Scalar Mesons in Lattice QCD Calculations

Scalar Collaboration

International Symposium on

Hadron Spectroscopy, Chiral Symmetry and

Relativistic Description of Bound Systems

24-26 Feb. 2003 Nihon University

SCALAR Collaboration

- T. Kunihiro, YITP, Kyoto Univ.
- S. Muroya, Tokuyama Women's Coll.
- A. Nakamura, RIISE, Hiroshima Univ.
- C. Nonaka, Dept. Phys., Duke Univ.
- M. Sekiguchi, Fac. of Eng. Kokushikan Univ.
- H. Wada, Fac. of Sci. and Eng.. Nihon Univ.

Objective of Scalar Collaboration

- Confidence level of Sigma Meson (and other scalar meson, κ) has been increasing.
- Using Lattice QCD, we have been (and will be) addressing the following Question about scalar mesons:

  Are you a Pole in QCD ?

- We study its features in QCD.

Lattice QCD Calculation

- Relativistic Formulation
  - Quarks are described by Dirac Fermions
- Not a Model
- Apart from numerical limitations, there is no approximation.
- No bound state Calculation
  - No potential
  - No B-S
  - It measures the mass gap in a given channel.

Lattice QCD Calculation (cont'd)

- Euclidean Path Integral

$$Z = \int dU d\bar{\psi} d\psi \, e^{-\bar{\psi} D \psi - S_G}$$

$$G(x,y) = \frac{1}{Z} \int dU d\bar{\psi} d\psi \, H(y) H^\dagger(x) e^{-\bar{\psi} D \psi - S_G}$$

$$\longrightarrow e^{-m|x-y|}$$

$H(x)$: Hadron Operator. For σ meson $H(x) = \bar{\psi}(x)\psi(x)$

$H^\dagger(x)|0>$ State with Quantum Numbers specified by H

You should trust Lattice QCD

because it is the First Principle Calculation!

You should not trust Lattice QCD

until the following conditions are satisfied:

- **Enough Statistic**
  - Gauge configurations are generated by Monte Carlo, and there are statistical errors like Experiments.
- **Continuum Limit**
  - Lattice spacing $a \to 0$
- **Infinite Volume Limit**
  - Lattice Volume $(La)^4$ is large enough to include hadron.
- **Chiral Extrapolation**
  - u and d quark masses on the lattice are large, and extrapolated to zero.

Operator for σ Meson

- I=0, scalar

$$\sigma(x) \equiv \sum_{c=1}^{3} \bar{\psi}_c(x)\psi^c(x)$$

$$= \sum_{c=1}^{3}\sum_{\alpha=1}^{4} \frac{\bar{u}_\alpha^c(x)u_\alpha^c(x) + \bar{d}_\alpha^c(x)d_\alpha^c(x)}{\sqrt{2}}$$

c = 1,2,3 ・・・ color

α =1,2,3,4 ・・・ Dirac spin

$$\psi \equiv \begin{pmatrix} u \\ d \end{pmatrix}$$

Propagator

$$G(y,x) = \left\langle \sigma(y)\sigma(x)^{\dagger} \right\rangle$$

$$= \frac{1}{Z}\int DUD\bar{u}DuD\bar{d}Dd \sum_{a,b=1}^{3}\sum_{\alpha,\beta=1}^{4} \frac{\bar{u}_{\beta}^{b}(y)u_{\beta}^{b}(y)+\bar{d}_{\beta}^{b}(y)d_{\beta}^{b}(y)}{\sqrt{2}}$$

$$\left(\frac{\bar{u}_{\alpha}^{a}(x)u_{\alpha}^{a}(x)+\bar{d}_{\alpha}^{a}(x)d_{\alpha}^{a}(x)}{\sqrt{2}}\right)^{\dagger} e^{-S_g-\bar{u}Wu-\bar{d}Wd}$$

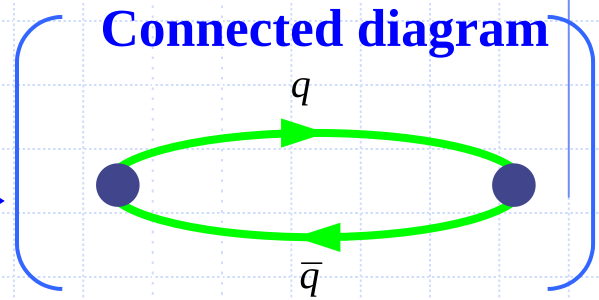
$$= \frac{1}{Z}\int DUD\bar{u}DuD\bar{d}Dd \sum_{a,b=1}^{3}\sum_{\alpha,\beta=1}^{4} \frac{1}{2}\Big[\ \bar{u}_{\beta}^{b}(y)u_{\beta}^{b}(y)\bar{u}_{\alpha}^{a}(x)u_{\alpha}^{a}(x)$$

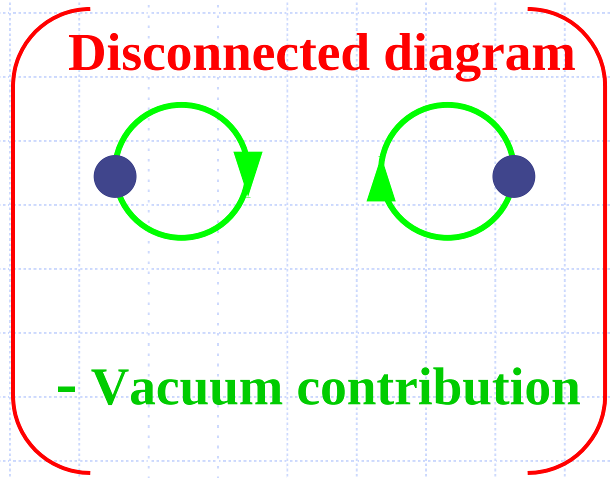
$$+\bar{d}_{\beta}^{b}(y)d_{\beta}^{b}(y)\bar{d}_{\alpha}^{a}(x)d_{\alpha}^{a}(x)+\bar{u}_{\beta}^{b}(y)u_{\beta}^{b}(y)\bar{d}_{\alpha}^{a}(x)d_{\alpha}^{a}(x)$$

$$+\bar{d}_{\beta}^{b}(y)d_{\beta}^{b}(y)\bar{u}_{\alpha}^{a}(x)u_{\alpha}^{a}(x)\ \Big] e^{-S_g-\bar{u}Wu-\bar{d}Wd}$$

Propagator (cont'd)

$$
\begin{aligned}
G(x,y) &= -\left\langle Tr\left(W^{-1}(x,y)W^{-1}(y,x) \right)\right\rangle \\
&\quad + 2\left\langle Tr\left(W^{-1}(y,y) \right)Tr\left(W^{-1}(x,x) \right)\right\rangle \\
&\quad - 2\left\langle Tr\left(W^{-1}(y,y) \right)\right\rangle\left\langle Tr\left(W^{-1}(x,x) \right)\right\rangle
\end{aligned}
$$





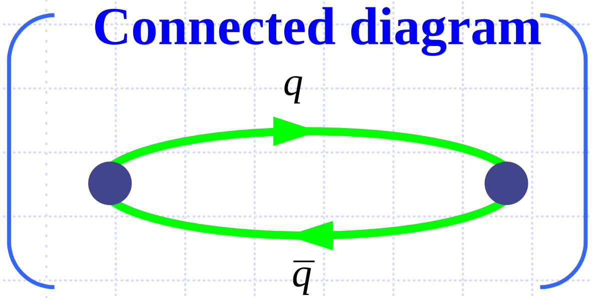
$W^{-1}(x,y)$: Inverse of Fermion Matrix, i.e., Quark Propagators

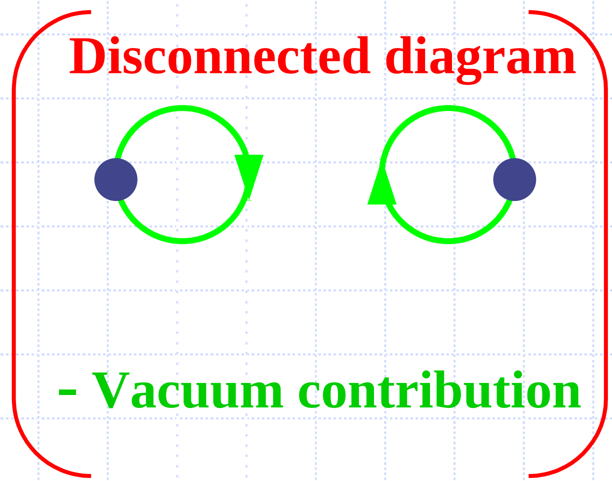
Propagator (cont'd)

$$G(x,y) = -< Tr W^{-1}(x,y) W^{-1}(y,x) >$$
$$+2 < (\sigma(x) - <\sigma>)(\sigma(y) - <\sigma>) >$$

where

$$\sigma(x) \equiv Tr W^{-1}(x,x) = \bar{\psi}(x)\psi(x)$$





Lattice QCD simulations of σ

- There have been many Lattice Simulations of scalar without the disconnected diagram; "Valence Sigma" σ_V
- deTar and Kogut
  - Phy.Rev. D36, (1987) 2828.
  - Screening masses
- Kim and Ohta
  - hep-lat/9609023,hep-lat/9712014
  - KS fermions, β=6.5, $48^3 \times 64$
  - *a*=0.054fm, 48*a*=2.6fm,

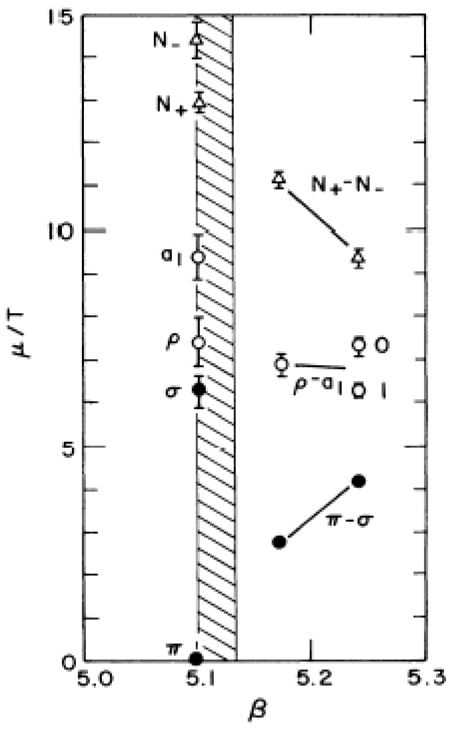
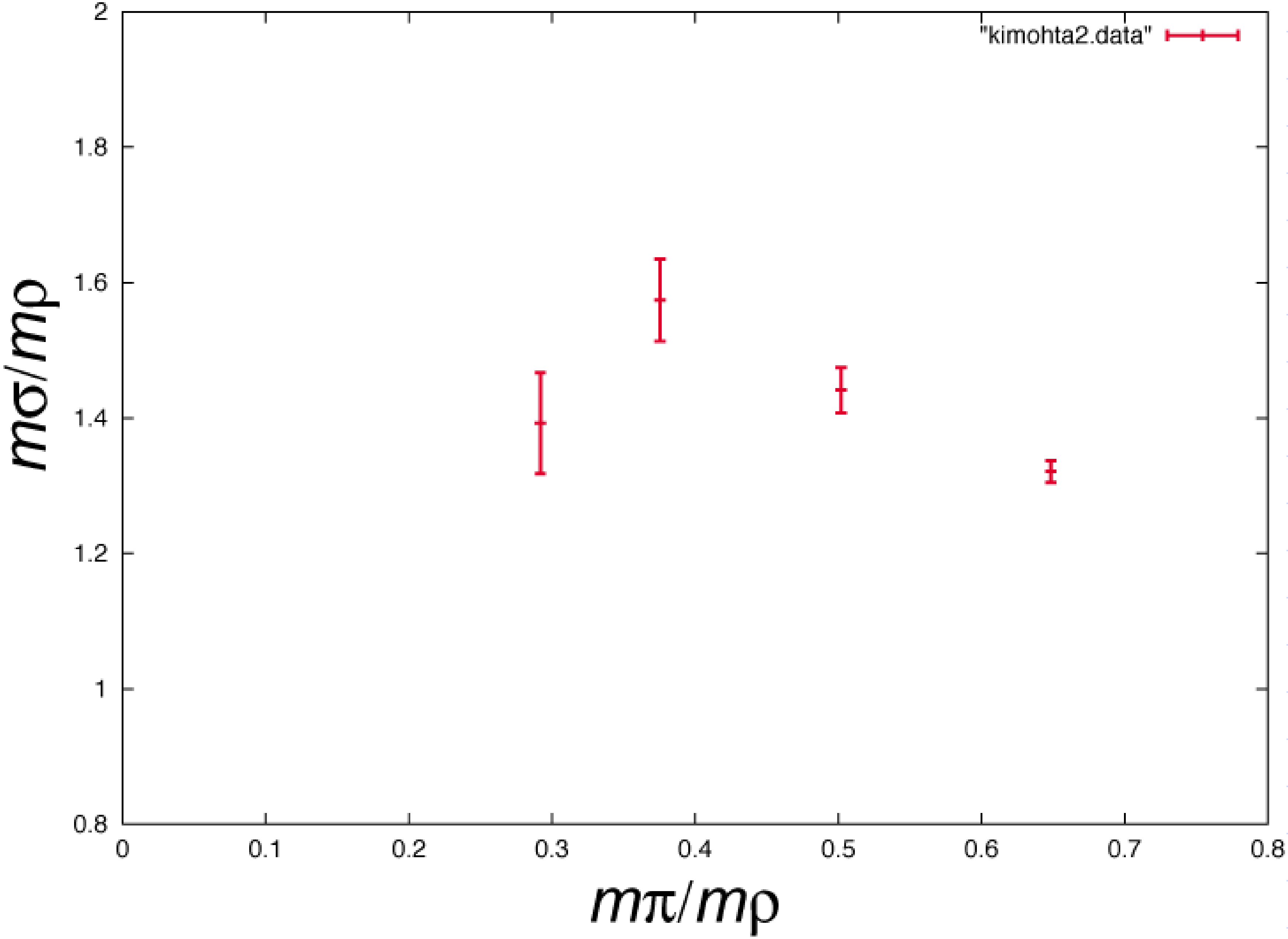


FIG. 10. Screening masses, expressed in units of the temper- ature, as extrapolated to the chiral limit, for the π-, σ-, ρ-, and a₁-meson plasmon modes, and the lowest even-parity (N_+) and


"kimohta2.data"
mσ/mρ
mπ/mρ
2
1.8
1.6
1.4
1.2
1
0.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Lattice QCD simulations of σ (cont'd)

- W. Lee and D. Weingarten
  - Phys. Rev. **D61** (1999) 014015
  - Quench
  - Mixing of Glue-ball and
- UKQCD C.McNeile and C.Michael
  - Phys. Rev. **D63** (2001) 114503
  - Full QCD
- Alford and Jaffe, Nucl.Phys. B578 (2000)367.
  - Quench
  - $\sigma = qq\overline{qq}$ $E(q\bar{q}q\bar{q}) < E(q\bar{q} + q\bar{q})$

W. Lee and D. Weingarten, Phys. Rev. **D61** (1999) 014015

Mixing of $q\bar{q}$ and glueball (I=0, $J^{PC}=0^{++}$)

Quenched approximation
Wilson fermion
Plaquette gauge action

$f_0(1710)$ ・・・ lightest scalar glueball (73.8 (9.5)%)
$f_0(1500)$ ・・・ $s\bar{s}$ quarkonium (98.4 (1.4)%)
$f_0(1390)$ ・・・ $n\bar{n}$ quarkonium (main)

n stands for $u\bar{u} - d\bar{d} / \sqrt{2}$

Lee and Weingarten (cont'd)

$$\begin{pmatrix} m_g & E(\mu_s) & \sqrt{2}rE(\mu_s) \\ E(\mu_s) & m_\sigma(\mu_s) & 0 \\ \sqrt{2}rE(\mu_s) & 0 & m_\sigma(\mu_n) \end{pmatrix} \Rightarrow \begin{pmatrix} f_0(1710) & & \\ & f_0(1500) & \\ & & f_0(1390) \end{pmatrix}$$

Input:

f0(1710)	1697(4)MeV
f0(1500)	1505(9)MeV
f0(1390)	1404(24)MeV
$m_\sigma(\mu_n)$	1470(25)MeV
$r \equiv E(\mu_n)/E(\mu_s)$	1.198(72)

(Only r is given by Lattice.)

Output:

m_g	1622(29) MeV
$m_\sigma(\mu_s)$	1514(11) MeV
$E(\mu_s)$	64(13) MeV

Lattice:

m_g	1654(47) MeV (World Average)
$m_\sigma(\mu_s)$	1322(42) MeV
$E(\mu_s)$	43(31) MeV

C.McNeile and C.Michael (UKQCD), Phys. Rev. **D63** (2001) 114503

Mixing of the Iso-singlet scalar (I=0, $J^{PC}=0^{++}$) and Glueball

Mass with Full QCD
<< mass with quench

$M_\sigma < M_\pi$

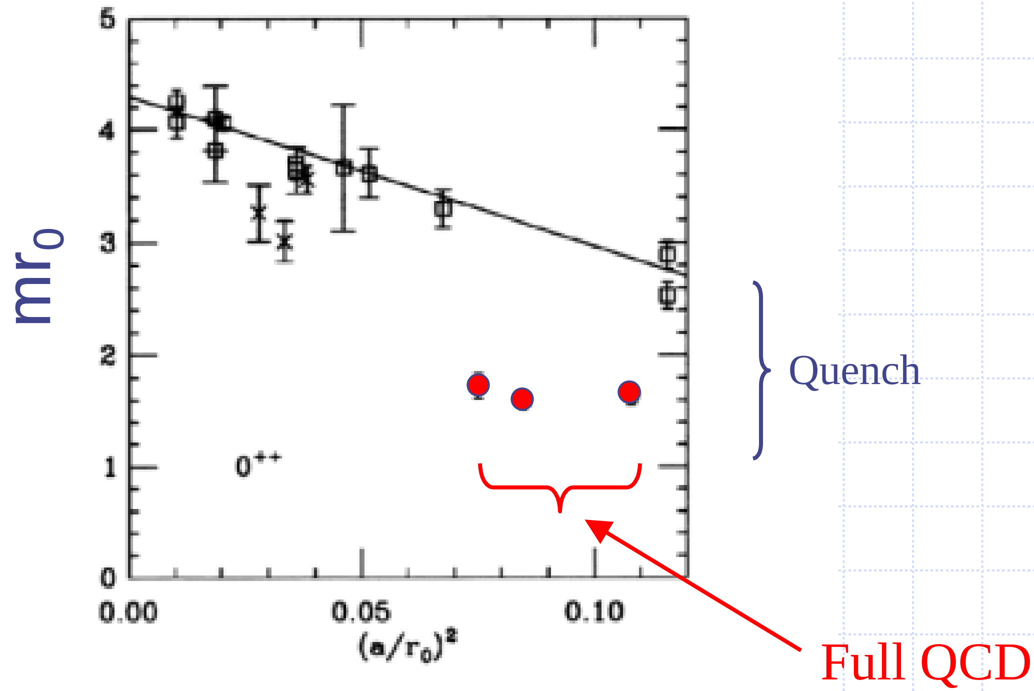


FIG. 5. The scalar mass versus a^2. The quenched results [12,10,13,14] are for the scalar glueball and are shown by boxes. The results from $N_f=2$ flavors of sea quark are from glueballs [15] (crosses from SESAM) and the lightest flavor singlet scalar we find here (circles).

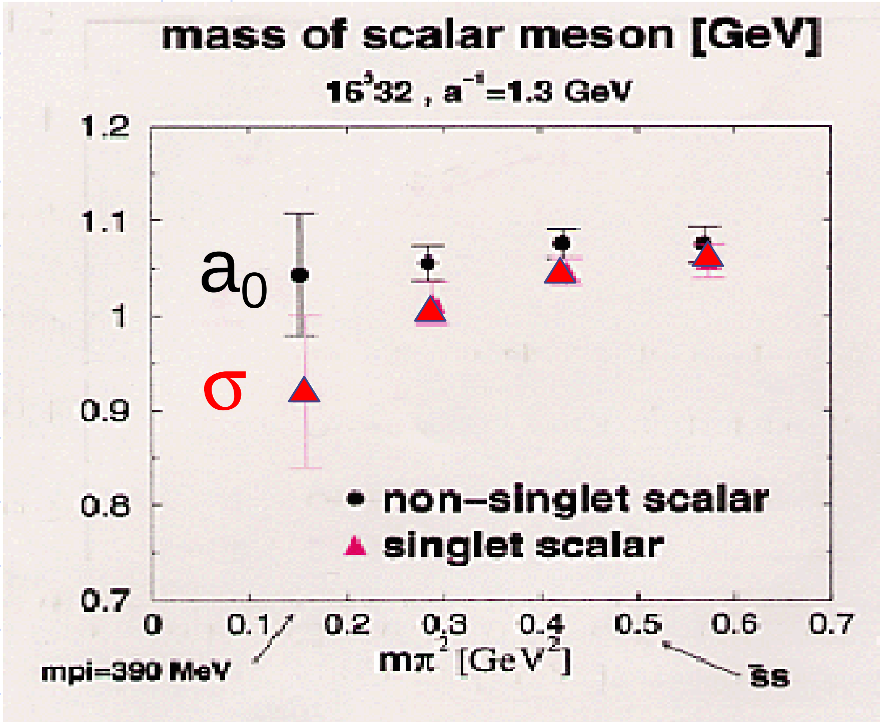
r_0: Sommer factor $\quad r_0^{-1} = 394 \pm 20 MeV$

Lattice QCD simulations of σ
- current going projects -

- Riken-Columbia-Brookhaven
  - Domain Wall Fermions
  - Quench
  - hep-lat/0209132 (Lattice02 Proceedings)
- Scalar Collaboration
  - Wilson Fermions
  - Full QCD
  - hep-lat/0210012 (Lattice02 Proceedings)

Riken-Brookhaven-Columbia

- Domain-wall fermions: Good Chiral nature
- Quench: Check the sickness of the quench calculations by quenched chiral perturbation theory.



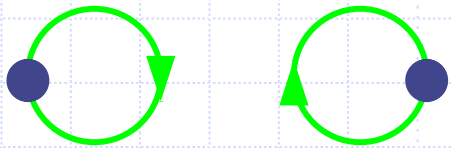
Details of our Calculation (1)

Wilson Fermions (2 flavors)

Plaquette Gauge Action

Full QCD Update by Hybrid Monte Carlo (SX5 at RCNP)

Disconnected Part by Z_2 Noise Method (SR8000 at KEK)



Details of our Calculation (2) - Simulation parameters

Lattice size : $8^3 \times 16$

$\beta = 4.8$

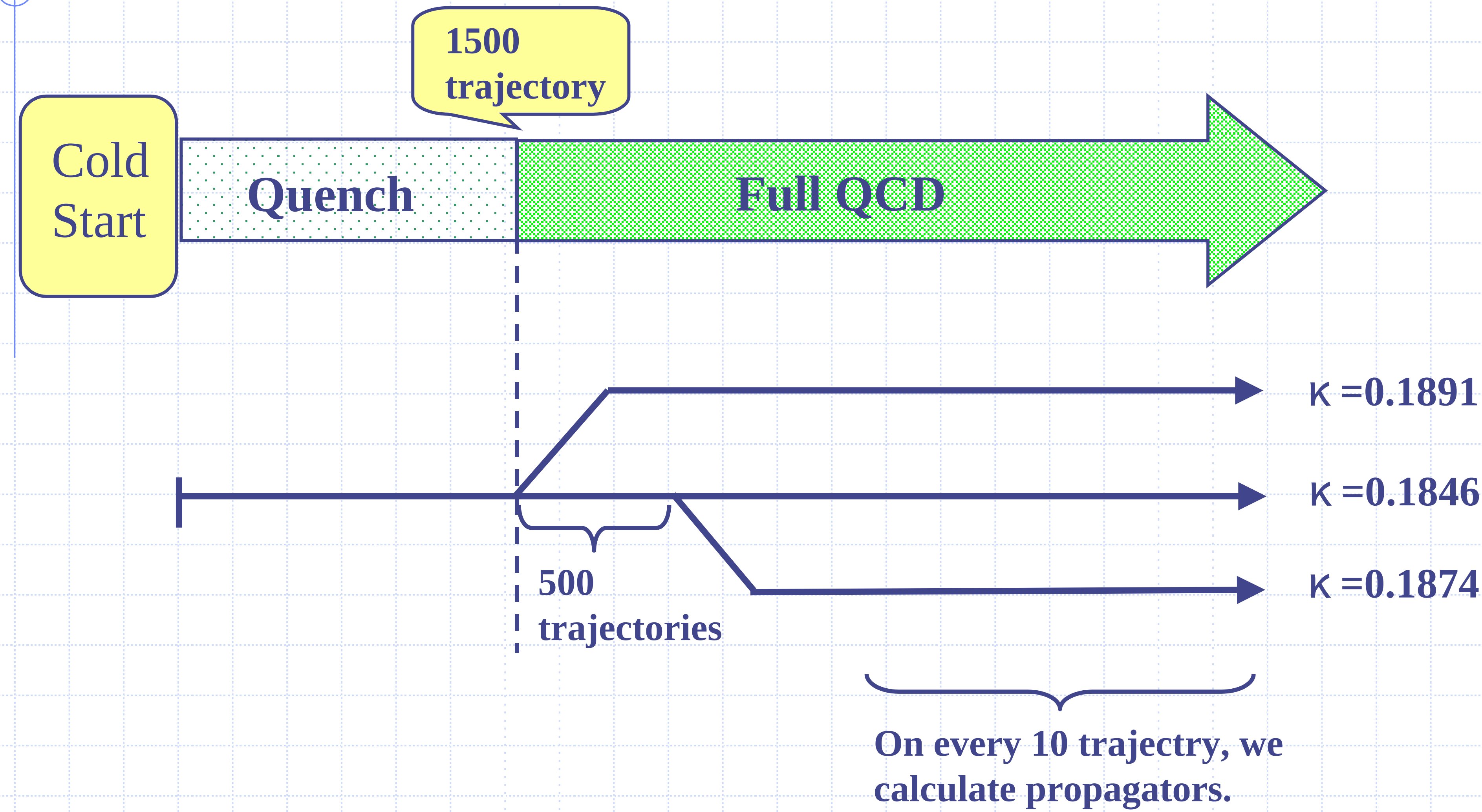
$\kappa = 0.1846,\ 0.1874,\ 0.1891$

well established by CP-PACS,
a = 0.197(2) fm , $\kappa_c = 0.19286(14)$
(CP - PACS, Phys. Rev. D60(1999)114508)

Wilson Fermions & Plaquette gauge action

Number of the Z2 noise = 1000 , 500

Details of our Calculation (3)



Details of our Calculation (4)

[κ = 0.1846]

1470 configurations from 720th trajectory

[κ = 0.1874]

970 configurations from 710th trajectory

[κ = 0.1891]

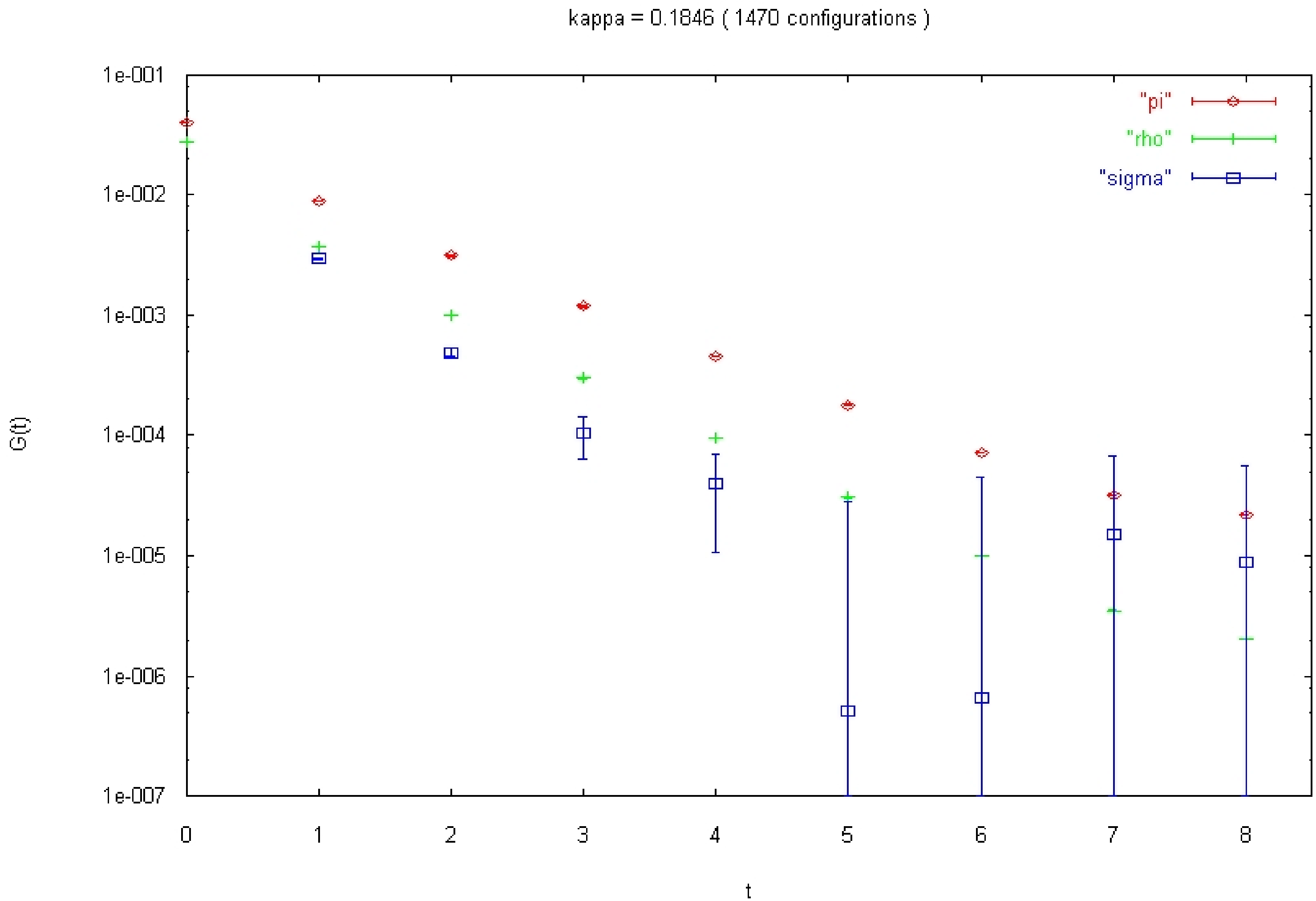
400 configurations from 500th trajectory

(Separation between configurations are 10 trajectories)

Details of our Calculation (5)

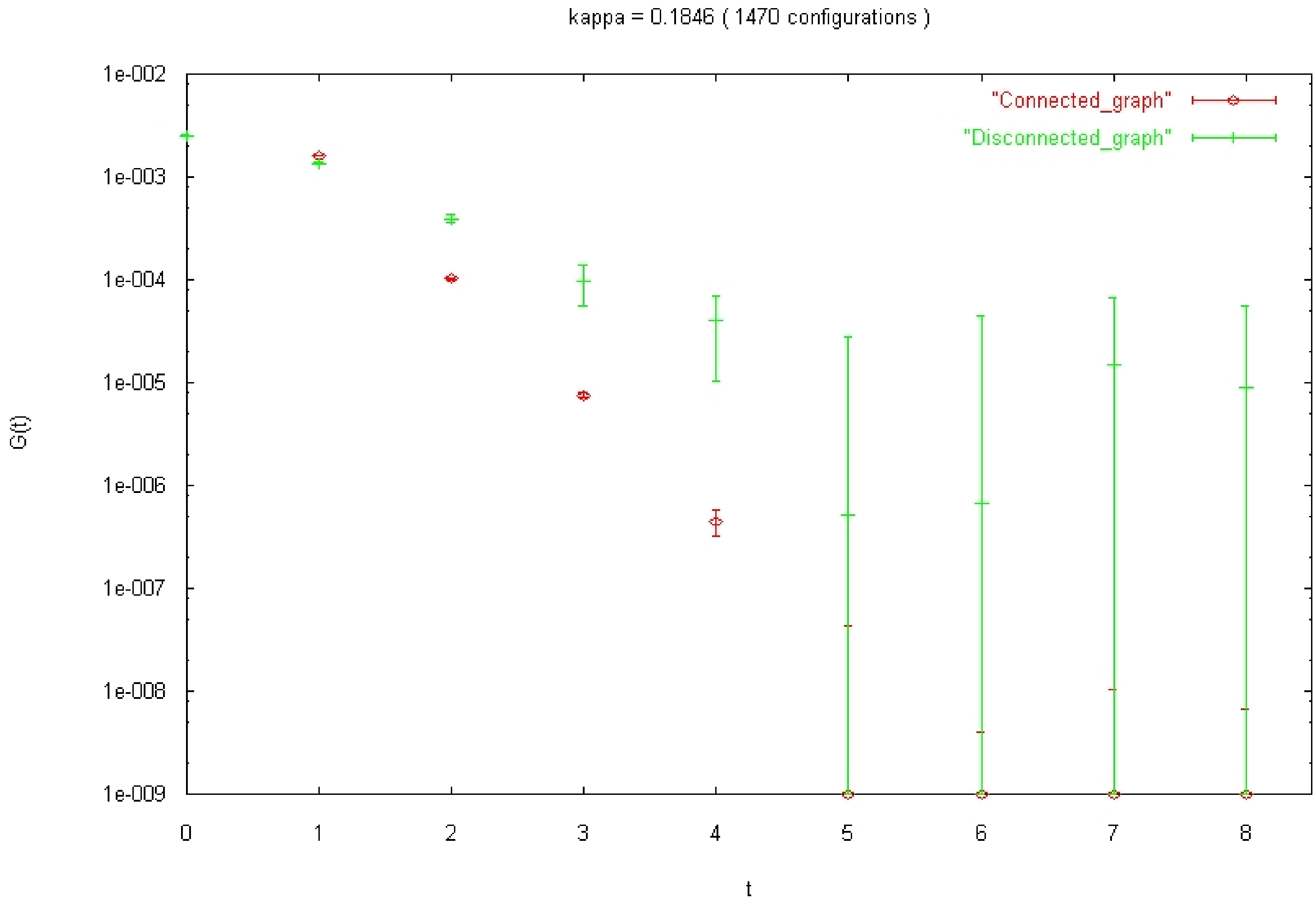
κ		m_π/m_ρ (Our Results)	m_π/m_ρ (CP-PACS)
0.1846		0.825±0.001	0.8291±0.0012
0.1874		0.760±0.002	0.7715±0.0017
0.1891		0.692±0.005	0.7026±0.0032

π , ρ , σ mesons (κ =0.1846)

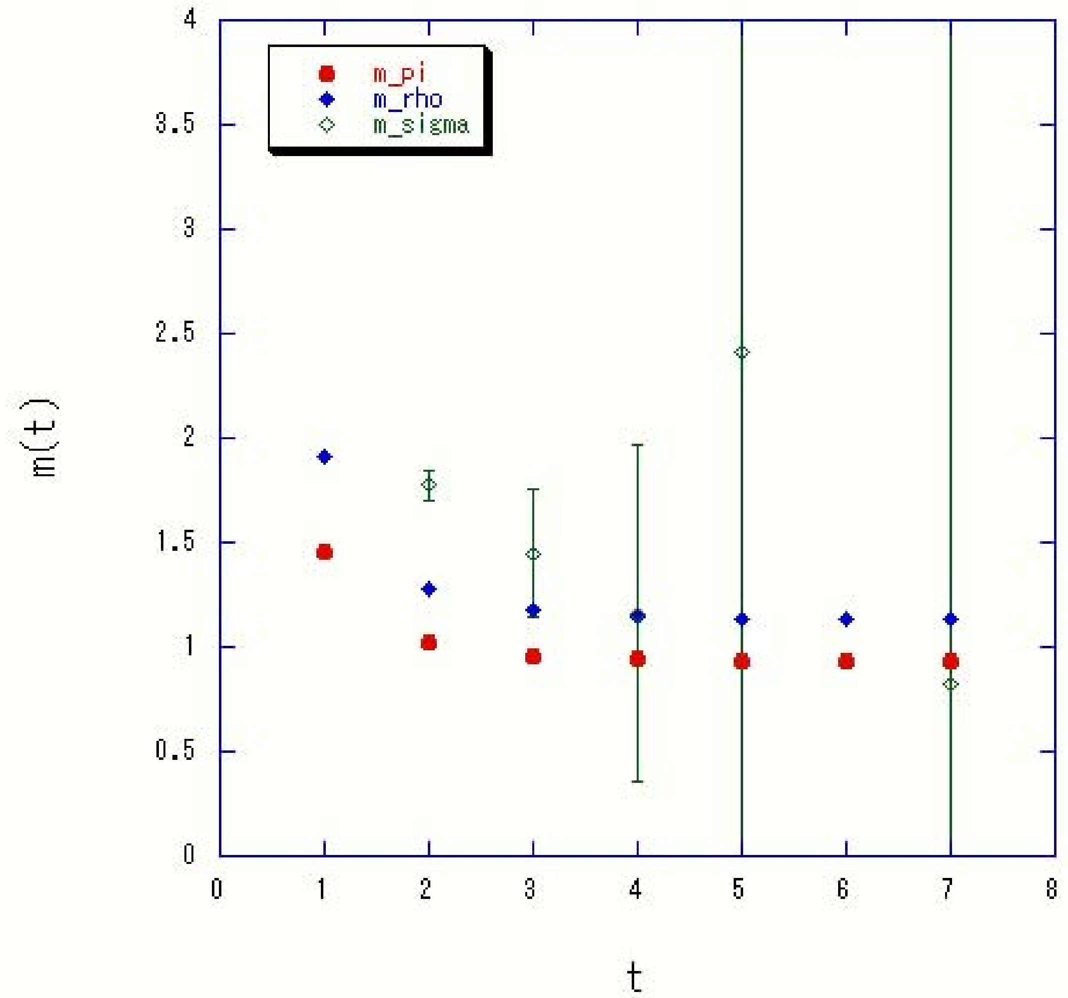


σ meson propagators

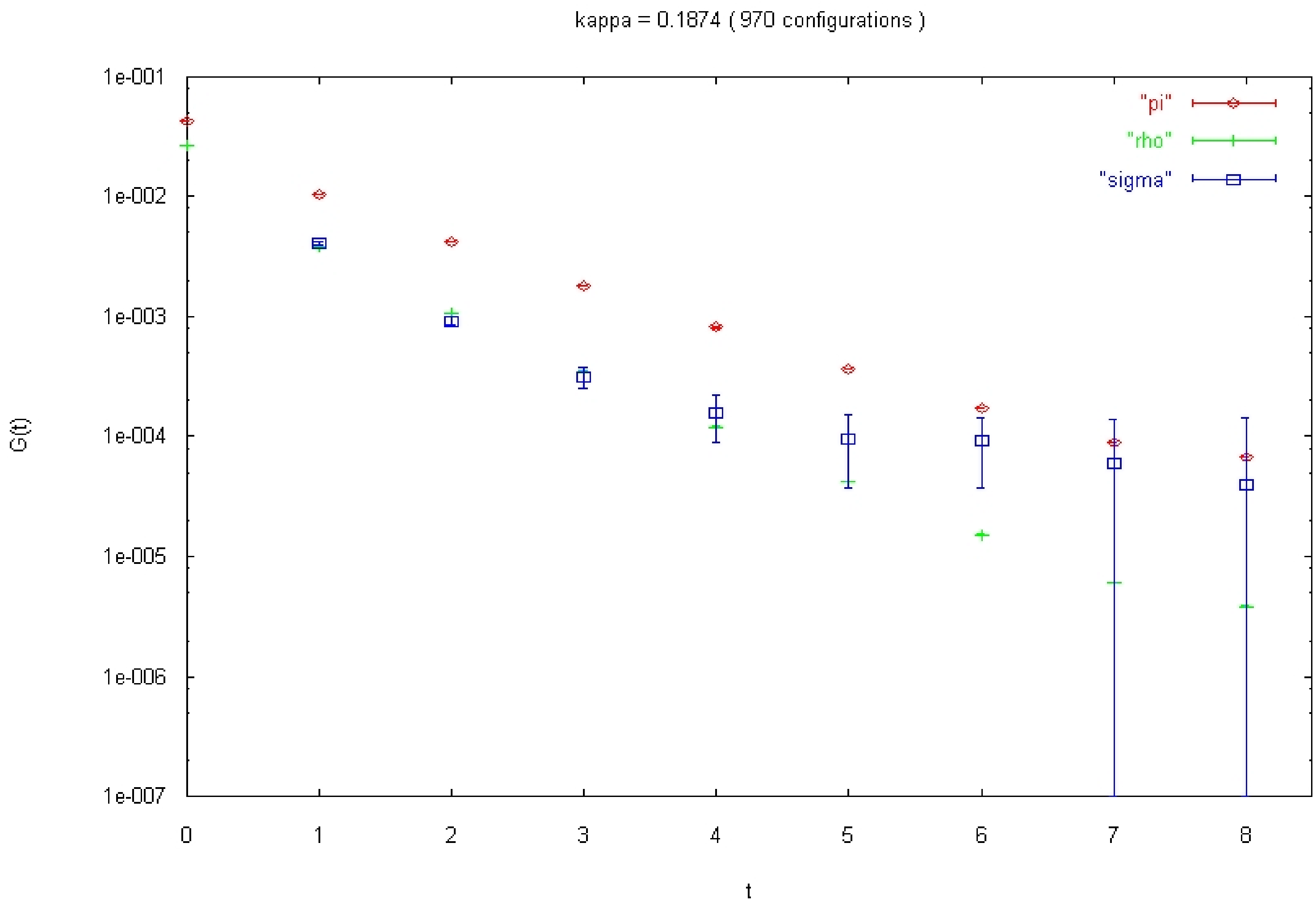
Connected and Disconnected Parts (κ =0.1846)



Effective mass (κ =0.1846)

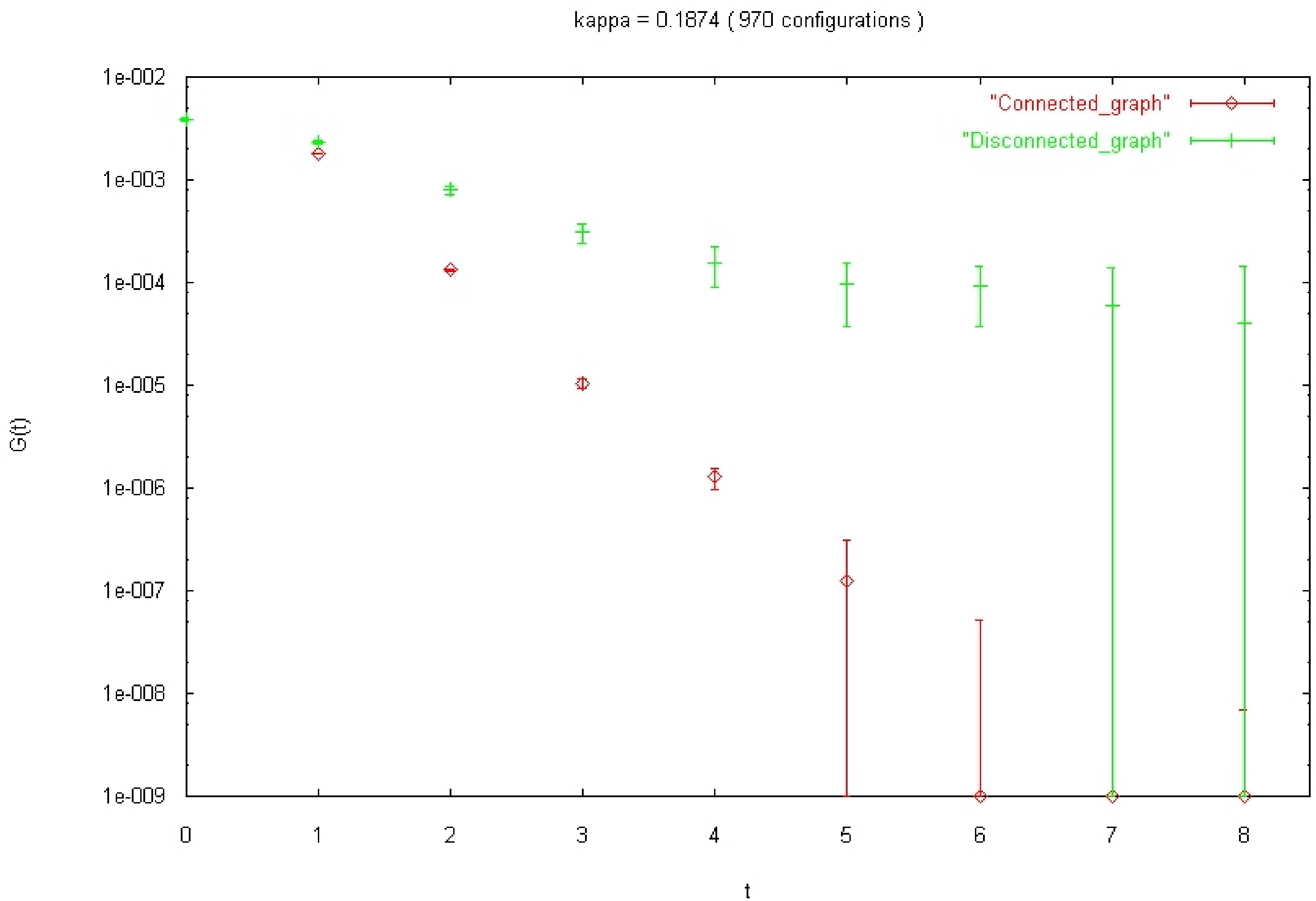


π , ρ , σ mesons (κ =0.1874)

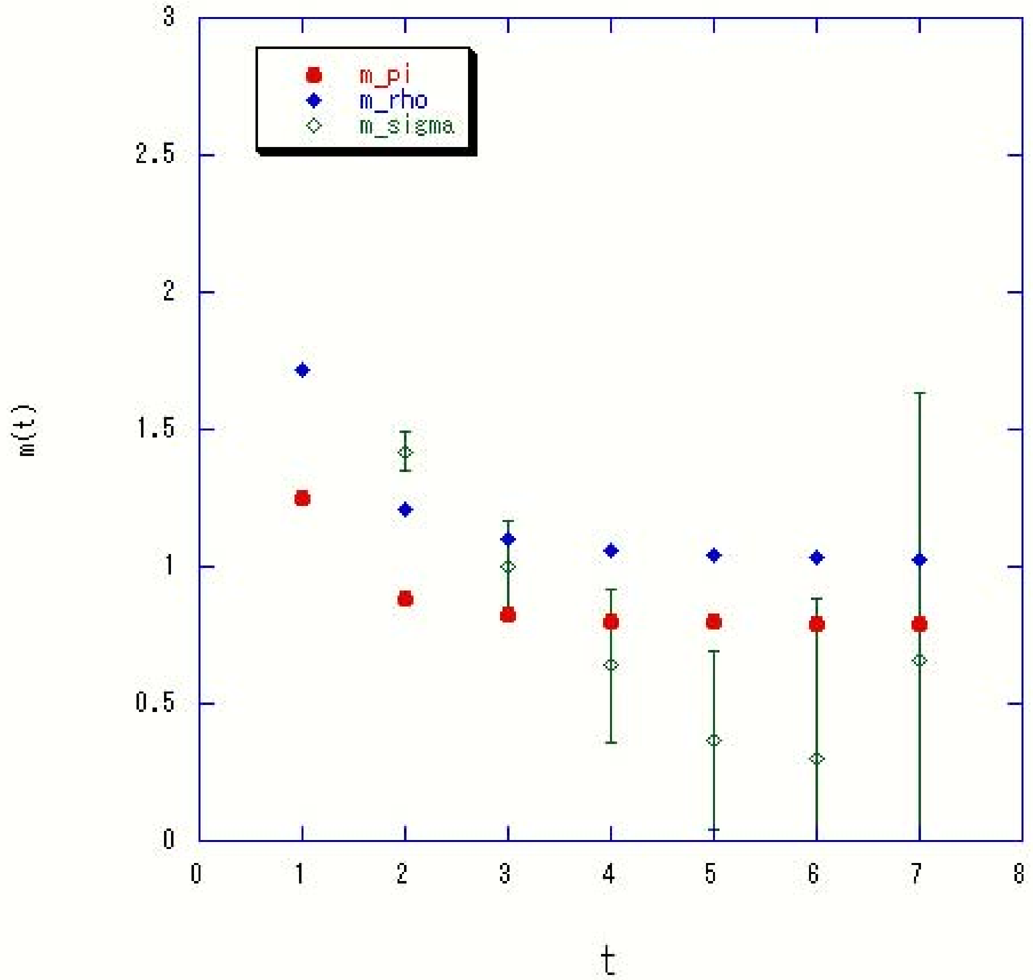


σ meson propagators

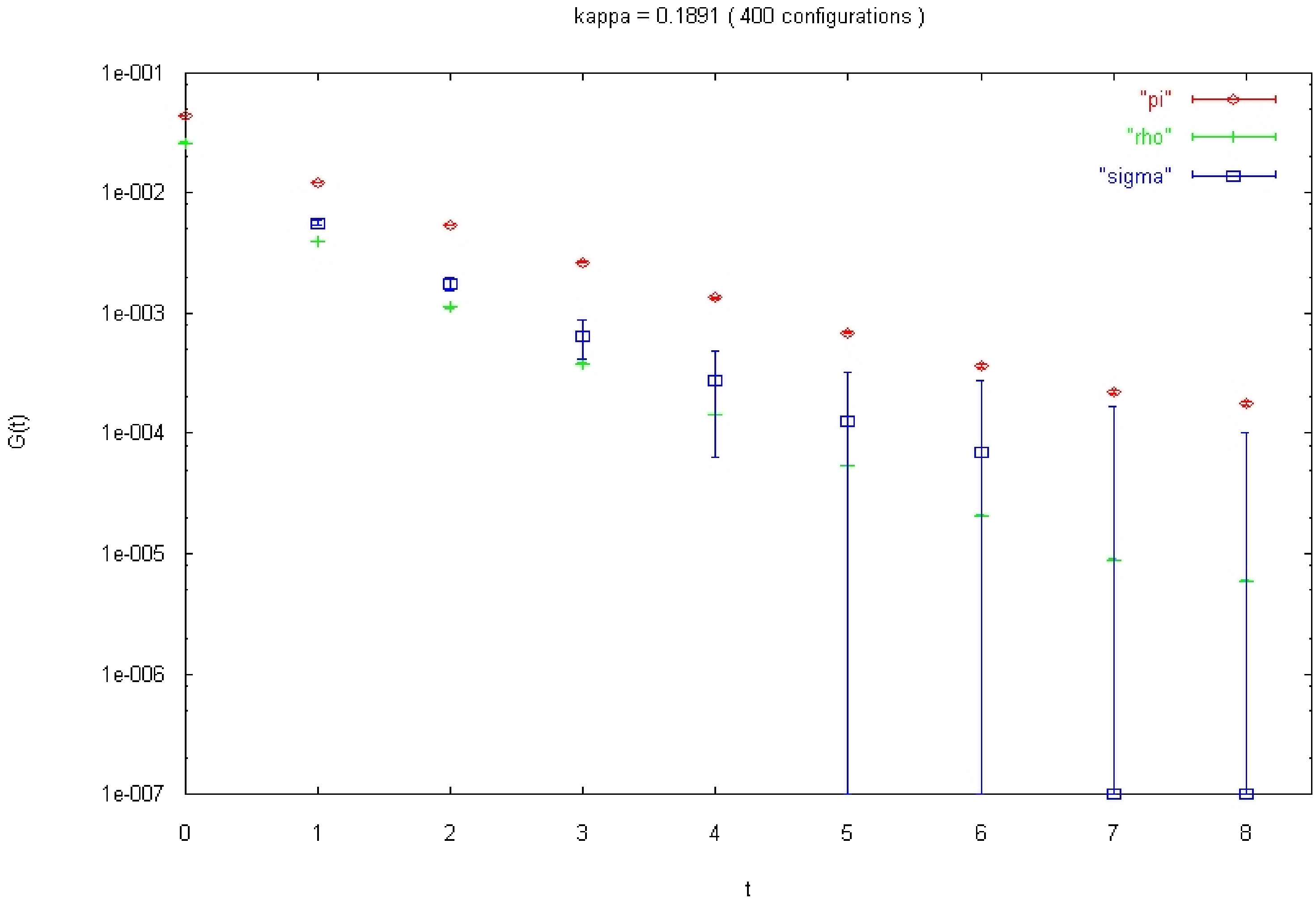
Connected and Disconnected Parts (κ =0.1874)



Effective mass (κ =0.1874)

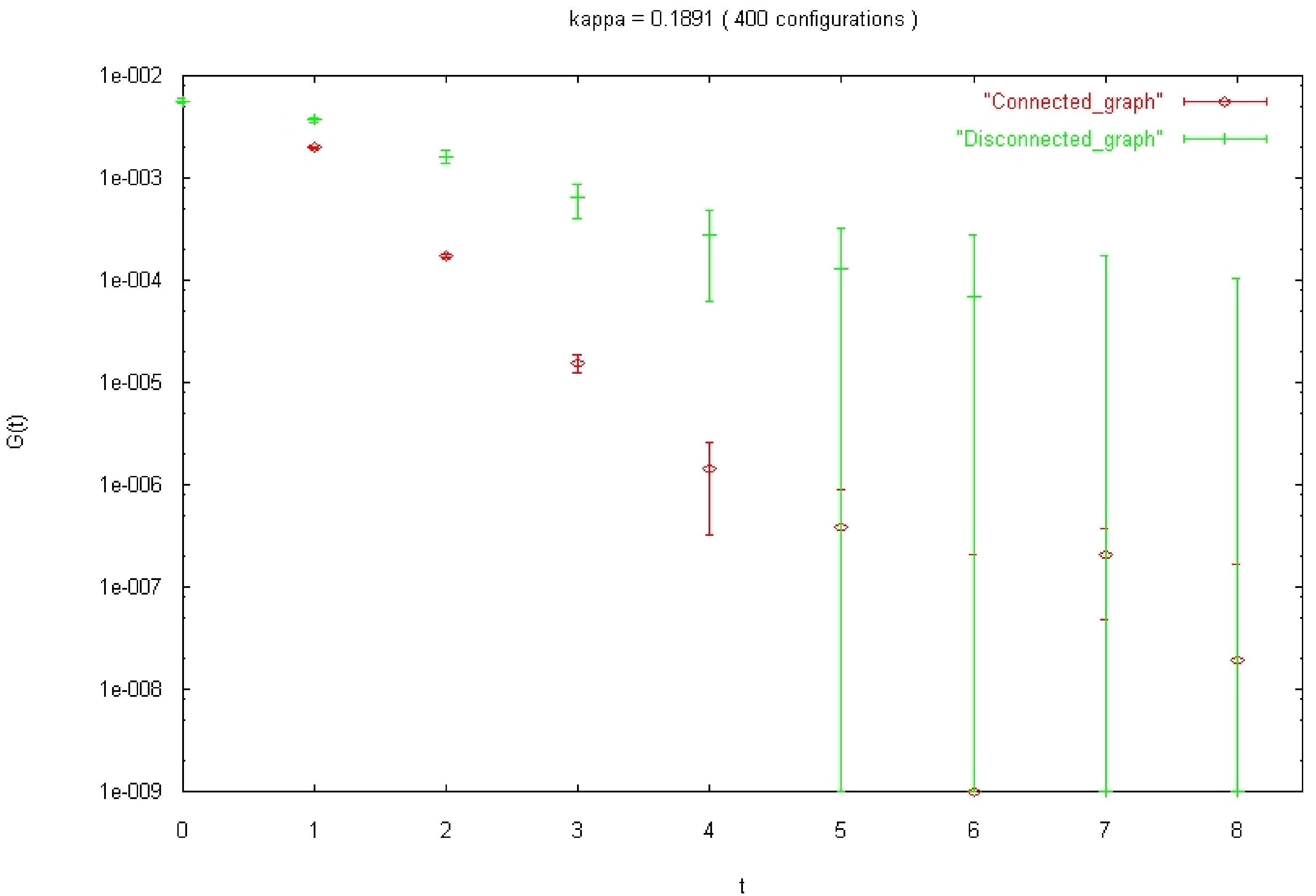


π , ρ , σ mesons (κ =0.1891)

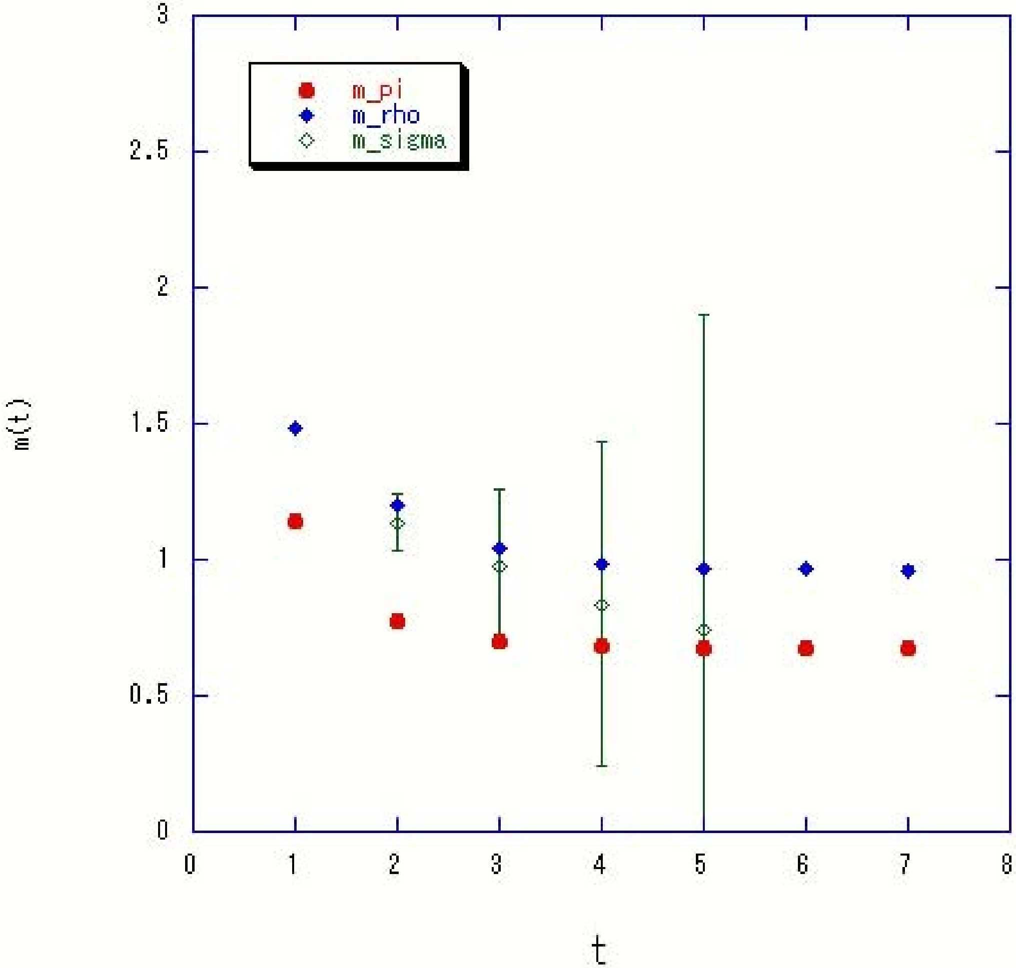


σ meson propagators

Connected and Disconnected Parts (κ =0.1891)



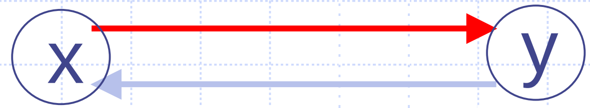
Effective mass (κ =0.1891)

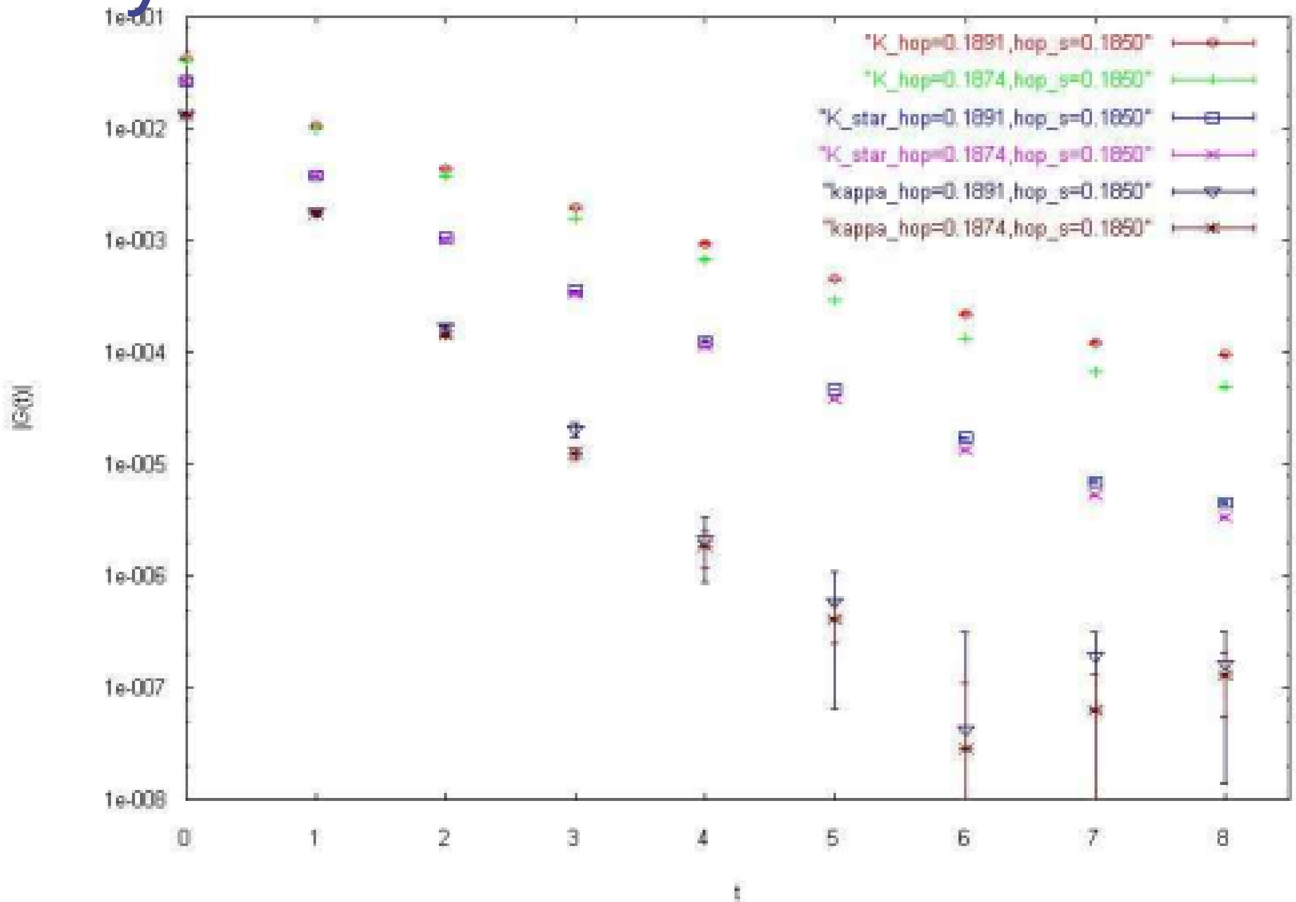


$$m_\sigma \approx m_\rho$$

κ meson

- strange scalar $\kappa^+ = \bar{\psi}_s \psi_u$
- Connected only





Summary

- Although σ propagators are noisy and we need high statistics, present data suggest that σ appears as a pole of QCD.
- Disconnected diagram dominates at large t.
- $m_\sigma \approx m_\rho$
- Analysis of κ will come soon.
- Lattice QCD Study of Scalar mesons was crazy before, but is now recognized as a meaningful work. In one year, it becomes matured and produces reliable data on the scalar mesons.