# $\kappa, \sigma$ Mesons in $J / \psi$ Decays Analyzed by VMW Method 

T. Komada, S. Ishida, M.Y.Ishida ${ }^{A}$ K. Ukai ${ }^{B}$, K. Takamatsu ${ }^{B}$, T. Tsuru ${ }^{B}$, I. Yamauchi ${ }^{C}$, K. Yamada, BES Collaboration ${ }^{D}$<br>Nihon Univ., Tokyo Institute of Technology ${ }^{A}$, KEK $^{B}$, Tokyo Metropolitan College of Technology ${ }^{C}$, IHEP $^{D}$

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## 1. Introduction

[ Two contrasting viewpoints of Level Classification]

|  | Non-Relativistic Q.M. | Relativistic Q.M. |
| :---: | :---: | :---: |
| Model | Non-Rel. Q.M. | NJL model |
| Approx. Symm. | LS-Symm. | Chiral Symm. |
| Evidence | Bases for PDG | $\pi$-octet as NG Boson |

(Recent Progress) It seems to be established !?
Existence of light $\sigma(600)$ , as chiral partner of $\pi(140)$,
$\Rightarrow$ Seemingly contradictory two viewpoints
; Serious Problem in Hadron Spectroscopy !

New level classification scheme
Unifying the above two viewpoints was proposed recently

$$
\tilde{U}_{S F}(12) \otimes O_{L}(3,1) \text { sheme }
$$

$\uparrow$ relativistic extension

$$
S U(6) \otimes O_{L}(3) \text { sheme }
$$

$\rightarrow$ Existence of chiral particles/states are predicted
This was reviewed by Prof. S.Ishida in this symposium
One of the Candidate for Chiral States

$$
\underline{\sigma-\text { nonet }} \quad S^{(N)}\left({ }^{1} S_{0}\right) 0^{++} ; \underline{\sigma(600)}, \kappa \underline{\kappa(900)}, a_{0}(980), f_{0}(980)
$$

$$
\text { [Porpose ] • } J / \psi \rightarrow K^{*}(892)+(\underbrace{K \pi}_{\underline{\kappa \pi}})
$$

- $J / \psi \rightarrow \omega+(\underbrace{\pi \pi}_{\underline{\sigma}})$
- S. Ishida and M. Ishida, PLB539 (2002), 249.
- S. Ishida et al., PTP104 (2000), 785.

$$
\underline{\sigma}
$$

2. Method of Analysis of $K \pi-, \pi \pi-$ Production Processes
[Strong interaction and right bases of $S$-matrix ]
(Basic consideration of Strong interaction )
$H^{\text {strong }}\left(\bar{\phi}_{i}\right)=$ "residual interaction" of QCD $\bar{\phi}_{i}$ : color-neutral bound states of $q, \bar{q}, g$ (bare state)

Unitarity of $S$-Matrix $\quad ; \quad S^{\dagger} S=S S^{\dagger}=1 \quad \Leftarrow \quad H_{I}^{\dagger}=H_{I}$

History of picture on strong interaction of $\pi \mathrm{N}$ System

|  | Chew-Low | Theory | After quark physics |
| :---: | :---: | :---: | :---: |
| Switch off $\mathcal{H}^{I}$ | Basic fields | $\pi, N$ | $\bar{\pi}=(q \bar{q}), \bar{N}, \underline{\bar{\Delta}}=(q q q)$ "zero" $\Gamma$ |
| Switch on $\mathcal{H}^{I}$ | Resonance | $\Delta=(N \pi+N \pi \pi)$ | $\pi_{\text {phys. }}, N_{\text {phys. }} ; \Delta_{\text {phys. }}$ "finite" $\Gamma$ |
|  | Compl. Set of $S$-bases | $\|\pi\rangle,\|N\rangle,\|\pi N\rangle, \cdots$ | $\|\bar{\pi}\rangle,\|\bar{N}\rangle, \underline{\|\bar{\Delta}\rangle}\|\bar{\pi} \bar{N}\rangle, \underline{\|\bar{\pi} \bar{\Delta}\rangle}, \cdots$ |

As is summarized in above Table, there are two pictures; the one is Chew-Low theory and the other one is of quark physics. In the former the basic fields are only $\pi$ and $N$, while in the latter they include also the bare $\bar{\Delta}$ field with zero-width as a three-quark stable bound state $\bar{\Delta}=(q q q)$. After switching on $\mathcal{H}_{\mathrm{I}}^{\text {str. }}$, in the former the physical $\Delta$ particle appears as a resonance of $\pi N$ system, while in the latter the $\Delta$ becomes $\Delta_{\text {phys. }}$ with finite width. These two pictures may be phenomenologically consistent with each other in so far as concerned with interactions of the $\pi N$ system. However, we recognize presently the latter as a true one from the general and fundamental viewpoint.
( Present Problem ; $K \pi$, $\pi \pi$ systems)
VMW Method ] : Right bases of $S$-matrix

| Basic field | $\begin{array}{r} \bar{\pi}, \bar{K} \quad ; \quad \bar{\sigma}, \bar{\kappa}, \bar{f}_{0}(q \bar{q}) \\ \Gamma: \text { Zero } \end{array}$ |
| :---: | :---: |
| Complete set of base | $\begin{aligned} & \|\bar{\pi}\rangle,\|\bar{K}\rangle,\|\bar{\sigma}\rangle,\|\bar{\kappa}\rangle,\|\bar{f}\rangle \\ & \|\bar{\pi} \bar{\pi}\rangle, \ldots \end{aligned}$ |

We must take into account the $\kappa(\sigma)$ as a basic field as well as $K$ and $\pi(\pi)$

## 3. Method of Analysis of $J / \psi \rightarrow K^{*}(892) K \pi$

We shall use the VMW method to analyze relevant data, where all resonant particles are treated as the basic fields describing the $S$-matrix elements. The propagator of resonant particles is given by the conventional Feynman propagator with substitution of $i \varepsilon$ by $i \sqrt{s} \Gamma(s)$, here $\Gamma(s)$ and $s$ being, respectively, width and 4 -momenta squared of the resonant particles.
( The three mechanisms of relevant decay process )

(a) $J / \psi \rightarrow K^{*}(892) R_{K \pi} \rightarrow K^{*}(892) \quad(K \pi)$
( $\left.R_{K \pi}=\kappa, K_{0}^{*}(1430), K_{2}^{*}(1430), K_{2}^{*}(1922)\right)$

(c) $J / \psi \rightarrow K^{*}(892) K \pi$

In the case (a) of intermediate $K \pi$ resonances, we take into account the possible cases of $R_{K \pi}$ with, $J^{P}=0^{+}, 2^{+}$, decaying into the $S$ and $D$ wave states of $K \pi$ system, respectively.

The most simple $\mathcal{L}$ agrangians relevant for production and decay of respective intermediate resonances, and the amplitudes in the respective cases of $S$ - and $D$-wave decays are given, respectively, by
Mechanism (a) (intermediate $K \pi$ resonance)
(S-wave)

$$
\begin{aligned}
\mathcal{L}_{S} & \sim \xi_{\kappa} \psi_{\mu} K_{\mu}^{*} \kappa+g_{\kappa} \kappa K \pi+\cdots \\
\mathcal{F}_{S} & =S_{h_{\psi} h_{K^{*}}}\left(r_{\kappa} e^{i \theta_{\kappa}} \Delta_{\kappa}\left(s_{K \pi}\right)+r_{K_{0}^{*}} e^{i \theta_{K_{0}^{*}}} \Delta_{K_{0}^{*}}\left(s_{K \pi}\right)+r_{K \pi} e^{i \theta_{K \pi}}\right)
\end{aligned}
$$

(D-wave)

$$
\begin{aligned}
& \mathcal{L}_{D} \sim \xi_{K_{2}^{*}} \psi_{\mu} K_{\nu}^{*} K_{2 \mu \nu}^{*}+g_{K_{2}^{*}} K_{2 \mu \nu}^{*}\left(\partial_{\mu} K \partial_{\nu} \pi+\cdots\right) \\
& \mathcal{F}_{D}=D_{h_{\psi} h_{K^{*}}} r_{K_{2}^{*}} e^{i \theta_{K_{2}^{*}}} \Delta_{K_{2}^{*}}^{D}\left(s_{K \pi}\right) \\
& \Delta_{K_{2}^{*}}^{D}\left(s_{K \pi}\right)=\frac{m_{K_{2}^{*}} \Gamma_{K_{2}^{*}}\left(\frac{F_{D}\left(s_{K \pi}\right)}{F_{D}\left(s_{m_{2}^{*}}^{2}\right)}\right)}{m_{K_{2}^{*}}^{2}-s_{K \pi}-i \sqrt{s_{K \pi}} \Gamma_{K_{2}^{*}}\left(s_{K \pi}\right)\left(\frac{F_{D}\left(s_{K \pi}\right)}{F_{D}\left(s_{m_{K_{2}^{*}}^{*}}\right.}\right)^{2}}, \Gamma_{K_{2}^{*}}\left(s_{K \pi}\right)=\frac{\mathbf{p}^{5} g_{K_{2}^{*}}^{2}}{8 \pi s_{K \pi}}, \quad F_{D}\left(s_{K \pi}\right)=\frac{1}{s_{K \pi}+m_{K_{2}^{*}}^{2}}
\end{aligned}
$$

In the case (b) of intermediate $K^{*} \pi$ resonances with $J^{P}=1^{+}$decaying into the $S$-wave $K^{*} \pi$ system the corresponding formulas are given by

Mechanism (b) (intermediate $K^{*} \pi$-resonance)

$$
\begin{aligned}
\mathcal{L}_{K_{1}} & \sim \xi_{K_{1}} \psi_{\mu} K_{1 \mu} K+g_{K_{1}} K_{1 \mu} K_{\mu}^{*} K \\
\mathcal{F}_{K_{1}} & =B_{h_{\psi} h_{K^{*}}} \quad r_{K_{1}} e^{i \theta_{K_{1}}} \Delta_{K_{1}}\left(s_{K^{*} \pi}\right)
\end{aligned}
$$

Mechanism (c) (direct $K^{*} K \pi$ )
In the case of (c) the corresponding formulas are

$$
\begin{aligned}
\mathcal{L}_{\text {direct } K \pi} & \sim \xi_{K \pi} \psi_{\mu} K_{\mu}^{*} K \pi \\
\mathcal{F}_{\text {direct } K \pi} & =S_{h_{\psi} h_{K^{*}}} r_{K \pi} e^{i \theta_{K \pi}}
\end{aligned}
$$

In $\mathcal{F}_{S}, \mathcal{F}_{D}, \mathcal{F}_{K_{1}}$ and $\mathcal{F}_{\text {direct } K \pi}$ the $S, D$ and $B$ are the factors due to helicitycombinations among relevant particles given by

$$
\begin{aligned}
S_{h_{\psi} h_{K^{*}}}= & \varepsilon_{\mu}^{h_{\psi}} \tilde{\varepsilon}_{\mu}^{h_{K^{*}}} \\
D_{h_{\psi} h_{K^{*}}}= & \varepsilon_{\mu}^{h_{\psi}} \tilde{\varepsilon}_{\nu}^{h_{K^{*}}} \frac{1}{4}\left\{\left(r_{\mu}-\frac{q_{\mu}\left(m_{K}^{2}-m_{\pi}^{2}\right)}{s}\right)\left(r_{\nu}-\frac{q_{\nu}\left(m_{K}^{2}-m_{\pi}^{2}\right)}{s}\right)-\frac{4 \mathbf{p}_{K}^{2}}{3}\left(\delta_{\mu \nu}+\frac{q_{\mu} q_{\nu}}{s}\right)\right\} ; \\
& r_{\mu}=\left(p_{K}-p_{\pi}\right)_{\lambda} ; \\
B_{h_{\psi} h_{K^{*}}}= & \varepsilon_{\mu}^{h_{\psi}} \tilde{\varepsilon}_{\nu}^{h_{K^{*}}}\left(\delta_{\mu \nu}+\frac{p_{K^{*} \pi \mu} p_{K^{*} \pi \nu}}{s_{K^{*} \pi}}\right)
\end{aligned}
$$

Then the amplitude $\mathcal{F}$ squared for the process is given by

$$
\overline{|\mathcal{F}|^{2}}=\frac{2}{3}\left\{\frac{1}{3} \sum_{h_{\psi}, h_{K^{*}}}\left|\mathcal{F}_{S}+\mathcal{F}_{D}+\mathcal{F}_{K_{1}}\right|^{2}\right\}+\frac{2}{3}\left\{\frac{1}{3} \sum_{h_{\psi}}\left|\sum_{K_{P}^{*}} \mathcal{F}_{K_{P}^{*}}\right|^{2}+\left|\mathcal{F}_{K_{S}}\right|^{2}\right\}
$$

where summation is taken for $h_{\psi}, h_{K^{*}}=+,-, 0$ and the factor $\frac{2}{3}$ comes from the fact that the initial $J / \psi$ has only $\pm$ polarization. The 2 nd term is the possible contribution from the background processes; The former $\mathcal{F}_{K_{P}^{*}}$ represents the contribution from $P$-wave resonances $K_{\mathrm{P}}^{*}\left(=K^{*}(892), K^{*}(1410)\right) J / \psi \rightarrow(K \pi)_{\mathrm{BG}} K_{\mathrm{P}}^{*}$ with $m_{K \pi}$ being within the required region of $K^{*}(892)$ (where $(K \pi)_{\mathrm{BG}}$ are possibly coming from $\kappa$ and non-resonant $K \pi$ ) while the latter does $J / \psi \rightarrow K^{*}(892) K_{S}$ ( $K_{S} \rightarrow \pi \pi$, where one $\pi$ was misidentified with $K$ ). Corresponding formulas are given by

$$
\begin{aligned}
\mathcal{F}_{K_{P}^{*}} & =A_{h_{\psi}} r_{K_{P}^{*}} e^{i \theta_{K_{P}^{*}}^{*} \Delta_{K_{P}^{*}}\left(s_{K \pi}\right)} \\
A_{h_{\psi}} & =\varepsilon_{\mu}^{h_{\psi}}\left(\delta_{\mu \nu}+\frac{q_{\mu} q_{\nu}}{s}\right) r_{\nu} \\
\mathcal{F}_{K_{S}} & =S_{h_{\psi} h_{K^{*}}} r_{K_{S}} \Delta_{K_{S}}\left(s_{K \pi}\right)
\end{aligned}
$$

The differential decay width is given by

$$
\begin{aligned}
& \Gamma=\frac{1}{64 \pi^{3} M_{\psi}^{2}} \int d \sqrt{s_{K \pi}} \int_{-1}^{1} d \cos \theta \quad \mathbf{q}\left(s_{K \pi}\right) \mathbf{p}\left(s_{K \pi}\right) \overline{|\mathcal{F}|^{2}} \\
&=\frac{1}{64 \pi^{3} M_{\psi}^{3}} \int d \sqrt{s_{K \pi}} \int d \sqrt{s_{K^{*} \pi}} \\
& \sqrt{s_{K \pi}} \sqrt{s_{K^{*} \pi}} \overline{|\mathcal{F}|^{2}}
\end{aligned}
$$



Kinematical variables
[ VMW method]
Parameters


## 4. Results of analysis

With $\kappa\left(J^{P}=0^{+}\right)$

(a) $K \pi$ mass spectrum

$$
\begin{gathered}
m_{\kappa}=882 \pm 24 \mathrm{MeV} \\
\Gamma_{\kappa}=335 \pm 82 \mathrm{MeV} \\
\operatorname{BR}\left(\frac{J / \psi \rightarrow \kappa K^{*}}{J / \psi \rightarrow K^{*} K \pi}\right)=0.115
\end{gathered}
$$



Fig. 1. Results of Analysis. The experimental data and fitted curves are given. The respective contributions of resonances and of direct process are also shown. The data of angular distribution are taken into account in the two ways (depending on $K \pi$ mass); one with a restriction $m_{K \pi} \leq 1$ ( GeV ) and the other with $m_{K \pi}$ in whole region.

With $\kappa\left(J^{P}=0^{+}\right)$

$$
\begin{aligned}
& \chi^{2} / N_{F}=375.8 /(203-37)=2.26 \\
& \chi^{2}\left(m_{K \pi}\right) / N_{D}(\text { bin })=2.27(82), \chi^{2}\left(m_{K^{*} \pi}\right) / N_{D}(\text { bin })=1.40(81), \\
& \chi^{2}\left(\text { ang. dist.) } / N_{D}(\text { bin })=1.91(40)\right.
\end{aligned}
$$

Table 1: Obtained values of the parameters.

| Process $\left(J^{P}\right)$ | mass $(\mathrm{MeV})$ | $\Gamma(\mathrm{MeV})$ | $\mathrm{g}(\mathrm{GeV})$ | r | $\theta$ (Deg.) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\kappa \rightarrow K \pi$ | $\underline{881.67}$ | $\underline{335.27}$ | 4.829 | 6759.92 | 0.00 |
| $K^{*}(892)\left(1^{-}\right) \rightarrow K \pi$ | 904.03 | 49.00 | 6.204 | -17355.73 | 0.00 |
| $K_{0}^{*}(1430)\left(0^{+}\right) \rightarrow K \pi$ | 1430.00 | 250.33 | 4.551 | -10731.95 | 55.73 |
| $K^{*}(1410)\left(1^{-}\right) \rightarrow K \pi$ | 1445.00 | 190.00 | 6.320 | -6255.44 | -0.37 |
| $K_{2}^{*}(1430)\left(2^{+}\right) \rightarrow K \pi$ | 1437.02 | 89.00 | 6.957 | 34722.19 | 122.04 |
| $K_{2}^{*}(1922)\left(2^{+}\right) \rightarrow K \pi$ | 1940.00 | 200.00 | 5.637 | -8622.34 | 1.43 |
| $K_{1}(1270)\left(1^{+}\right) \rightarrow K^{*} \pi$ | 1266.34 | 86.39 | 3.452 | -7190.82 | 185.08 |
| $K_{1}(1400)\left(1^{+}\right) \rightarrow K^{*} \pi$ | 1427.68 | 185.22 | 4.772 | 10337.64 | 101.13 |
| $K \pi$ B.G. around 700 MeV | 726.00 | 146.00 | 0.527 | 7140.70 | - |
| Direct $K \pi$ |  |  |  | -1357.88 | 276.55 |

## 5. Conclusion

- $J / \psi \rightarrow K^{*}(892) K \pi$ process ( $58 \mathrm{M} J / \psi$-Data in BESII)
was analyzed by VMW method
- $K \pi$ and $K^{*} \pi$ Mass spectra and $K \pi$ Angular distributions

With $\kappa$

$$
\chi^{2} / N_{F}=375.8 /(203-37)=\underline{2.26}
$$

Without $\kappa$

$$
\chi^{2} / N_{F}=882.0 /(203-32)=5.16
$$

## $\kappa$ is indispensable to reproduce low-mass peak

$$
\begin{aligned}
& m_{\kappa}=882 \pm 24 \mathrm{MeV} \quad \Longleftrightarrow \\
& \Gamma_{\kappa}=335 \pm 82 \mathrm{MeV} \quad \text { Comparable }
\end{aligned}
$$

$$
\begin{gathered}
\text { The analysis of } \\
K \pi \text { scattring Phase shists } \\
m_{\kappa}=905_{-30}^{+65} \mathrm{MeV} \\
\Gamma_{\kappa}=545_{-110}^{+235} \mathrm{MeV} \\
\hline m_{\kappa}=877 \pm 85 \mathrm{MeV} \\
\Gamma_{\kappa}=346 \pm 89 \mathrm{MeV} \\
\text { Wu Ning (BESII) } \\
\hline
\end{gathered}
$$

S. Ishida et al., PTP98(1997)621.

This symposium

