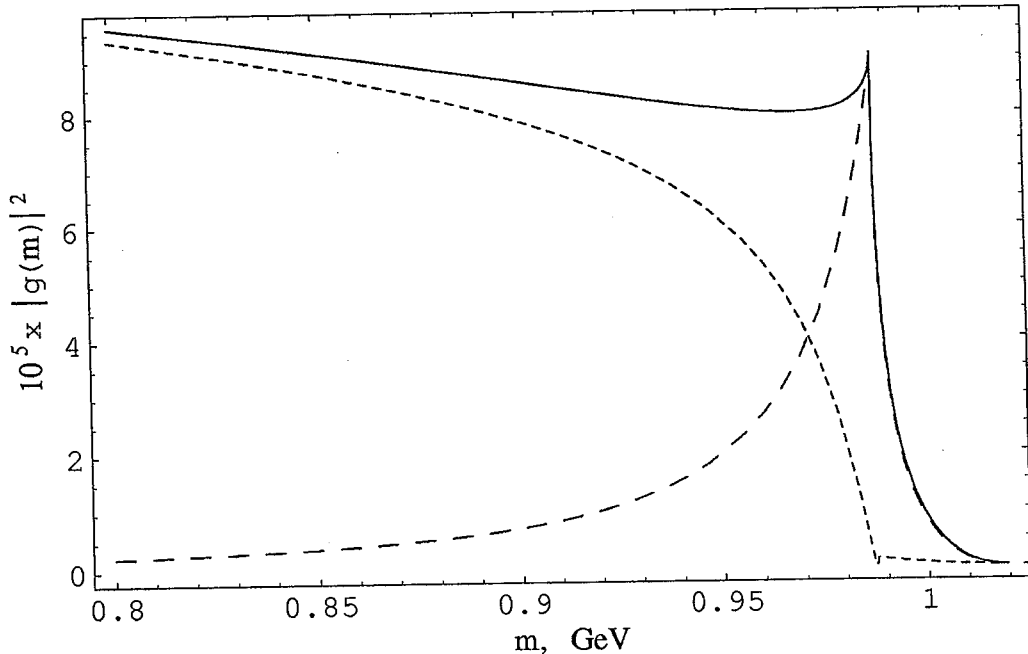


FIG. 4. The function  $|g(m)|^2$  is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

To demonstrate the threshold character of this effect we present Fig. 5 and Fig. 6 in which the function  $|g(m)|^2$  is shown in the case of  $K^+$  meson mass is 25 MeV and 50 MeV less than in reality.

<sup>4</sup>The forms of  $g_R(m)$  and  $g(m) = g_R(m)/g_{RK^+K^-}$  everywhere over the  $m$  region are in Refs. [4] and [19] respectively.

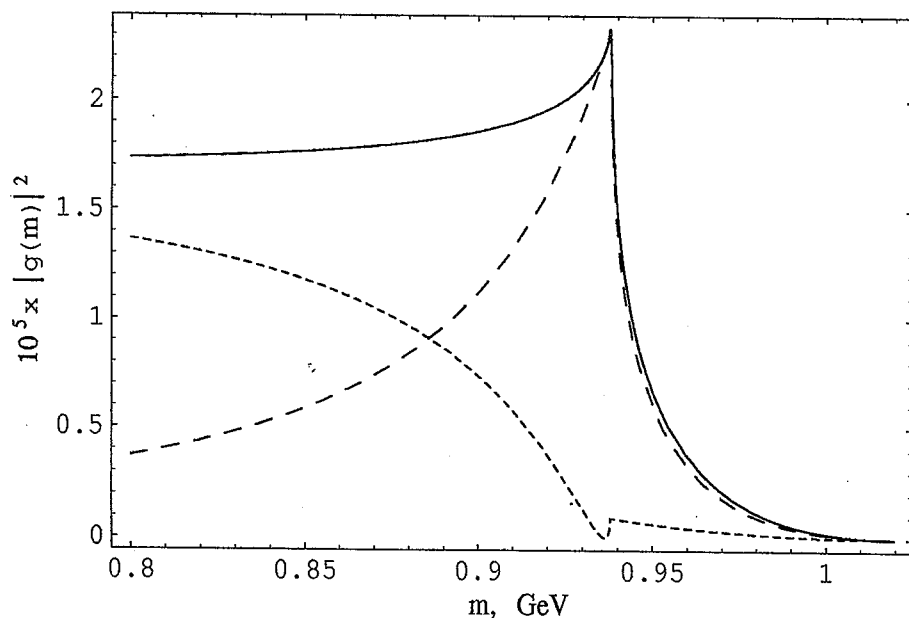


FIG. 5. The function  $|g(m)|^2$  for  $m_{K^+} = 469$  MeV is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

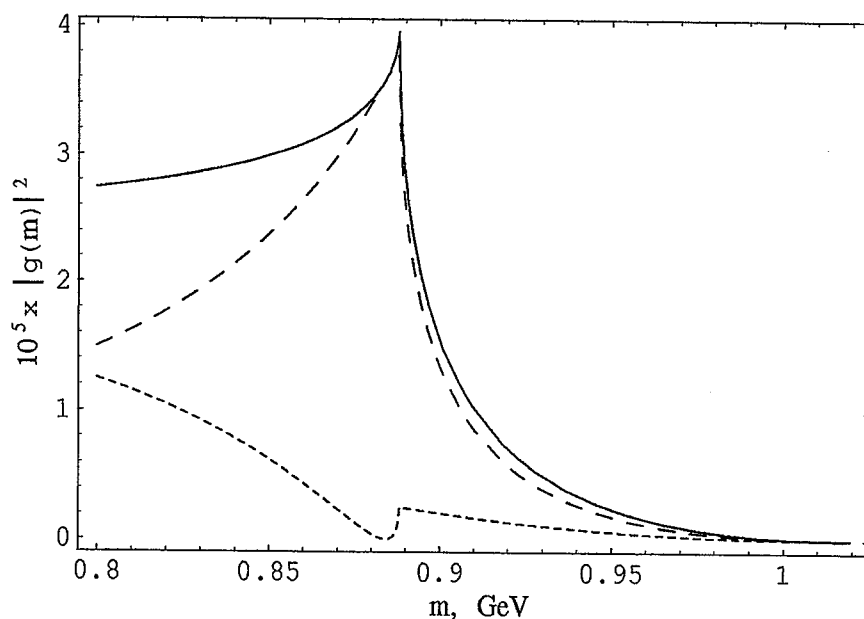


FIG. 6. The function  $|g(m)|^2$  for  $m_{K^+} = 444$  MeV is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

One can see from Figs. 5 and 6 that the function  $|g(m)|^2$  is suppressed by the  $(\omega(m))^2$  law in the region 950-1020 MeV and 900-1020 MeV respectively <sup>5</sup>.

In the mass spectrum this suppression is increased by one more power of  $\omega(m)$ , see Eq. (1), so that we cannot see the resonance in the region 980-995 MeV. The maximum in the spectrum is effectively shifted to the region 935-950 MeV and 880-900 MeV respectively.

In truth this means that  $a_0(980)$  and  $f_0(980)$  resonances are seen in the radiative decays of  $\phi$  meson owing to the  $K^+K^-$  intermediate state, otherwise the maxima in the spectra would be shifted to 900 MeV.

So the mechanism of production of  $a_0(980)$  and  $f_0(980)$  mesons in the  $\phi$  radiative decays is established.

### III. THE LARGE $N_C$ EXPANSION OF THE $\phi \rightarrow \gamma a_0$ AND $\phi \rightarrow \gamma f_0$ AMPLITUDES

Both real and imaginary parts of the  $\phi \rightarrow \gamma R$  amplitude are caused by the  $K^+K^-$  intermediate state. The imaginary part is caused by

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<sup>5</sup>The actual absence of a background at a soft photon energy region  $\omega(m) < 112$  MeV ( $m > 900$  MeV) owes to gauge invariance also.

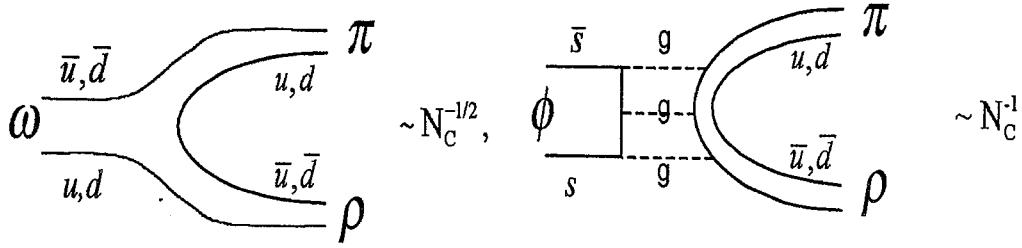
the real  $K^+K^-$  intermediate state while the real part is caused by the virtual compact  $K^+K^-$  intermediate state, i.e., we are dealing here with the four-quark transition <sup>6</sup>. Needless to say, radiative four-quark transitions can happen between two  $q\bar{q}$  states as well as between  $q\bar{q}$  and  $q^2\bar{q}^2$  states but their intensities depend strongly on a type of the transitions. A radiative four-quark transition between two  $q\bar{q}$  states requires creation and annihilation of an additional  $q\bar{q}$  pair, i.e., such a transition is forbidden according to the Okuba-Zweig-Izuka (OZI) rule, while a radiative four-quark transition between  $q\bar{q}$  and  $q^2\bar{q}^2$  states requires only creation of an additional  $q\bar{q}$  pair, i.e., such a transition is allowed according to the OZI rule.

Let us consider this problem from the large  $N_C$  expansion standpoint, using the G.'t Hooft rules [20]:  $g_s^2 N_C \rightarrow const$  at  $N_C \rightarrow \infty$  and a gluon is equivalent to a quark-antiquark pair ( $A_j^i \sim q^i \bar{q}_j$ ).

Fig. 7 reminds us of the large  $N_C$  expansion of some well-known decay amplitudes.

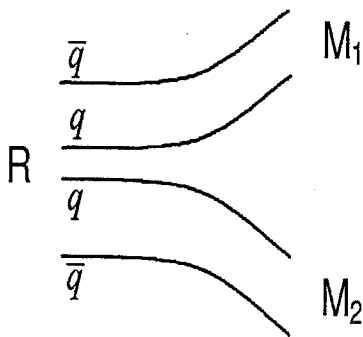
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<sup>6</sup>It will be recalled that the imaginary part of every hadronic amplitude describes a multi-quark transition.

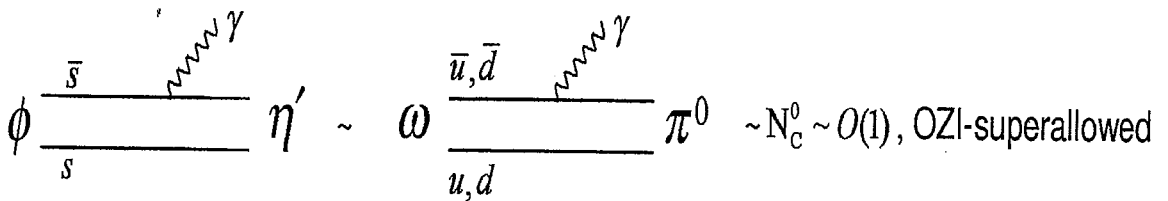


OZI-allowed

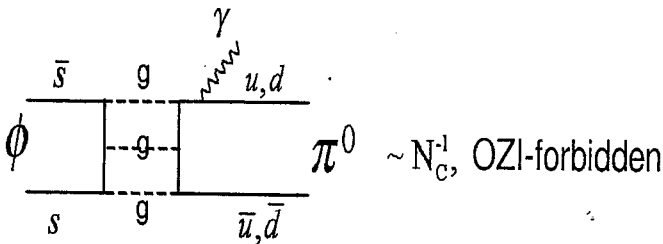
OZI-forbidden



$\sim N_c^0 \sim O(1)$ , OZI-superallowed



$\sim N_c^0 \sim O(1)$ , OZI-superallowed



$\sim N_c^{-1}$ , OZI-forbidden

FIG. 7. The large  $N_C$  expansion of some well-known decay amplitudes. The graphs with the obvious permutations of the gamma quantum are understood.

Let us begin our consideration with the  $q\bar{q}$  model.

In the two-quark model  $a_0^0(980) = (u\bar{u} - d\bar{d})/\sqrt{2}$ , the large  $N_C$  expansion of the  $\phi = s\bar{s} \rightarrow \gamma a_0(980)$  amplitude starts with the OZI forbidden transition of the  $1/N_C$  order:  $s\bar{s}$  annihilation and  $u\bar{u}$ ,  $d\bar{d}$  creation, see Fig. 8.

But its weight is bound to be small, because this term does not contain the  $K^+K^-$  intermediate state, which emerges only in the next to leading term of the  $(1/N_C)^2$  (!) order for creation and annihilation of additional  $q\bar{q}$  pairs, see Fig. 8.

Note that  $\phi = s\bar{s} \rightarrow \gamma s\bar{s} \rightarrow \gamma \eta'(958)$  transition ( as intensive experimentally as  $\phi \rightarrow \gamma a_0(980)$  ) does not require creation of an additional  $q\bar{q}$  pair at all (the OZI superallowed transition) and has the  $(N_C)^0$  order, see Fig. 7.

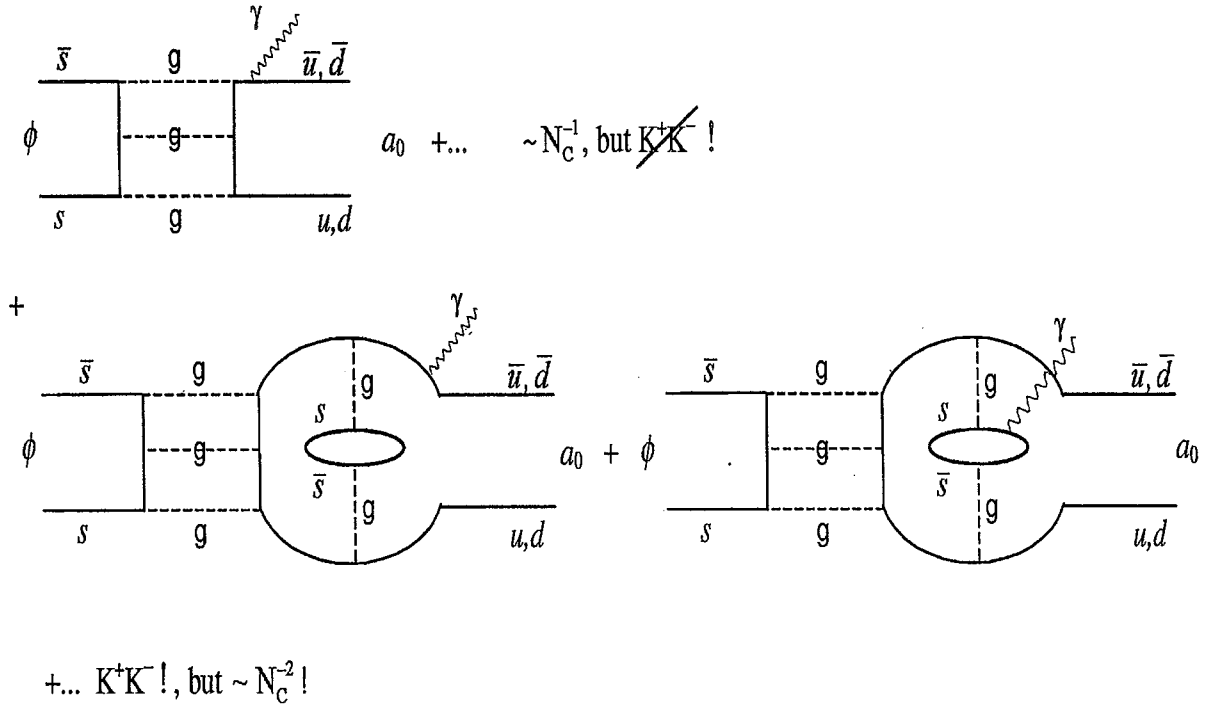


FIG. 8. The large  $N_C$  expansion of the  $\phi \rightarrow \gamma a_0(980)$  amplitude in the two-quark model  $a_0^0(980) = (u\bar{u} - d\bar{d})/\sqrt{2}$ . The graphs with the obvious permutations of the gamma quantum are understood.

In the two-quark model  $f_0(980) = (u\bar{u} + d\bar{d})/\sqrt{2}$ , which involves the  $a_0$ - $f_0$  mass degeneration, the large  $N_C$  expansion of the  $\phi = s\bar{s} \rightarrow \gamma f_0(980)$  amplitude starts also with the OZI forbidden transition of the  $1/N_C$  order:  $s\bar{s}$  annihilation and  $u\bar{u}$ ,  $d\bar{d}$  creation, see Fig. 9, whose weight also is bound to be small, because this term does not contain the  $K^+ K^-$  intermediate state, which emerges only in the next to leading term of the  $(1/N_C)^2$  order, see Fig. 9.



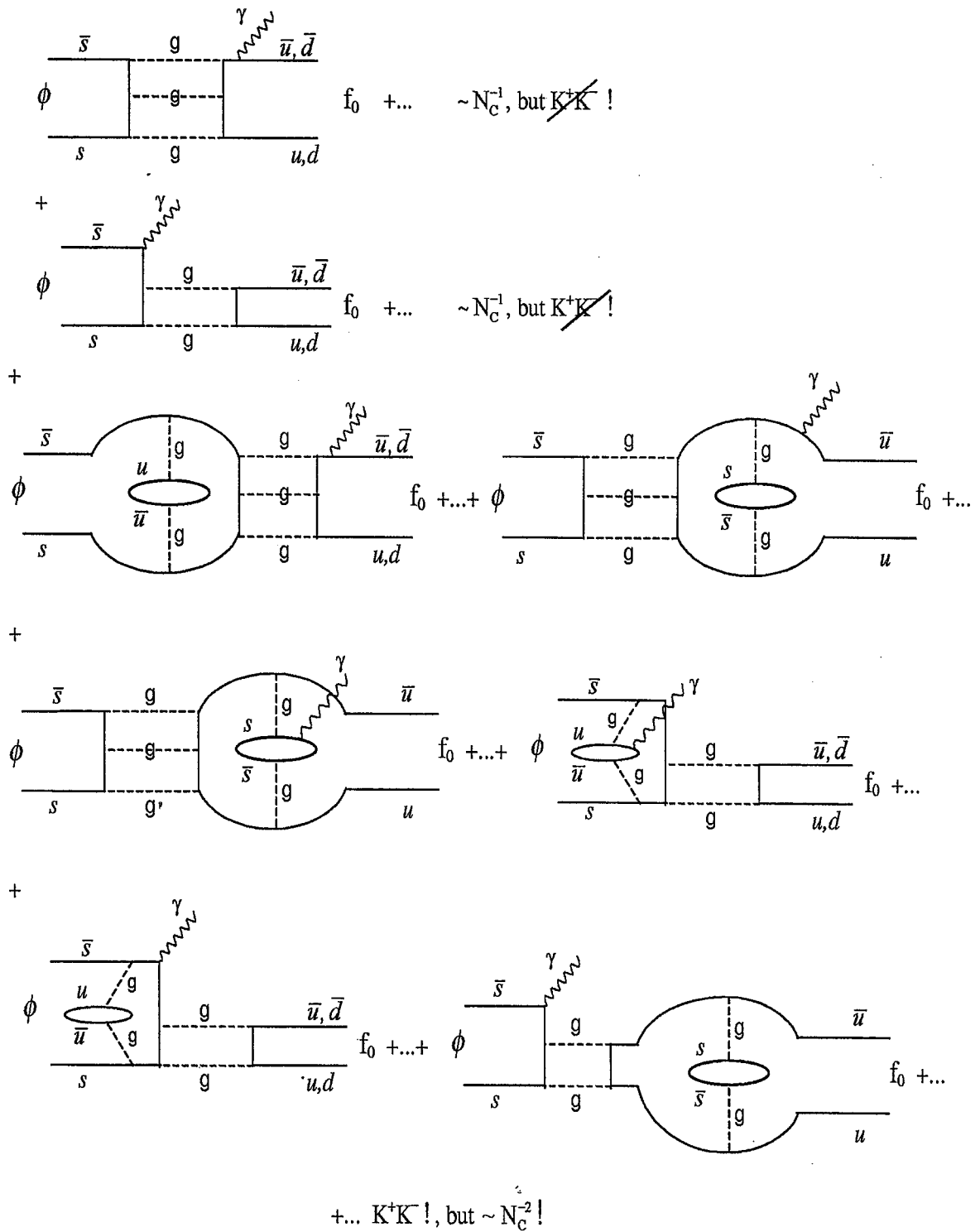


FIG. 9. The large  $N_C$  expansion of the  $\phi \rightarrow \gamma f_0(980)$  amplitude in the two-quark model  $f_0(980) = (u\bar{u} + d\bar{d})/\sqrt{2}$ . The graphs with the obvious permutations of the gamma quantum are understood.

In the two-quark model  $f_0(980) = s\bar{s}$ , which has the serious trouble with the  $a_0$ - $f_0$  mass degeneration, the  $(N_C)^0$  order transition without creation of an additional  $q\bar{q}$  pair <sup>7</sup>, see Fig. 10, is bound to have a small weight in the large  $N_C$  expansion of the  $\phi = s\bar{s} \rightarrow \gamma f_0(980)$  amplitude, because this term does not contain the  $K^+K^-$  intermediate state, which emerges only in the next to leading term of the  $1/N_C$  order, i.e., in the OZI forbidden transition, see Fig. 10. Emphasize that the mechanism without creation and annihilation of the additional  $u\bar{u}$  pair cannot explain the  $S_{f_0}(m)$  spectrum because it does not contain the  $K^+K^-$  intermediate state!

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<sup>7</sup>In this regard the  $(N_C)^0$  order mechanism is similar to the principal mechanism of the  $\phi \rightarrow \gamma\eta'(958)$  decay, see Fig. 7.

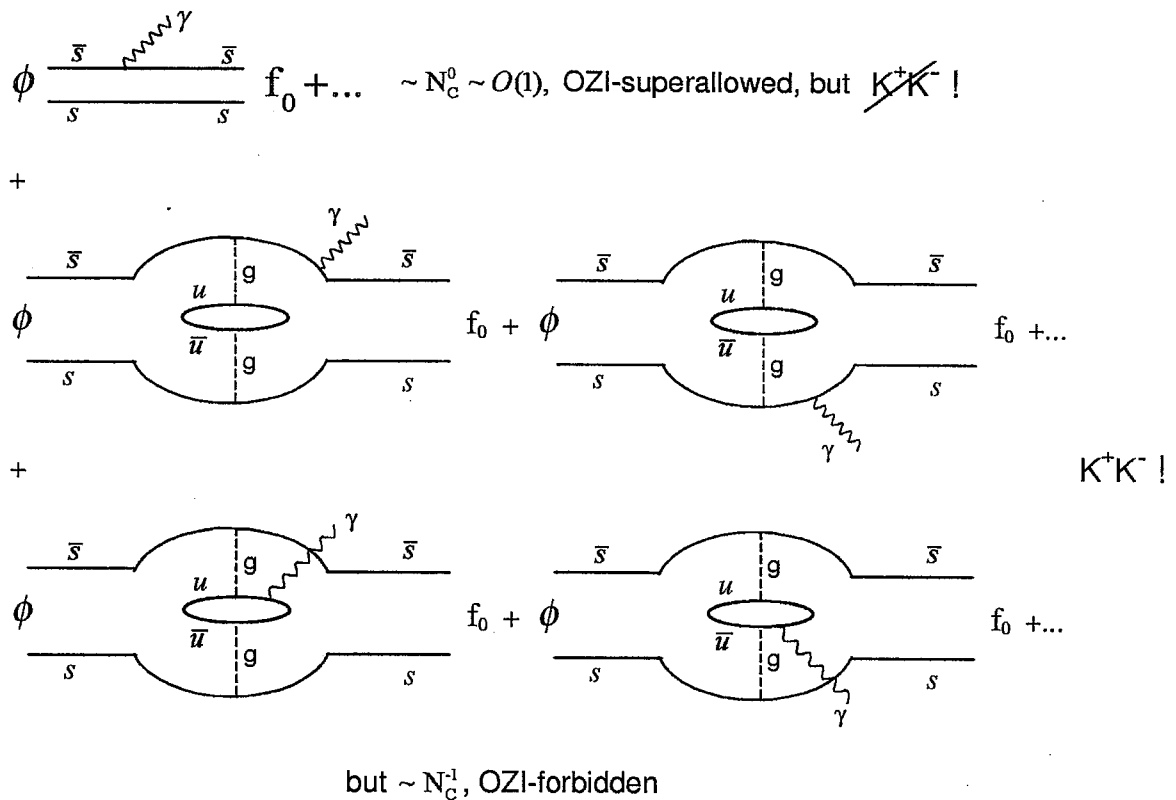


FIG. 10. The large  $N_C$  expansion of the  $\phi \rightarrow \gamma f_0(980)$  amplitude in the two-quark model  $f_0(980) = s\bar{s}$ . The graphs with the obvious permutations of the gamma quantum are understood.

But if  $a_0^0(980)$  and  $f_0(980)$  mesons are compact  $K\bar{K}$  states  $a_0^0(980) = (u\bar{s}s\bar{u} - d\bar{s}s\bar{d})/\sqrt{2}$  and  $f_0(980) = (u\bar{s}s\bar{u} + d\bar{s}s\bar{d})/\sqrt{2}$ , i.e., four-quark states similar ( but need not be the same! ) to states of the MIT-bag model, the large  $N_C$  expansions of the  $\phi = s\bar{s} \rightarrow \gamma a_0(980)$  and  $\phi = s\bar{s} \rightarrow \gamma f_0(980)$  amplitudes start with the OZI allowed transitions of the  $(1/N_C)^{-1/2}$  order, which require only creation the additional  $u\bar{u}$  pair for the  $K^+K^-$  intermediate

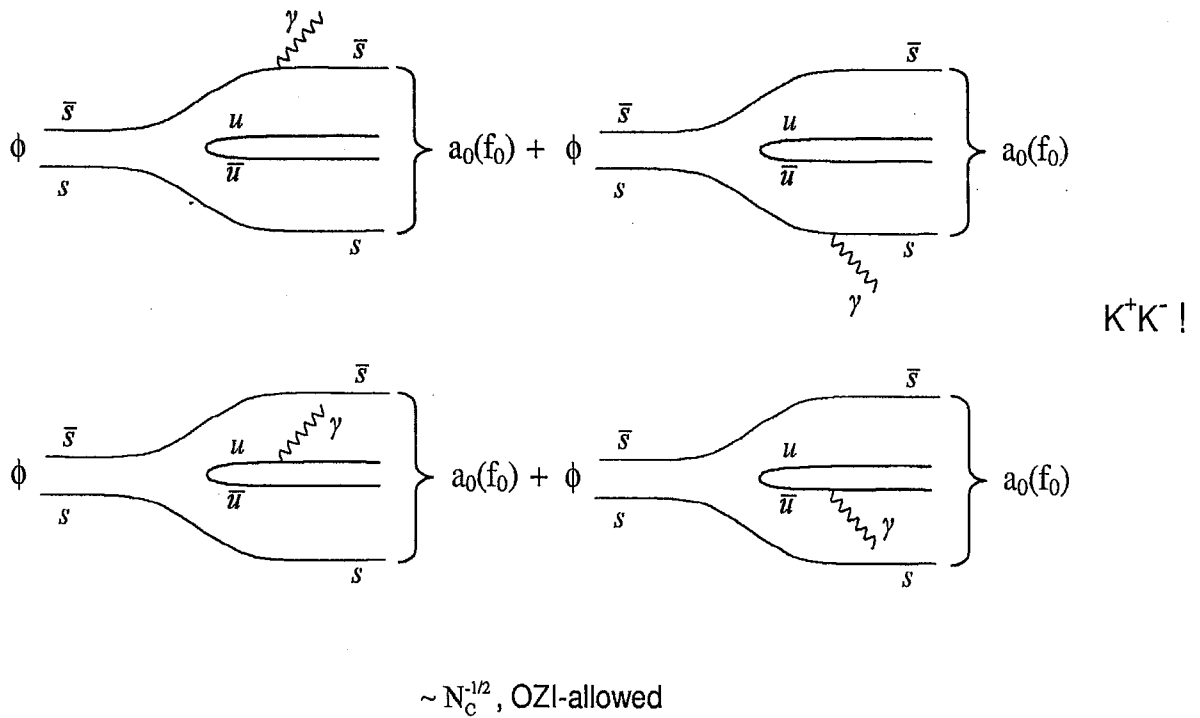


FIG. 11. The large  $N_C$  expansion of the  $\phi \rightarrow \gamma a_0(980)$  and  $\phi \rightarrow \gamma f_0(980)$  amplitudes in the four-quark model  $a_0^0(980) = (u\bar{s}s\bar{u} - d\bar{s}s\bar{d})/\sqrt{2}$  and  $f_0(980) = (u\bar{s}s\bar{u} + d\bar{s}s\bar{d})/\sqrt{2}$ .

state, see Fig. 11<sup>8</sup>. It will be recalled that a OZI allowed hadronic decay amplitude, for example, the  $\rho \rightarrow \pi\pi$  amplitude, has the

<sup>8</sup>In the case of the  $K\bar{K}$  bound states with the binding energy close to 20 MeV, i.e., in the extended molecule case, the contribution of the virtual intermediate  $K^+K^-$  states in the  $K^+K^-$  loop is suppressed by the momentum distribution in the molecule, and the real part of the loop amplitudes are negligible [7]. It leads to the branching ratios much less than the experimental ones. In addition, the  $S_R(m)$  spectra in the  $K\bar{K}$  molecule case are much narrower than the experimental ones, see the behavior of the imaginary part contribution in Fig. 4.

$(1/N_C)^{-1/2}$  order, see Fig. 7.

#### IV. CONCLUSION

In summary the fine threshold phenomenon is discovered, which is to say that the  $K^+K^-$  loop mechanism of the  $a_0(980)$  and  $f_0(980)$  scalar meson production in the  $\phi$  radiative decays is established at a physical level of proof. The case is rarest in hadron physics. This production mechanism is the four-quark transition what constrains the large  $N_C$  expansion of the  $\phi \rightarrow \gamma a_0(980)$  and  $\phi \rightarrow \gamma f_0(980)$  amplitudes and gives the new strong (if not crucial) evidences in favor of the four-quark nature of  $a_0(980)$  and  $f_0(980)$  mesons.

#### V. ACKNOWLEDGEMENT

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