Covariant Classification of Hadrons and Chiral States

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II. Covariant framework for hadrons

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I. Introduction

[Purpose of This Talk]

(Classification by Non-Rel. Q.M.)

$$SU(6)_{SF} \otimes O(3)_L$$

successful for 4 decades but

Recently
(Necessity for Covariant Classif.) has become strong.

theor. **QCD** Chiral Symmetry

phenom. π meson Nambu-Goldstone Boson

chiral $\updownarrow$ partner for Spont. Br. of chir. Symm.

σ meson ; confirmed

(Covariant Hadron Class. and Existence of **Chiral States**)

We proposed manifestly covariant scheme

symm. $\tilde{U}(12)_{SF} \otimes O(3.1)_L$; $\tilde{U}(12)_{SF} \supset \tilde{U}(4)_{DS} \otimes SU(3)_F$

Hadron WF Tensors in $\tilde{U}(12)_{SF} \otimes O(3,1)$ space

predicted

$\longrightarrow$ Existence of

Chiral Particle/states Out of

Non-Rel. scheme

(Candidates : σ meson nonet,
$N^{\oplus}_{1/2}(1440)$ (Roper R.),
$\Lambda^{\ominus}_{1/2}(1405)$)

[Lorentz Covariance for Local Particle]

Lorentz Transf. $\quad X'_\mu = \Lambda_{\mu\nu} X_\nu = (\delta_{\mu\nu} + \varepsilon_{\mu\nu}) X_\nu$

Wave Funct. $\quad \Phi(x) \to \Phi'(x') = S(\Lambda)\Phi(x)$

$$S(\Lambda) = \left(1 + \frac{i}{2}\varepsilon_{\mu\nu}\Sigma_{\mu\nu}\right)$$

generator $\Sigma_{\mu\nu}$: $\quad \underbrace{J_i \equiv \frac{1}{2} e_{ijk}\Sigma_{jk}}_{\text{Rotation}}, \quad \underbrace{K_i \equiv i\,\Sigma_{i4}}_{\text{Boost}}$

(Dirac Spinor Case)

$$J_i = \frac{1}{2}\sigma_i \otimes \overset{\overset{1}{\shortparallel}}{\rho_0} \qquad K_i = \frac{i}{2}\rho_1 \otimes \sigma_i$$

$$S(\Lambda) = R(\theta_i) = e^{-i\theta_i J_i}$$

Only σ-WF is necessary

<u>Non-Rel. Pauli Spinor</u>

$$S(\Lambda) = B(\mathbb{P}) = e^{-i\mathbb{b}\cdot\mathbb{K}}$$

$$(\mathbb{b} \equiv \mathbb{P}\cosh^{-1} P_0/m)$$

Both σ- and ρ- WF are necessary

<u>Covariant Dirac Spinor</u>

$$\Psi_\alpha = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \chi_1 \\ \chi_2 \end{pmatrix} \begin{matrix} \sigma \\ \sigma \end{matrix} \; \rho$$

$$\underset{4\times4}{\gamma} = \underset{2\times2}{\rho} \otimes \underset{2\times2}{\sigma}$$

ρ-WF plays a Role for Ceter of Mass Motion

II. Covariant Framework for Hadrons

(Attributes of Hadrons)

mass | Lorentz T. Property | quark-composite Str.

Klein-Gordon Eq. $(\Box - \mathcal{M}^2)\Phi = 0$ | Def. of generator $R(\theta)$, $B(\mathbb{p})$ | Flavor-Spinor indices $A(\alpha, a)$, $B(\beta, b)$

(Wave Function) x_μ, y_μ ; $x_{i\mu}$ Lorentz 4-vector

Meson : $\Phi_A^{\ B}(x, y) \sim \psi_A \bar{\psi}^B$

Baryon : $\Phi_{A_1 A_2 A_3}(x_1, x_2, x_3) \sim \psi_{A_1} \psi_{A_2} \psi_{A_3}$

$\boxed{\text{Tensors in } \tilde{U}(12)_{SF} \otimes O(3,1) \text{ space}}$

[Klein-Gordon Eq. and Mass Term]

$$\left[\frac{\partial^2}{\partial X_\mu^2} - \mathcal{M}^2 \left(r_\mu, \frac{\partial}{\partial r_\mu} ; \frac{\partial}{i \partial X_\mu} \right) \right] \Phi(X, r, \cdots) = 0$$

$\psi_N(P_N, r)$; mass^2 eigen-function

$$\mathcal{M}^2 \psi^{(\pm)}_{N(P_N, r)} = M_N^2 \psi^{(\pm)}_{N(P_N, r)}$$ Positive freq. Negative freq.

$$\Phi(X, r, \cdots) = \sum_N \sum_{\substack{\mathbb{P}_N \\ (P_{N,0} > 0)}} \left[e^{iP_N X} \psi_N^{(+)}(P_N, r, \cdots) + e^{-iP_N X} \psi_N^{(-)}(P_N, r, \cdots) \right]$$

(Light q. System) If $\mathcal{M}^2$ indep. of $A(B)$ indices,

$\mathcal{M}^2$-spectra ; $\tilde{U}(12)_{SF}$ and Chiral Symm.

[Expansion of WF in (BW-Spinor ⊗ Oscillator)]

$$\Psi^{\beta\cdots}_{J,\alpha\cdots}(P_N, r, \cdots) = \sum_{i,j} C^J_{i,j} \; \overset{(i)}{W}{}^{\beta\cdots}_{\alpha\cdots}(P_N) \; \overset{(j)}{O}(P_N, r \cdots)$$

respective [Complete Sets of Eigen Funct] of $\widetilde{U}(4)_{D.S.} \otimes O(3,1)$

Dirac spinor   Lorentz space

(Eigen-funct. for $\widetilde{U}(4)_{DS}$)

Bargmann-Wigner Spinor function

$$\left(\frac{\partial^2}{\partial X_\mu^2} - M^2 \right) W^{\beta\cdots}_{\alpha\cdots}(X) = 0$$

$$W^{\beta\cdots}_{\alpha\cdots}(X) = \sum_{\mathbb{P}, P_0>0} \left(e^{ipX} W^{(+)\beta\cdots}_{\alpha\cdots}(p) + e^{-ipX} W^{(-)\beta\cdots}_{\alpha\cdots}(p) \right)$$

meson. $W_\alpha^\beta(P) \propto W_\alpha(P)\overline{W_{\bar q}}(P)$: bi-Dirac Spinors

(Eigen funct. for $O(3,1)$)

Yukawa Oscillator function

$\Downarrow$ (definite metric; relative-time freezing $\langle P_\mu r_\mu \rangle = \langle P_\mu p_\mu \rangle = 0$)

$\mathbb{P}=0$; $O(3,1) \approx O(3)$: 3-dimensional H.O.

(Phenomen. advantage of this Base)

① Constituent quarks behave like free Dirac-particle

② Global level spectra (Regge trajectory and so on) are well described

[Spinor WF and Chiral States]

(Dirac Spinor) BW spinor with single-index

$$\psi_{q,\alpha}(X) = \sum_{\mathbb{P}} \Big[e^{ipx} \underset{u_\alpha(p)}{W^{(+)}_{q,\alpha}(P)} + e^{-ipx} \underset{u_\alpha(-p)}{W^{(-)}_{q,\alpha}(P)} \Big] \quad : \text{quarks}$$

$$\bar{\psi}_{\bar{q},\alpha}(X) = \sum_{\mathbb{P}} \Big[e^{ipx} \underbrace{\overline{W}^{(+)}_{\bar{q},\alpha}}_{\bar{v}_{\bar{q},\alpha}(-P)}(P) + e^{-ipx} \underbrace{\overline{W}^{(-)}_{\bar{q},\alpha}(P)}_{\bar{v}_{\bar{q},\alpha}(P)} \Big] \quad : \text{anti-quarks}$$

$$\mathbb{P} = 0 \; ; \qquad W^{(+)}_q = \begin{pmatrix} \chi \\ 0 \end{pmatrix}, \rho_3 = + \quad \Bigg| \quad W^{(-)}_q = \begin{pmatrix} 0 \\ \chi \end{pmatrix}, \rho_3 = -$$

Members of Complete S.

$$\overline{W}^{(+)}_{\bar{q}} = (0, \bar{\chi}), \bar{\rho}_3 = + \quad \Big| \quad \overline{W}^{(-)}_{\bar{q}} = (\bar{\chi}, 0), \bar{\rho}_3 = -$$

(Meson Spinor) bi-Dirac Spinor

$$W_\alpha^{\ \beta}(P) = W_q(P)_\alpha \; \overline{W}_{\bar{q}}(P)^\beta$$

$$\mathbb{P} = 0; \quad (\rho_3, \bar{\rho}_3) = (+, +) \quad \text{boosted Pauli Spinor}$$

$$\left.\begin{matrix} (+, -) \\ (-, +) \\ (-, -) \end{matrix}\right\} \text{"Chiral States"}$$

(Baryon spinor) All $\underline{4\text{-type}}$ BW spinors required

tri-Dirac spinor

$$\overset{(B)}{W}_{\alpha\beta\gamma}(P) = W_{q,\alpha}(P)\, W_{q,\beta}(P)\, W_{q,\gamma}(P) \quad : \text{for Baryons}$$

$$P = 0; \; (\rho_1, \rho_2, \rho_3) = (+, +, +) \quad \text{boosted Pauli Spinor}$$

$$\left.\begin{matrix} (+, +, -) \\ (+, -, -) \end{matrix}\right\} \text{"Chiral States"}$$

All $\underline{3\text{-type}}$ BW spinors required

$$\overset{(\bar{B})}{W}_{\alpha\beta\gamma}(P) = \overset{(B)}{W}\{ W_{q,\alpha} \to W_{\bar{q},\alpha} \} \quad : \text{for anti-Baryons}$$

[Transf. Rule for Hadrons and Chiral Symmetry]

<u>any</u> Transf. Rule for Composite Hadrons in Covar. Class. Scheme

is derived <u>Automatically</u> from T. R. for Quarks

【Chiral T.】 $\psi_q \to \psi_q' = e^{i\alpha\gamma_5}\,\psi_q$

Baryon : $W_{\alpha_1\alpha_2\alpha_3} \to W'_{\alpha_1\alpha_2\alpha_3} = \prod_{i=1,2,3}\left(e^{i\alpha\gamma_5^{(i)}}\right) W_{\alpha_1\alpha_2\alpha_3}$

Meson : similarly

(infinit. gen.) γ_5

$$\left[\begin{array}{l} \overset{\rho_3\ +}{u(p)} \to u'(p) = -\gamma_5\, u(p) = \overset{\rho_3\ -}{u(-p)} \\ \underset{\bar{\rho}_3\ +}{v(p)} \to v'(p) = -\gamma_5\, v(p) = \underset{\bar{\rho}_3\ -}{v(-p)} \end{array}\right.$$

$\Rightarrow$ <u>Members</u> of Complete set of BW-spinors <u>change with each other</u>

(Chiral Symmetry)

<u>If</u> <u>$\mathcal{M}^2$ is Lorentz Scalar indep. of Spinor indices</u>,

<u>(mass)^2</u>-Spectra become <u>Chiral Symmetric</u>,

in addition to $\tilde{U}(4)_{DS}$ Symmetry and

<u>$\tilde{U}(12)_{SF}$ Symmetry</u> (for Light-Quarks)

"<u>This</u> is only a phenom. assumption to be checked experimentally"

III. Level Structure of Mesons and Baryons

(Ground states) $\tilde{U}(12)_{SF}$

Meson : $(\underline{12} \times \underline{12}^*) = \underline{144}$

Chiral states

	$P_s^{(N)}$	$V_\mu^{(N)}$	$P_s^{(E)}$	$V_\mu^{(E)}$	$S^{(N)}$	$A_\mu^{(N)}$	$S^{(E)}$	$A_\mu^{(N)}$
J^{PC}	0^{-+}	1^{--}	0^{-+}	1^{--}	0^{++}	1^{++}	$\underline{0^{+-}}$	1^{+-}
							exotic	

Baryon : $(\underline{12} \times \underline{12} \times \underline{12})_{sym.} = \underline{364} = \underline{182}_B \oplus \underline{182}_{\bar{B}}$

$\underline{182}$ —
- $\underline{56}$ $\Delta_{3/2}^{\oplus}$ $N_{1/2}^{\oplus}$
- $\underline{70}$ $\Delta_{1/2}^{\ominus}$ $N_{3/2}^{\ominus}$ $N_{1/2}^{\ominus}$ $\Lambda_{1/2}^{\ominus}$
- $\underline{56}'$ $\Delta_{3/2}^{\oplus}$ $N_{1/2}^{\oplus}$

Chiral states

(Excited states) $\tilde{U}(12)_{SF} \otimes O(3)_L$

$(\text{mass})^2 \quad M_N^2 = M_0^2 + N\Omega$

meson : $M_N = m_q^{(N)} + m_{\bar{q}}^{(N)}$

Baryon: $M_B = m_{q_1}^{(N)} + m_{q_2}^{(N)} + m_{q_3}^{(N)}$

($m_q^{(N)}$: mass of q in N-th Excited States

(Effective Chiral Symm.)

Phenom. Rule for Eff. χ. Symm. $\quad m_q^{(N)2} \ll \Lambda^2_{conf.} \approx 1 \ \text{GeV}^2$

$\Rightarrow \quad n\bar{n} \qquad n\bar{c} \qquad n\bar{b}$

$\quad N \le 1 \text{ or } 2 \qquad N \le 0 \text{ or } 1 \qquad N \le 0 \text{ or } 1$

For the Lower-mass states with these N values we expect <u>the Existence of Chiral States</u>

IV. Interactions among Hadrons

$\boxed{\text{Int. vertex}} = \boxed{\text{spect./overlap. Int.}} \otimes \boxed{\text{Element. q. Int.}}$

In Old Treat.

$$\overline{B}^{A_1A_2D}(p')\,B_{A_1A_2C}(p)\,\overline{M}^{C}_{D}(q),\quad \underbrace{(\overline{B}^{A_1}(p')B_{A_1}(p))}_{\propto(\overline{\psi}^{\alpha}(p')\psi_\alpha(p))}(\overline{B}^{A_2}B_{A_2}),\quad (\overline{B}^{D}(p')B_C(p)\overline{M}^{C}_{D}(q))$$

(Overlap-Int.) comes from QCD should be Xiral Symm. but Non-symm.

In New Treat.

(Revised Overl. I.)

$$(\overline{\psi}(p_1)\psi(p_2)) \Rightarrow (\widetilde{\psi}(v_1)\psi(v_2)) \equiv (\overbrace{\psi^*\gamma_4\overset{(0)}{\gamma}}^{\overline{\psi}}\psi)$$

Xiral Symm. X O

$$\overset{(0)}{\gamma} \equiv [-i\hat{v}_1\gamma - i\hat{v}_2\gamma]/2$$

$$\hat{v} = \begin{cases} v & \text{for } q \\ -v & \text{for } \bar{q} \end{cases}$$

$$\xrightarrow{(v_1=v_2)} = (\overline{\psi}\psi)$$

$$(\widetilde{\psi}(v_1)\,\psi(v_2)) = \overline{\psi}^{(+)}(v_1)\psi^{(+)}(v_2) - \overline{\psi}^{(-)}(v_1)\psi^{(-)}(v_2)$$

$$\begin{cases} \Lambda^{(\pm)}(v) \equiv \dfrac{1 \mp iv\gamma}{2}, & \Lambda^{(\pm)}\psi_q^{(\pm)} = \psi_q^{(\pm)}, \quad \Lambda^{(\mp)}\psi_q^{(\pm)} = 0, \\ & \Lambda^{(\pm)}\psi_{\bar{q}}^{(\mp)} = \psi_{\bar{q}}^{(\mp)}, \quad \Lambda^{(\mp)}\psi_{\bar{q}}^{(\mp)} = 0. \\ \psi(v) = \psi_q(v) + \psi_{\bar{q}}(v) & \end{cases}$$

Rest Frame

$$\psi_q(0) \equiv \begin{pmatrix} \varphi_q \\ \chi_q \end{pmatrix};\quad \psi_q^{(+)}(0) = \begin{pmatrix} \varphi_q \\ 0 \end{pmatrix},\quad \psi_q^{(-)}(0) = \begin{pmatrix} 0 \\ \chi_q \end{pmatrix} \quad \begin{matrix}\rho_3 \\ + \\ -\end{matrix}$$

$$\psi_{\bar{q}}(0) \equiv \begin{pmatrix} \chi_{\bar{q}} = \chi_q^* \\ \varphi_{\bar{q}} = -\varphi_q^* \end{pmatrix};\quad \psi_{\bar{q}}^{(+)}(0) = \begin{pmatrix} 0 \\ \varphi \end{pmatrix},\quad \psi_{\bar{q}}^{(-)}(0) = \begin{pmatrix} \chi_{\bar{q}} \\ 0 \end{pmatrix} \quad \begin{matrix}\bar{\rho}_3 \\ + \\ -\end{matrix}$$

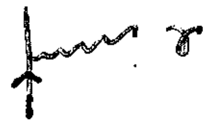
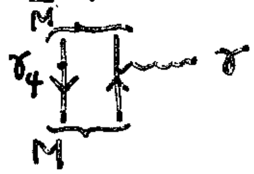
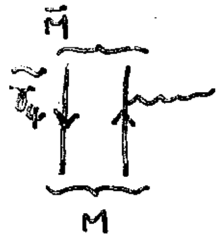
$$(\widetilde{\psi}(0)\psi(0)) = (\varphi_q^*\varphi_q) + (\chi_q^*\chi_q) \;\cdots\; \begin{matrix} +(-) \\ +(-) \end{matrix}$$

$$-[(\chi_{\bar{q}}^*\chi_{\bar{q}}) + (\varphi_{\bar{q}}^*\varphi_{\bar{q}})] \;\cdots\; \begin{matrix} +(-) \\ +(-) \end{matrix} \quad \text{O},$$

(But +(-) −(+) Non- , X ,)

ρ_3- line Rule	ρ_3 at respect. rest frame is conserved on quark line

(Electro-Magnetic Int.)

			Chiral Symm.
$\bar{q}q\gamma$	$(\bar{\Psi}\gamma_\mu\Psi)A_\mu$		O
$\bar{M}M\gamma$	$(\bar{M}\gamma_\mu M)A_\mu + \cdots\cdots$		X
	$\Downarrow$		
	$(\tilde{\bar{M}}\gamma_\mu M)A_\mu + \cdots\cdots$		O

(Achasov Problem) for $f_0 = (s\bar{s})$ and $a_0 = (n\bar{n})$

Exper. $Br(\phi \to f_0\gamma) < Br(\phi \to a_0\gamma)$

Seemingly violating OZI-Rule,

While in Covar. Classif. scheme with $(\tilde{\bar{\Psi}}\Psi)$

Xiral overlap Int. both Processes are forbidden in Xiral limit

, and allowed through Spont. broken hadronic Process

No Violation of OZI-Rul

(Strong Interaction)

is "Residual" Interaction among all color-neutral bound states

$\rightarrow$

$\boxed{\text{Strong Interactions are presumably to be given by Spont.ly broken Xral Symm. Overlapping.}}$

$\Rightarrow$ Concrete forms of Effective Strong Int. $\overbrace{\text{are derived}}^{\text{automatically}}$!!

V. Candidates for Chiral States

$(q\bar{q})$-mesons

$S^{(N)}({}^1S_0)\ 0^{++},\ \sigma(600)\quad \kappa(900)\quad a_0(980)\quad f_0(980)$

$V_\mu^{(E)}({}^3S_1)\ 1^{--},\ \rho'(1250)\quad \omega'(1250)$ (→ T. Maeda)

$P_S^{(E)}({}^1S_0)\ 0^{-+},\ \eta(1295)$

$S^{(E)}({}^1P_1)\ \underline{1^{-+}},\ \pi_1(1400)$ (→ T. Tsuru)

$A_\mu^{(E)}({}^3P_1)\ \underline{1^{-+}},\ \pi_1(1600)$

⇒ (K. Yamada)

$(q\bar{Q})$-mesons

$D_1^\chi\ ({}^3S_1)\ 1^+\qquad D_1^\chi \to D^* + \pi$

$B_0^\chi\ ({}^1S_0)\ 0^+\qquad B_0^\chi \to B + \pi$

(→ I. Yamauchi)

Baryons

$N(1440)^+_{1/2}$ Roper Res. $\underline{56'}^{\oplus}$ $(N^{\oplus}_{1/2})$

$\Lambda(1405)^-_{1/2}$ $\underline{70}^{\ominus}$ $(\Lambda^{\ominus}_{1/2})$

(→ M. Ishida)

VI. Concluding Remarks

- Covariant Level Classific. Scheme of Hadrons has been reviewed.
- Chiral Particles/states out of Non-Relat. Scheme are predicted to exist in low mass in $q\bar{q}$, $q\bar{Q}$; qqq, etc.
  → Several promising Candidates
- Interactions among Composite Hadrons, which are Xiral Symmetric, are described and a Selection Rule : ρ_3 conservation of quark-line is obtained
  → very intersting & Useful in seeking for Chiral particles
- Both experimental and theoretical investigation for chiral states are required for New development of Hadron Physics.