Properties of σ/κ - Production Amplitude

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I. Introduction

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I. Introduction

Evidences for σ/κ in $\pi\pi/K\pi$ Production Proc.

σ $D^+ \to \pi^-\pi^+\pi^+$

$J/\psi \to \omega\pi^+\pi^-$

$\Upsilon'' \to \Upsilon\pi^+\pi^-$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$, $p\bar{p} \to \pi^0\pi^0\pi^0$

κ $D^+ \to K^-\pi^+\pi^+$

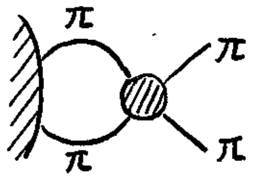
$J/\psi \to \overline{K^{*0}} K^+\pi^-$

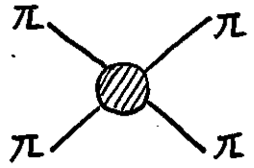
σ/κ-peak directly fitted by Breit-Wigner formula
$\pi\pi/K\pi$-Production amplitudes $\mathcal{F}$ analyzed independently of $\pi\pi/K\pi$-Scattering amplitude $\mathcal{T}$
phase shift δ

$\Uparrow$ **criticized**

conventional method "Universality of $\pi\pi$-scattering"

considering only $\pi\pi/K\pi$ dynamics

$\mathcal{F} = \alpha\mathcal{T}$ 

$\Leftarrow$ $\mathcal{T}$ 

α : real Pennington, α : complex Bugg / Anisovich

$\mathcal{F}$ and $\mathcal{T}$ show common phase motion because of Watson F.S.I. theorem.

<u>$\mathcal{F}$ commonly fitted together with $\mathcal{T}$ (or δ)</u>

Relation between $\mathcal{F}$ and $\mathcal{T}$

features of $\mathcal{T}$

- threshold suppression of spectra : applicable to $\mathcal{F}$?
- phase motion : universal in all $\mathcal{F}$?

common fit of $\mathcal{F}$ with $\mathcal{T}$ is necessary or unnecessary?

II. $\pi\pi/K\pi$ – Scattering (𝒯 by LσM)

$$\mathcal{L} = \frac{1}{2}\left((\partial_\mu \sigma)^2 + (\partial_\mu \phi)^2\right) - \frac{\mu^2}{2}(\sigma^2 + \phi^2) - \frac{\lambda}{4}(\sigma^2+\phi^2)^2 + f_\pi m_\pi^2 \sigma$$

$$\sigma = f_\pi + \sigma' \quad \Rightarrow \quad g_{\sigma\pi\pi} = f_\pi \lambda = \frac{m_\sigma^2 - m_\pi^2}{2 f_\pi}$$

$$\mathcal{T}_{\pi\pi}^{I=0} = \underbrace{3A(s,t,u)}_{\text{main term}} + A(t,s,u) + A(u,t,s)$$

$$3A(s,t,u) = \underbrace{\frac{3(-2g_{\sigma\pi\pi})^2}{m_\sigma^2 - s}}_{O(p^0):\ \text{large}} \underbrace{- 6\lambda}_{O(p^0):\ \text{large}} = \underbrace{3\frac{s-m_\pi^2}{f_\pi^2}}_{O(p^2):\ \text{small}} + \frac{3}{f_\pi^2}\frac{(s-m_\pi^2)^2}{m_\sigma^2 - s}$$

Tomozawa-Weinberg ampl.

Strong Cancel.

$$\delta_{tot} = \delta_\sigma + \delta_{BG} \quad (+\,\delta_{f_0(980)} + \cdots)$$

- No direct σ-Breit Wig. phase motion δ_σ is observed because of the cancellation guaranteed by χ sym.

- $m_{\pi\pi}$ spectra suppressed near threshold region

⇑

$$\boxed{\text{derivative coupling property of } \pi \text{ as Nambu-G. boson}}$$

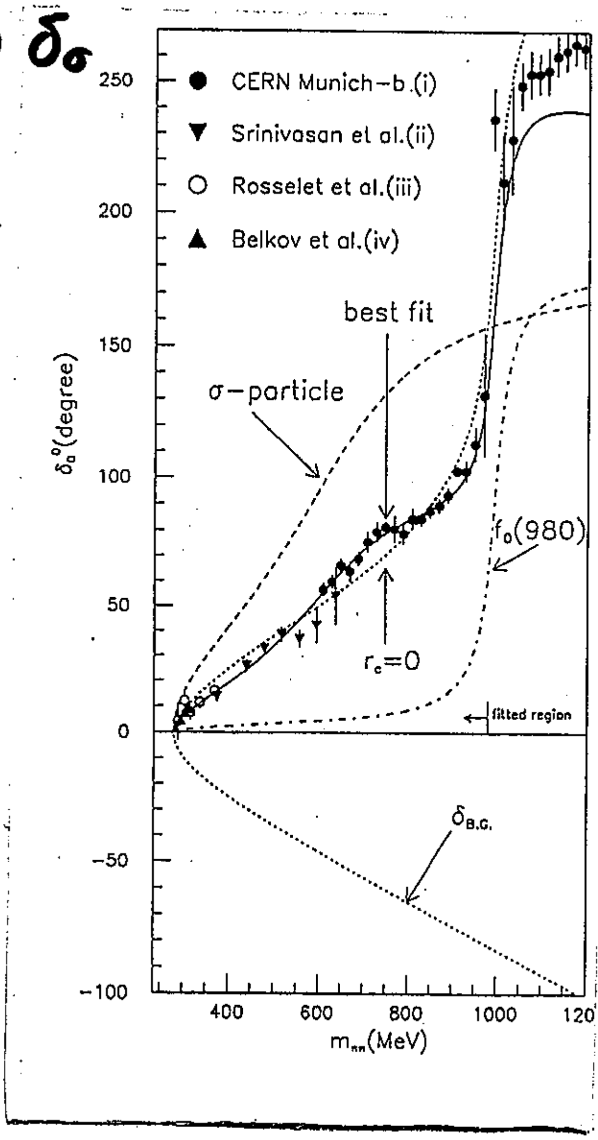
∵) $\mathcal{T} \sim p_1 \cdot p_2 \sim m_\pi^2$ near thres.

pion momentum

Adler 0 condition

“ $p_{1\mu} \rightarrow 0_\mu$, $\mathcal{T} \rightarrow 0$ ”

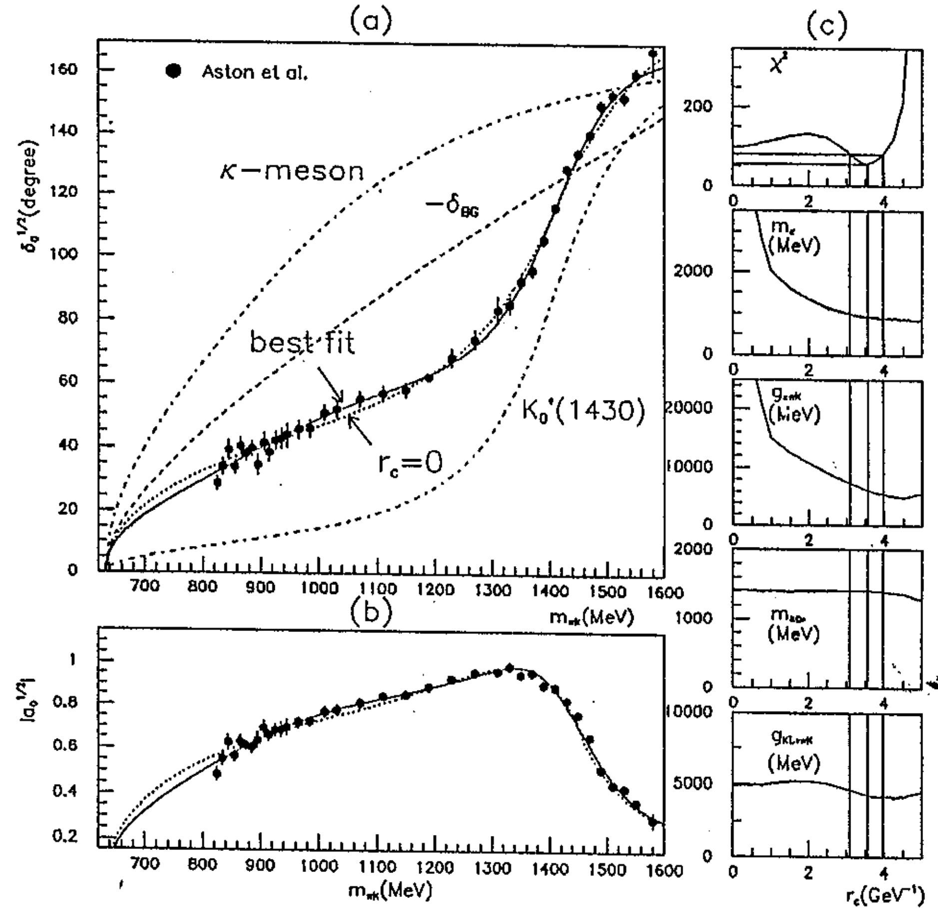
(Other pion momenta remain unchanged.)



[$I = \frac{1}{2}$ Phase Shift Analysis and $\kappa(900)$]

$m_\kappa = 905^{+65}_{-30}$ MeV

$\Gamma_\kappa = 545^{+235}_{-110}$ MeV



Chiral Cancellat between δ_κ & $\delta^N_{K\pi}$ taken into accoun

Fig. 12. Fits to I=1/2 $K\pi$ S-wave scattering amplitude; (a) phase shift $\delta_0^{1/2}$, and (b) magnitude of amplitude $|a_0^{1/2}|$. The solid line in each figure is the best fit with r_c=3.57 GeV^{-1}, while the dotted line is fit with r_c=0 GeV^{-1}. Two dash-dotted lines in (a) represent κ (upper) and $K_0^*(1430)$ (lower) resonance contributions to the best fit, and dashed line is the sign-reversed of the repulsive background δ_{BG}. (c) χ^2, M_κ, g_κ, $M_{K_0^*}$, and $g_{K_0^*}$ behavior as functions of core radius r_c. Vertical lines represent r_c=3.57, 3.1 and 3.975 GeV^{-1}, corresponding to the best fit and the fit with ± 5 sigma deviations.

Table VI. Resonance parameters of κ(900), $K_0^*(1430)$ and core radius. The errors correspond to five standard deviations from the best fit. Two kinds of width, $\Gamma^{(p)}$ and $\Gamma^{(d)}$, defined as $\Gamma^{(p)} = \Gamma_R^i\ (s = M^2)$ (Eq. (2.5)), $\Gamma^{(d)} = N^{-1}\int ds\Gamma(s)/[(s-M^2)^2 + s\Gamma(s)^2]$; $N = \int ds 1/[(s-M^2)^2 + s\Gamma(s)^2]$, considering the broadness of relevant widths.

	M_κ	$g_{K\pi}$	$\Gamma^{(p)}_{K\pi}$	$\Gamma^{(d)}_{K\pi}$
κ(900)	905^{+65}_{-30} MeV	6150^{+1200}_{-650} MeV	545^{+235}_{-110} MeV	470^{+155}_{-90} MeV
$K_0^*(1430)$	1410^{+10}_{-15} MeV	4250^{+380}_{-70} MeV	220^{+40}_{-5} MeV	220^{+40}_{-5} MeV

Table VIII. Comparison between the fit in the case with $r_c \neq 0$ and the fit in the case with $r_c = 0$. In the latter analysis with $r_c = 0$ the small δ^{tot} in the low energy region is explained as a background phase $[\delta_{BG}^{pos.}]$. In the case with $r_c \neq 0$ the sum of the large positive δ_κ and the large negative δ_{BG} gives a small positive phase, corresponding to the $[\delta_{BG}^{pos.}]$ in the case with $r_c = 0$. The χ^2-value becomes much smaller in the former than in the latter. The latter is essentially equivalent to the original analysis[65] without repulsive δ_{BG}.

improvement

	$r_c \neq 0(\chi^2/N_f = 57.0/41)$	$r_c = 0(\chi^2/N_f = 96.0/42)$
	$\delta^{tot} = \delta_{K_0^*(1430)} + [\delta_{\kappa(900)} + \delta_{BG}]^{pos.}$ $\kappa(900)$	$\delta^{tot} = \delta_{K_0^*(1430)} + [\delta_{BG}^{pos.}]$ positive BG
m_κ	905^{+65}_{-30}	> 2000
$\Gamma_\kappa^{(p)}$	545^{+235}_{-110}	> 2000
$\sqrt{s_{pole}}$/MeV	$(875 \pm 75) - i(335 \pm 110)$	—

III. Method of Analyses of $\begin{matrix}\pi\pi\\K\pi\end{matrix}$) Production Proc.

Strong Interaction : Residual interaction of QCD

interaction between color-neutral $\begin{matrix}q\bar{q}\\qqq\end{matrix}$) bound states ϕ_i

$$\{\ \phi_i\ \} = \{\ \pi, K, \underline{\rho, K^*, \sigma, \kappa, f_0(980), \cdots}\ \}$$

$$, \{\ N, \underline{\Delta, \cdots}\ \}$$

$\mathcal{H}^{strong}(\phi_i)$ described by ϕ_i-field

└→ Final State Interaction

induces strong phases $e^{i\theta_\sigma}, \cdots$

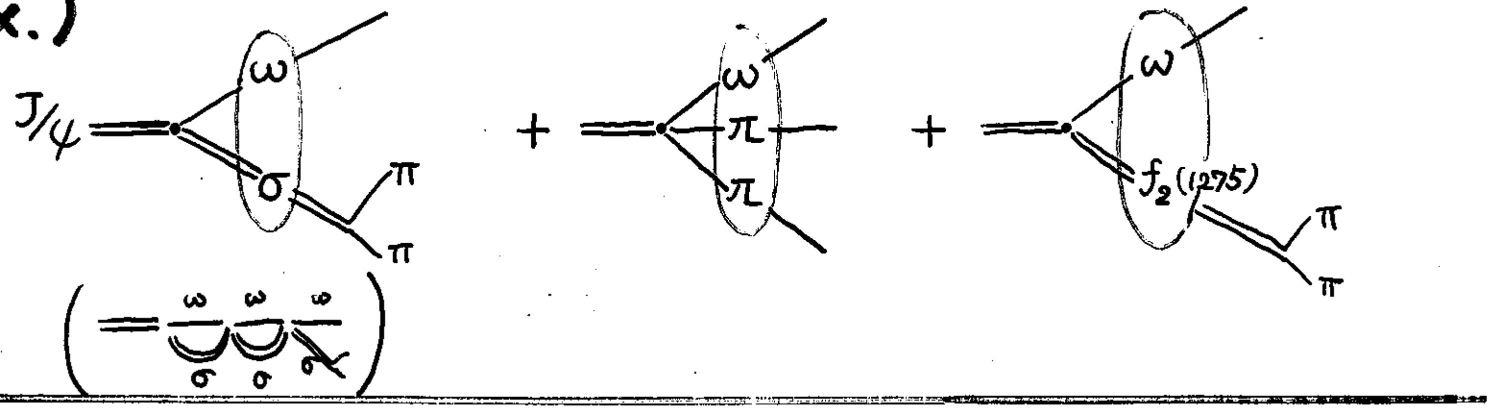
$\mathcal{S}$-matrix Bases : **configuration space of** multi-ϕ_i's states

$\phi_i = \rho, K^*, \sigma, \kappa \cdots$ have finite widths. Unstable Particles

intuitive method

$$\frac{1}{m_\sigma^2 - s - i\varepsilon} \Longrightarrow \frac{1}{m_\sigma^2 - s - i m_\sigma \Gamma_\sigma}$$

Ex.)



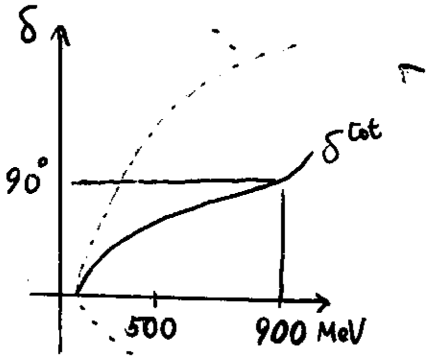
Prod. Coupl. Strong Phase

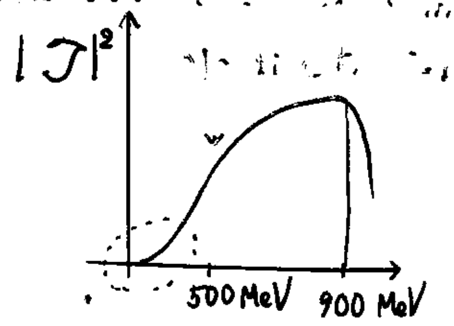
$$\mathcal{F} = \underbrace{r_\sigma e^{i\theta_\sigma} \frac{m_\sigma \Gamma_\sigma}{m_\sigma^2 - s - i m_\sigma \Gamma_\sigma(s)}}_{{}_{out}\langle \omega\sigma \mid J/\psi \rangle_{in}} + \underbrace{r_{2\pi}^{NR} e^{i\theta_{2\pi}^{NR}}}_{{}_{out}\langle \omega (2\pi)_{NR} \mid J/\psi \rangle_{in}} + \underbrace{r_{f_2} e^{i\theta_{f_2}} \frac{m_{f_2}\Gamma_{f_2} N(s,\cos\theta)}{m_{f_2}^2 - s - i m_{f_2}\Gamma_{f_2}(s)}}_{{}_{out}\langle \omega f_2 \mid J/\psi \rangle_{in}}$$

This form is consistent with Unitarity of Generalized S-matrix. (Generalized Unitarity)

VMW method

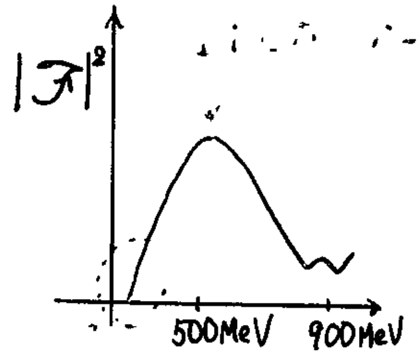
$\pi\pi$ - Scattering





$\pi\pi$ - Production

such as $J/\psi \longrightarrow \omega\pi\pi$
$D^+ \longrightarrow \pi^-\pi^+\pi^+$

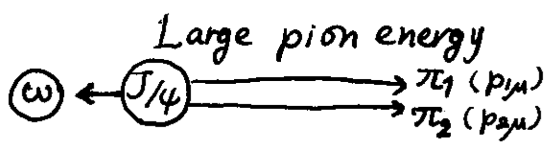


Situations are common for
πK - Scattering & πK - Production Processes

(Adler 0 and threshold behavior)

Production Processes with

at $s \simeq 4m_\pi^2$, $p_{1\mu} \approx p_{2\mu}$



$$E_{\pi 1} \simeq E_{\pi 2} \simeq \frac{M_{J/\psi}}{4} \gg m_\pi$$

$$\mathcal{F} \sim \frac{P_\psi \cdot p_1 \, P_\psi \cdot p_2}{M_\psi^2} + \cdots \sim E_{\pi 1} E_{\pi 2} \simeq \left(\frac{M_\psi}{4}\right)^2 \gg m_\pi^2$$

cf. Scattering $\mathcal{T} \sim p_1 \cdot p_2 + \cdots \sim m_\pi^2$

(Old method)

$$\underset{P_\psi \cdot p_1 P_\psi \cdot p_2}{\mathcal{F}} = \alpha(s) \underset{p_1 \cdot p_2}{\mathcal{T}}$$

(Adler 0 condition)

$$\mathcal{J} \sim p_1 \cdot p_2 \sim s$$

$$p_{1\mu} \longrightarrow 0_\mu \implies s = -(p_1+p_2)^2 \longrightarrow -p_2^2 = \underset{\text{chiral limit}}{0}$$

$$\implies \mathcal{J} \longrightarrow 0$$

$$\mathcal{F} \sim \frac{P\cdot p_1\, P\cdot p_2}{M_{J/\psi}^2} = E_{\pi 1} E_{\pi 2} \cong \left(\frac{M_\psi - m_\omega}{2}\right)^2 + \cdots s + \cdots$$

In $s \longrightarrow 0$ (threshold), $\mathcal{F}$ remains large.

No threshold suppression.

but in $p_{1\mu} \longrightarrow 0_\mu$ (including $E_{\pi 1} \rightarrow 0$), $\mathcal{F} \rightarrow 0$.

Adler 0 condition satisfied.

(Old method)

$$\underset{\substack{\uparrow \\ \text{No thres. suppression}}}{\mathcal{F}} = \underset{\substack{\downarrow \\ \text{rapidly varying funct.!}}}{\alpha(s)}\ \underset{\substack{\uparrow \\ \text{thres. suppression}}}{\mathcal{J}}$$

: $\alpha(s)$ slowly varying func.

$\left(\begin{array}{c}\text{artificial} \\ \alpha(s) \rightarrow \dfrac{\hat{\alpha}(s)}{s-s_0}\end{array}\right)$

$\mathcal{F} = \alpha \mathcal{J}$ is Not applicable to Production Processes with large energy release.

IV. Phases of Production Amplitudes

It is believed by many people that
all the $\pi\pi$-Production amplitudes $\mathcal{F}$ have
the same phase as that of $\pi\pi$-Scattering amplitude $\mathcal{T}$

(Old method) Au, Morgan, Pennington '87

$$\mathcal{F} = \alpha(s)\mathcal{T}$$

$\alpha(s)$: slowly varying real function

- Pole-universality
  $\mathcal{F}$ and $\mathcal{T}$ have common positions of poles
- Watson Final State Interaction Theorem

Reason of Overlooking σ for almost 20 years.

This belief comes from

- incorrect application of elastic Unitarity condition (applicable only to Scattering) to Production Processes.
- overlooking the Strong Phase allowed in Generalized Unitarity condition.

$$\mathcal{F} = r_\sigma e^{i\theta_\sigma} \frac{m_\sigma \Gamma_\sigma}{m_\sigma^2 - s - i m_\sigma \Gamma_\sigma(s)} + r_{2\pi}^{NR} e^{i\theta_{2\pi}^{NR}} + \cdots$$

$$\boxed{\textbf{Generally } \mathcal{F} \textbf{ have different phases from } \mathcal{T}}$$

- $\mathcal{F}$ have the same phase as $\mathcal{T}$ only in limited cases.

"limited cases"
- final $\begin{matrix}\pi\pi \\ K\pi\end{matrix}$ isolated.
- Processes with small energy release.

- Phase motion from σ-pole may be directly observed in some cases.

(Elastic Unitarity Constraint)

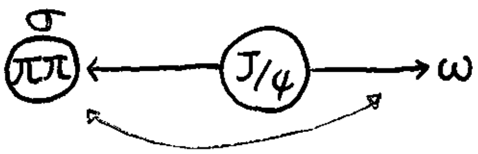
It is often argued that

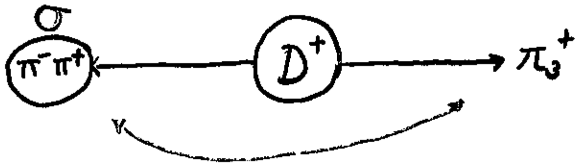
$$\mathrm{Arg}\, \mathcal{F}_{J/\psi \to \omega\pi\pi}(m_{\pi\pi} \simeq 2m_\pi) = \mathrm{Arg}\, \mathcal{T}_{\pi\pi\to\pi\pi}(m_{\pi\pi})$$

$$\mathrm{Arg}\, \mathcal{F}_{D^+ \to \pi^-\pi^+\pi^+}(m_{\pi^-\pi^+} \simeq 2m_\pi) = \mathrm{Arg}\, \mathcal{T}_{\pi\pi\to\pi\pi}(m_{\pi^-\pi^+})$$

$\pi\pi$-threshold region

since





$m_{\omega\pi} \simeq M_{J/\psi}$: large

$m_{\pi^+\pi_3^+} \simeq m_{\pi^-\pi_3^+} \simeq M_D$: large

$\Downarrow$

σ ($\pi\pi$) decouple from ω in $|\omega\pi\pi\rangle_F$

σ ($\pi^-\pi^+$) decouple from π_3^+ in $|\pi^-\pi^+\pi_3^+$

Anisovich / Bugg

⇧

Not Correct.

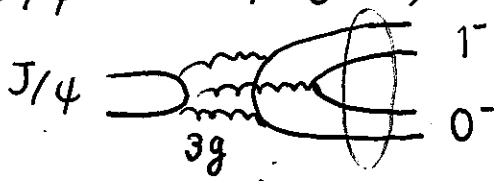
mass-scale $M_{J/\psi}$, M_D is Not large enough.

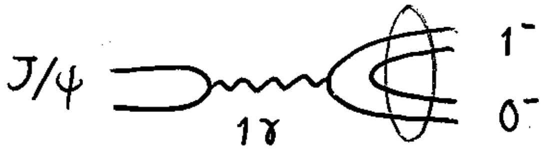
not in the region of perturbative QCD.

Large strong phases $\theta_{\omega\pi}$ ⋯ $\theta_{\pi^-\pi_3^+}$ ⋯ expected.

(Relative strong phases : examples)

- $J/\psi \longrightarrow 1^- 0^-$, $0^- 0^-$ M. Suzuki , N. N. Achasov





$(J/\psi \rightarrow 1^- 0^-)$ $\omega\pi^0, \rho\pi, K^*\overline{K}, \cdots$

$$\mathcal{L}_{int} = a\langle V_9 P_8\rangle + \epsilon\langle\{V_9, P_8\} T_3^{\,0}\rangle + a_\gamma\langle\{V_9, P_8\}\lambda_E\rangle$$

Relative phase of 3g-/1γ-ampl. $\delta_\gamma = \mathrm{Arg}\,\dfrac{a_\gamma}{a} = 80.3^\circ$

best fit $\left|\dfrac{a_\gamma}{a}\right| = 0.34$, $\left|\dfrac{\epsilon}{a}\right| = -0.22$, $\delta_\epsilon \simeq -2$

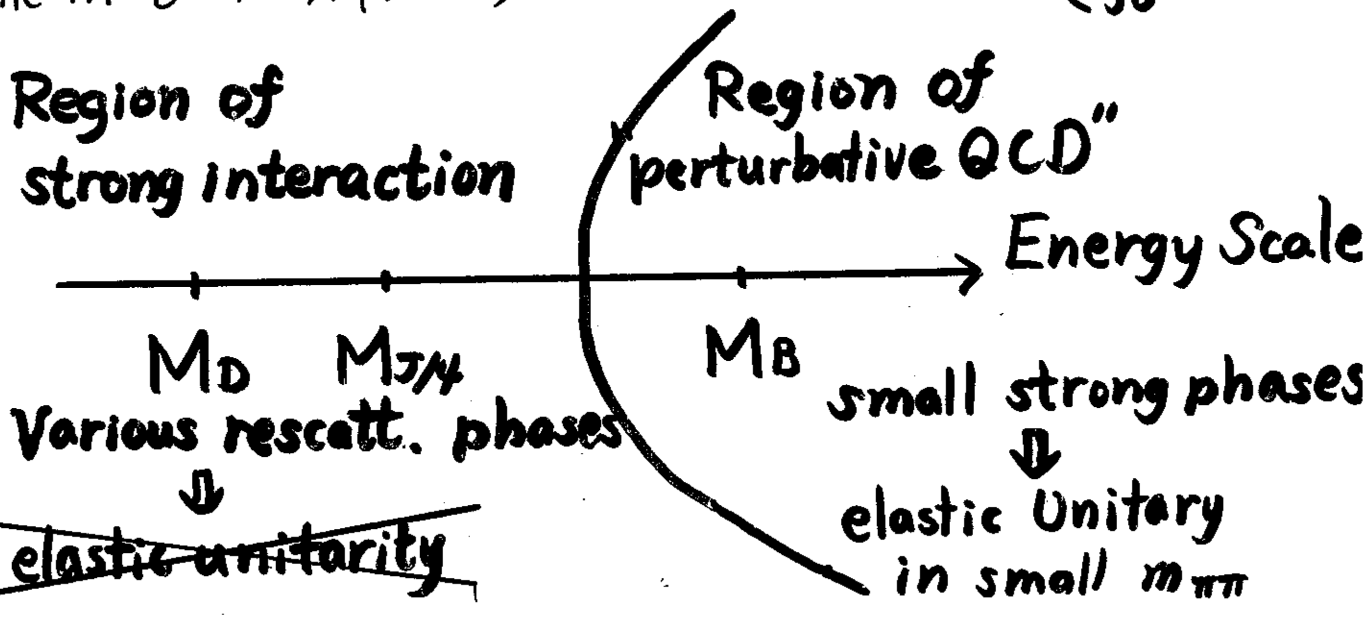
$(J/\psi \rightarrow 0^- 0^-)$ $\pi^+\pi^-, K^+K^-, K^0\overline{K^0}$

$$\mathrm{Arg}\,\frac{A_\gamma}{A_8} = 89.6 \pm 9.9^\circ$$

- $D \longrightarrow K\pi$ CLEO

$$\delta_{\frac{3}{2}}(m_D) - \delta_{\frac{1}{2}}(m_D) = (96 \pm 13)^\circ$$

(While in $B \rightarrow D\pi, D\rho, D^*\pi$, rather small relative phases obtained.) $< 30^\circ$



$\Rightarrow M_{J/\psi}$, M_D is not in the region of p. QCD.

In $J/\psi \longrightarrow \omega\pi\pi$ $\qquad D^- \rightarrow \pi^-\pi^+\pi_3^+$

$J/\psi \longrightarrow K^* K\pi$ $\qquad D^- \rightarrow K^-\pi^+\pi_3^+$

does not decouple from final 3-body channel !

$\Rightarrow$ Elastic Unitarity cannot be applied.
gives No constraint.

Bugg / Anisovich

$$\mathcal{J} = \mathcal{K} / (1 - i\rho\mathcal{K})$$

$$\mathcal{K} = \underbrace{(s - \tfrac{m_\pi^2}{2})}_{\text{Adler 0-factor}} \hat{\mathcal{K}}$$

$\Downarrow$ ~~Elastic Unitarity~~

~~only ππ-rescattering~~ considered.

$$\cancel{\mathcal{F} = N' / (1 - i\rho \underline{(s - \tfrac{m_\pi^2}{2})} \hat{\mathcal{K}})}$$

"Adler 0" in $\mathcal{F}$ ⇐ Bugg

His "Adler 0" has <u>No relation with the standard Adler self-consistency condition.</u>

"$p_{\sigma\mu} \to 0_\mu$, $\mathcal{F} \to 0$"

zero in total amplitude

($J/\psi \to \omega\pi\pi$)

Interesting possibility.

if the peak around $m_{\pi\pi} \simeq 500\,\mathrm{MeV}$ dominated by σ, we may possibly observe the σ-Breit-Wigner phase motion by using σ-$b_1(1235)$ interference in Dalitz Plot.

Ochs "No σ-Breit-Wigner phase motion in $J/\psi \to \omega\pi\pi$"

DM2 $\cos\theta$ distribution in $m_{\pi\pi} = 250 \sim 750\,\mathrm{MeV}$.

Assumptions:

- Partial Wave Expansion (PWE)

$$\frac{d\sigma}{d\Omega} = |S|^2 + 10\,(3\cos^2\theta - 1)\ \mathrm{Re}(SD^*) + O(|D|^2)$$

- D-wave is dominated by $f_2(1275)$-Breit-Wigner ampl. almost real.

$\Rightarrow$ $\mathrm{Arg}\,S < 90^\circ$ in $m_{\pi\pi} = 250 \sim 750\,\mathrm{MeV}$.

BES data

D-wave in this energy region mainly comes from $b_1(1235)$, Not $f_2(1275)$.

theoretical investigation

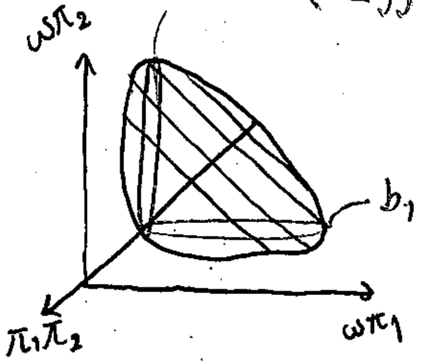
- PWE not applicable $m_{\pi\pi} \gtrsim 500\,\mathrm{MeV}$
- D-wave from $b_1(1235)$ show rather rapid phase motion in $m_{\pi\pi} \simeq 500\,\mathrm{MeV}$.

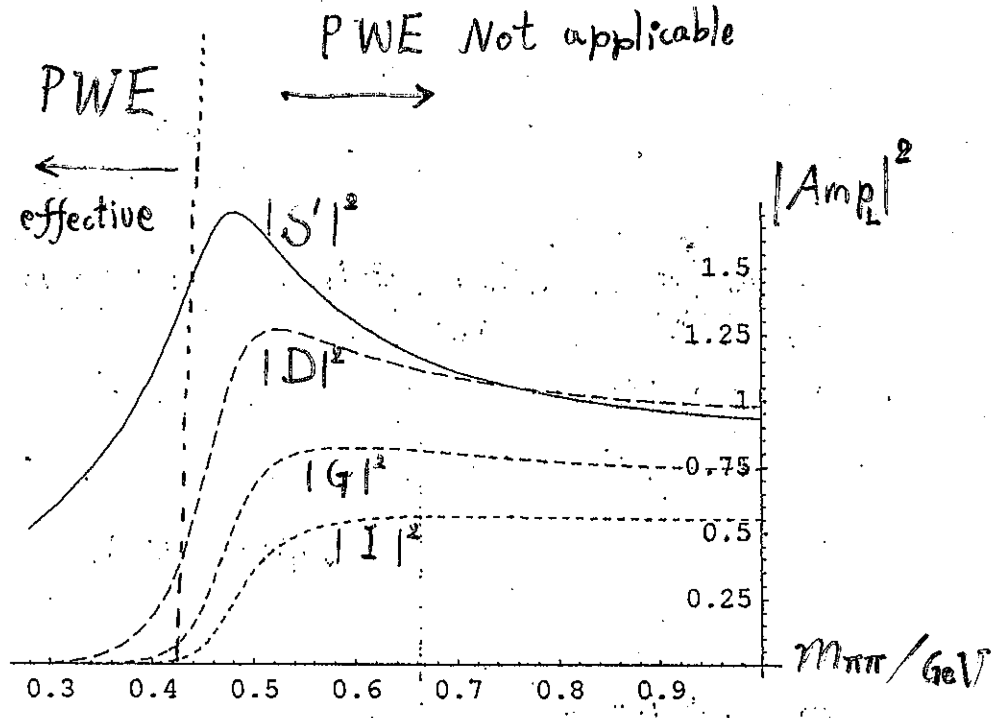
$\Downarrow$

Basic assumptions by Ochs are lost.

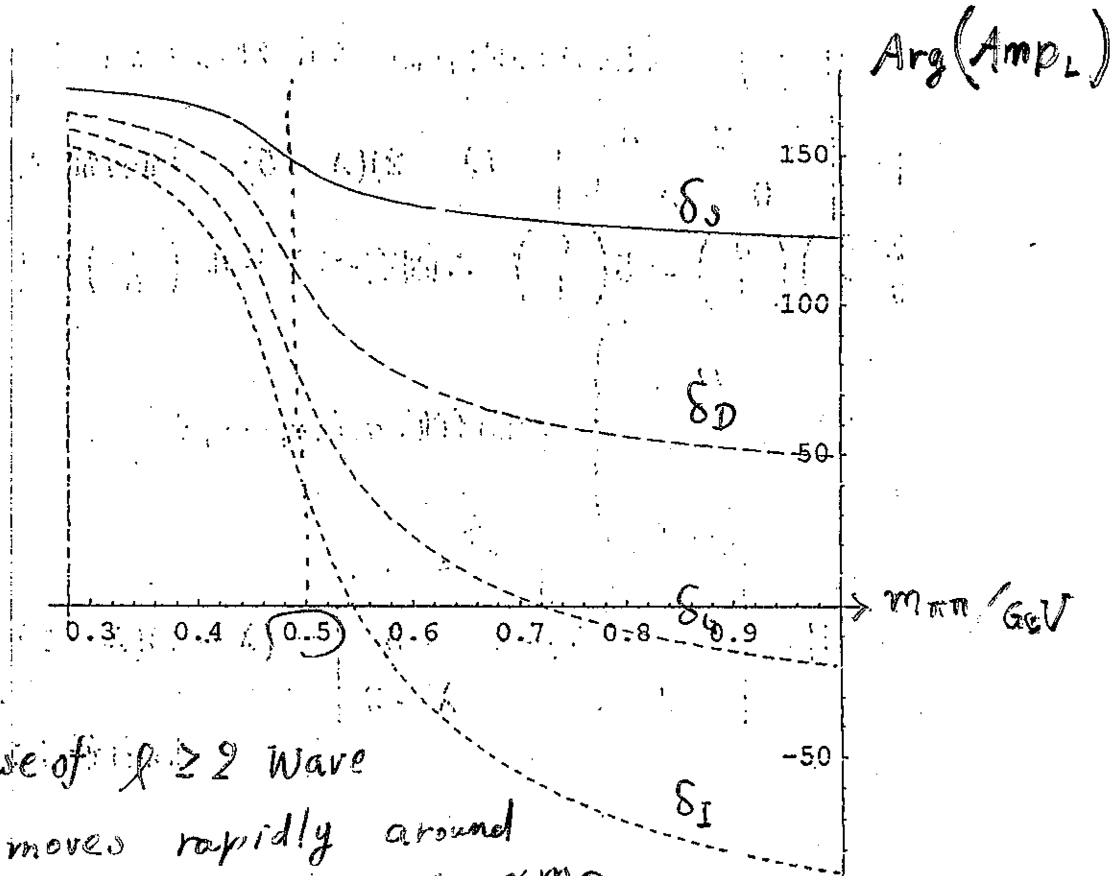
should be re-considered.

(Effect of $b_1(1235)$ in $J/\psi \to \omega\pi\pi$)





PWE of $b_1(1235)$- amplitude



Phase of $\ell \geq 2$ Wave moves rapidly around $m_{\pi\pi} \simeq m_\sigma$

($D^+ \to K^-\pi^+\mu^+\nu$ by FOCUS)

$D^+ \to \overline{K}^{*0}\mu^+\nu$ dominant.

- small S-wave $(K^-\pi^+)$-component.

with constant phase $\delta = \frac{\pi}{4}$ in $m_{K\pi} = 0.8 \sim 1.0$ GeV

necessary to redroduce exp. $\cos\theta$-distrib.

the same phase as the $K\pi$-scattering phase.

Minkowski & Ochs

critisize the κ-Breit-Wigner fit by E 791 of $D^+ \to K^-\pi^+\pi^+$.

"Such a result appears to contradict the above FOCUS result"

Not correct : overlooking the strong phase

- $D^+ \longrightarrow (K^-\pi^+)\mu^+\nu$ **FOCUS**

  isolated in Strong interaction level.

  $\Rightarrow$ Watson theorem applicable.

  $\Rightarrow$ $\mathcal{F}$ and $\mathcal{T}$ have common phase.

- $D^+ \longrightarrow (K^-\pi^+)\pi^+$ **E791**

  $D^+ \searrow \overline{K}^0\pi^+ \nearrow (K^-\pi^+)$ Not isolated

  $|\overline{K}^0\pi^+\rangle$ rescattering

  $\Rightarrow$ $\mathcal{F}$ has different phase from $\mathcal{T}$

Method of analyses adopted by E791 is correct.

Their result of $\kappa(800)$-existence is reliable.

V. Concluding Remark

Conventional Analyses of $\left.\begin{matrix}\pi\pi \\ K\pi\end{matrix}\right)$ Production Processes

common fit with $\begin{matrix}\pi\pi\text{-} \\ K\pi\text{-}\end{matrix}$ Scattering phase shift δ

based on "Universality of $\begin{matrix}\pi\pi\text{-} \\ K\pi\text{-}\end{matrix}$ scattering

- $\mathcal{F} = \alpha\,\mathcal{T}$ with slowly varying real $\alpha(s)$
- $\Rightarrow$
  - threshold suppression of mass spectra
  - universal phase motion : Watson theorem

    $\mathrm{Arg}\,\mathcal{F} = \mathrm{Arg}\,\mathcal{T} = \delta$ elastic unitarity.

but Actually in $\mathcal{F}$

- No threshold suppression in the processes with large energy release to $\left(\begin{matrix}\pi\pi. \\ K\pi.\end{matrix}\right.$
- $\mathrm{Arg}\,\mathcal{F} \neq \mathrm{Arg}\,\mathcal{T}$

  elastic unitarity cannot be applicable.

  various strong phases of generalized S-matrix elements.

$\Downarrow$

common fit is Not Correct.

$\mathcal{F}$ of J/ψ and D decays <u>must be independently analyzed from $\mathcal{T}$ (or δ)</u>

Especially Bugg's statement of "Adler 0"

"0-factor $(s-\frac{m_\pi^2}{2})$ in Im D of $\mathcal{F}$"

- has No relation with standard prescription of Adler 0.
- comes from the incorrect application of elastic unitarity.

C. Goebel

$\Delta\delta_\sigma \sim 180^\circ$ is observed in

$D^+ \to \pi^-\pi^+\pi^+$

large $(K\pi)_S$-phase motion

(different from $K\pi$-scattering^{LASS})

is suggested in $D^+ \to K^-\pi^+\pi^+$

$\Downarrow$

$\mathrm{Arg}\,\mathcal{F} \neq \mathrm{Arg}\,\mathcal{T}$

~~Universality $\mathcal{F} = \alpha\,\mathcal{T}$~~

~~Pennington / Bugg~~

[Comment on Bugg's method]

- Various strong interaction phases of generalized S-matrix elements overlooked.

- Incorrect application of Elastic Unitarity

$$\mathcal{T} = K/(1-i\rho K)$$

$$K = \left(s-\frac{m_\pi^2}{2}\right)\hat{K}$$

~~Elastic Unitarity~~

$$\xcancel{\mathcal{F} = N'/\left(1-i\rho\left(s-\frac{m_\pi^2}{2}\right)\hat{K}\right)}$$

Bugg's statement "Adler 0 in $\mathcal{F}$"

- Elastic Unitarity cannot be applied to J/ψ decay
- standard prescription of <u>Adler 0 condition</u> describes the 0 in total $\mathcal{F}$.

  " $p_{1\mu} \to 0_\mu$, $\mathcal{F} \to 0$ "

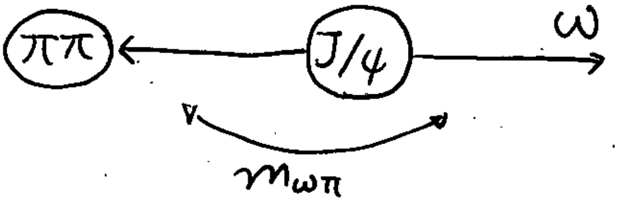
  (Other pion momenta remained unchanged)

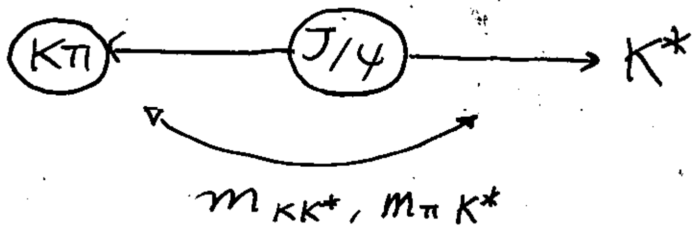
$\longleftrightarrow$ Bugg's 0 does not describe 0 in total $\mathcal{F}$

Bugg's 0-factor condition in Im $\mathcal{D}$ of $\mathcal{F}$ has <u>norelation</u> with the Adler 0 condition. comes from Incorrect application of <u>eles. unitarily.</u>

Not correct.

• "Elastic Unitarity condition" in Prod. Proc.





In low $m_{\pi\pi}$ mass region
$m_{\omega\pi} \sim M_{J/\psi}$: large

anticipation
$\Longrightarrow$ ω decouples from final $|\omega\pi\pi\rangle_F$

In low $m_{K\pi}$ region
$m_{KK^*}, m_{\pi K^*} \sim M_{J/\psi}$: large

anticipation
$\Longrightarrow$ K^* decouples from final $|K^* K\pi\rangle_F$

These anticipations by Bugg **are not Correct because $M_{J/\psi}$ is not large enough.**

not in the region of p. QCD.

$\Uparrow$

M. Suzuki N. N. Achasov.

$$\mathcal{F} = N'/D \qquad N' = \frac{G_{J/\psi}}{M^2 - s}$$

$$D = 1 - i\rho K$$

parameter with No physical meaning

$b_1 + b_2 s$

$$K \propto \frac{f(s)}{M^2 - s} \boxed{\left(s - \frac{m_\pi^2}{2}\right)} \times \boxed{e^{-\frac{s-M^2}{a}}}$$

"Bugg's 0-factor in Im D"
Incorrect.
Strong s-dependence
proportional to energy-squared!

obliged to include "F.F." with strong s-dependence

S-wave Amplitude : Blatt-Weiskopf type factor gives 1.

No F.F.

Mistake $\left(s - \frac{m_\pi^2}{2}\right) \longrightarrow$ Mistake $e^{-\frac{s-M^2}{a}}$

$\boxed{\textbf{Parameter fitting with No Physics.}}$

Especially their obtained param. of σ/κ unreliable!

$\left(s - \frac{m_\pi^2}{2}\right)$ $\Longrightarrow$ $\left.\begin{array}{l}\Gamma_\sigma \\ \Gamma_\kappa\end{array}\right)$ **enlarged!**

Incorrect factor

~~$\Gamma_\kappa = 800$ MeV~~ **too large**

Artifact from his Incorrect form
No relation with BES data themselves

$\Downarrow$

Removed **WuNing Komada** $\Longrightarrow$ $\Gamma_\kappa = 300 \sim 400$ **MeV determined directly from BES data.**