Illusory are the conventional anomalies in the conformal-gauge two-dimensional quantum gravity

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based on the work done in collaboration with

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M. Abe and N. Nakanishi, *Int. J. Mod. Phys.* **A14** (1999), 521.
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BRS formulation of conformal-gauge two-dimensional quantum gravity

coupled with $D\,(\neq 26$ in general) massless scalar fields.

Not intend to criticize non-field-theoretical approaches to string theory such as conformal field theory, path-integral approach without ghosts, etc. (Rather, those approaches are not equivalent to the genuine field-theoretical formalism based on the fundamental action involving ghosts.)

Conformal-gauge 2d QG reduces to a free field theory if B-field (= BRS daughter of FP antighost) is eliminated (or path-integrated) at the starting point. This fact is usually discarded because it contradicts the expectation from string theory (cf. Fujikawa, Kato-Ogawa).

Truth can be found by constructing explicitly exact solution of the model without eliminating B-field.

The problem is of very Deep level.

Basic aspects of operator-formalism approach and path-integral approach (= Feynman-diagrammatic approach)

Two essential differences:

① Operator formalism consists of two steps; operator algebra and its representation in terms of state vectors. Path-integral approach directly gives the solution without distinguishing operator level and representation level.

② Path integral can describe only the quantities which are expressed in terms of *T*-product,* while operator formalism can describe any kind of operator products.

T*-product is *different* from T-product.

T*-product is defined by the prescription that all differentiation should be made *after* taking T-product of fundamental field operators.

T-product generally violates field equations.*

Hence Noether theorem no longer holds inside T*-product. This violation of current conservation is often misidentified with current anomaly.

Amount of field-equation violation in path integral

From path-integral measure invariance under functional translation of φ:

$$\langle\ \mathrm{T}^*(\delta/\delta\varphi)S\cdot F\rangle - i\ \langle\ \mathrm{T}^*(\delta/\delta\varphi)F\rangle = 0$$

φ: generic field

S: action

F: arbitrary function of fields

Second term expresses the deviation from field equation $(\delta/\delta\varphi)S = 0$.

Operator formalism derivation:. From the equivalence between the field equation and the Heisenberg equation, one has $(\delta/\delta\varphi)S = -\partial_0\pi + i\,[H, \pi]$. Substituting it into $\langle\ \mathrm{T}(\delta/\delta\varphi)S\cdot F\rangle = 0$ and pulling out ∂_0 outside T-product, one encounters an equal-time commutator between π and F, which yields the second term

Thus as long as canonical conjugate, π, of φ exists, the T*-product effect can be reproduced by operator formalism.

The B-field of the model, however, does *not* have its canonical conjugate. Hence path integral *can* give a result *different* from operator formalism.

How to construct the solution in the operator-formalism approach

Given a system of field equations $(\delta / \delta\varphi_A)S = 0$ and equal-time commutation relations

Rewriting field equations into *commutator form*

$$[(\delta / \delta\varphi_A(x))S,\ \varphi_B(y)] = 0,$$

one sets up *q-number Cauchy problem* for $[\varphi_A(x), \varphi_B(y)]$. Solving it, one obtains an infinite-dimensional Lie algebra for field operators. Then one constructs its matrix representation by introducing state vectors. This is realized by constructing a set of Wightman functions (=vacuum expectation values of simple products) under certain conditions characterizing the vacuum (= energy positivity and generalized normal ordering).

Crucial mathematical point to be noted here is

This matrix algebra is required to be a representation of *Lie algebra* of fields but *not* of its *universal enveloping algebra* defined by algebraic products of fields.

Original field equations *may* not completely be satisfied at representation level:

"field-equation anomaly"

Field-equation anomaly cannot be described by path integral.

Conformal-gauge 2d QG in BRS formalism

gravitational field $g_{\mu\nu}$: gauge-fixed to $\rho\eta_{\mu\nu}$, where $\rho(x) > 0$

matter fields: D massless scalar fields ϕ_M (M = 0, 1, ... ,$D-1$). Parametrizing $g_{\mu\nu}$ by a traceless symmetric tensor $h^{\mu\nu}$, one has an action *independent* of ρ. FP-ghost c^λ is a vector, while B-field $\tilde{b}_{\mu\nu}$ and FP-antighost $\bar{c}_{\mu\nu}$ are traceless symmetric tensors. But it is possible to rewrite $h^{\mu\nu}, \tilde{b}_{\mu\nu}, \bar{c}_{\mu\nu}$ into *vector-like quantities* $h_\lambda, \tilde{b}^\lambda, \bar{c}^\lambda$.

With $x^\pm = (x^0 \pm x^1)/\sqrt{2}$, the Lagrangian density is given by

$$\pounds = [-\tfrac{1}{2}\tilde{b}^+ h_+ - i\bar{c}^+\partial_- c^+ + (+ \leftrightarrow -)] + \partial_+\phi_M \partial_-\phi^M + \pounds_I$$

with

$$\pounds_I = \tfrac{1}{2} h_+[-2i\bar{c}^+\partial_+ c^+ - i(\partial_+\bar{c}^+ \cdot c^+ + \partial_-\bar{c}^+ \cdot c^-) + \partial_+\phi_M \cdot \partial_+\phi^M])$$
$$+ (+ \leftrightarrow -) + O(h^2)$$

$O(h^2)$: higher-order terms of h_λ

Field equations:

$$h_\pm = 0,$$
$$\tilde{b}^\pm = \tfrac{1}{2}\,\delta\,\pounds_I / \delta\, h_\pm, \qquad \Rightarrow \quad \partial_\mp \tilde{b}^\pm = 0$$
$$\partial_\mp c^\pm = 0, \qquad \partial_\mp \bar{c}^\pm = 0,$$
$$\partial_+\partial_-\phi_M = 0$$

Any of $\tilde{b}^\pm, c^\pm, \bar{c}^\pm, \partial_\pm\phi_M$ is a function of $x^\pm$ *alone.*

This remarkable simplicity is valid only in operator formalism, but ***not*** in path-integral formalism because of the use of T*-product.

Exact solution in terms of Wightman functions

Nonvanishing truncated Wightman functions:
n-point functions consisting of
either ϕ_M, ϕ_N and $n-2$ B-fields
or $c^{\pm}$, $\bar{c}^{\pm}$ and $n-2$ B-fields

Thus if B-field is not considered, the solution is a free-field one.

Exact solution is completely consistent with BRS invariance and FP-ghost number conservation.

Subtlety arises for $\tilde{b}^{\pm} = \frac{1}{2}\, \delta\, \pounds_I / \delta\, h_{\pm}$

It is slightly (i.e., modulo $\partial_{\mp}\tilde{b}^{\pm} = 0$) violated at representation level; that is,
it suffers from *field-equation anomaly*:

$$\langle \tilde{b}^{\pm}(x_1)[\tilde{b}^{\pm}(x_2) - \frac{1}{2}\, \delta\, \pounds_I / \delta\, h_{\pm}(x_2)]\rangle$$

$$= -2(D-26)\left[\partial_{\pm}{}^{2} D^{(+)}(x_1 - x_2)\right]^2$$

Feynman-diagrammatic calculation

Results become very **crazy** *owing to T*-product's field- equation violation*

① In addition to $\langle T^* \bar{c}^{\pm}(x_1) c^{\pm}(x_2) \rangle$ and $\langle T^* \phi_M(x_1)\phi_N(x_2) \rangle$, there is a nonvanishing 2-point function

$$\langle T^* \tilde{b}^{\pm}(x_1) h_{\pm}(x_2) \rangle = -2i\delta^2(x_1 - x_2)$$

in apparent contradiction with $h_{\pm} = 0$

It induces many pathological effects which are absent in operator formalism.

Feynman rules based on $\pounds_I$ imply existence of one-loop diagrams for *n*-point function consisting of B-fields only; e.g.,

$$\langle T^* \tilde{b}^{+}(x_1) \tilde{b}^{+}(x_2) \rangle = 2(D-26)\left[\partial_+^{\,2} D_F(x_1 - x_2)\right]^2 \qquad (*)$$

$$\langle T^* \tilde{b}^{+}(x_1) \tilde{b}^{-}(x_2) \rangle = -\tfrac{1}{2}(D-2)\left[\delta^2(x_1 - x_2)\right]^2 \qquad (\#)$$

(*) shows *BRS anomaly appears for* $D \neq 26$.

② T*-product does not respect the fact that any of $\tilde{b}^{\pm}, c^{\pm}, \bar{c}^{\pm}, \partial_{\pm}\phi_M$ is a function of $x^{\pm}$ *alone.* Therefore, the Green's functions consisting of *both* + components *and* − components can remain **nonvanishing**; e.g., (#) and

$$\langle T^* c^{+}(x_1) \tilde{b}^{+}(x_2) \bar{c}^{-}(x_3) \rangle = -2\delta^2(x_1 - x_2)\partial_- D_F(x_2 - x_3)$$

Decoupling of right-moving and left-moving is **not** ***realized in path integral !!***

③ An infinite number of higher-order terms $O(h^2)$ in $\pounds_I$ ***do* contribute** to the higher-point functions containing more than one B-fields.

BRS Noether current

$$j_b^{\mp} = j_b'^{\mp} + (\widetilde{b}^{\pm} - ½\, \delta\, £_I / \delta\, h_{\pm}) c^{\pm}$$

with

$$j_b'^{\mp} \equiv -\widetilde{b}^{\pm} c^{\pm} + i\bar{c}^{\pm} c^{\pm} \partial_{\pm} c^{\pm}$$

If one defines BRS charge by using BRS *Noether* current, one encounters anomaly for $D \neq 26$. This is nothing but the result found by Kato and Ogawa.

(Finiteness effect of string length can be taken into account without bringing any essential change into the conclusion.)

But if BRS charge is defined by using $j_b'^{\mp}$, which, is completely equal to $j_b^{\mp}$ at operator level, then *there is no anomaly even for* $D \neq 26$.

Thus violation of BRS-charge nilpotency for $D \neq 26$, claimed by Kato and Ogawa, is *not an intrinsic result* but a consequence of unconsciously taking in field-equation anomaly.

FP-ghost number Noether current

$$j_c^{\mp} = -i\bar{c}^{\pm} c^{\pm}$$

It conservation law follows from $\partial_{\mp} c^{\pm} = 0$ and $\partial_{\mp} \bar{c}^{\pm} = 0$.

The "anomaly" is implied by the Feynman-diagrammatic result

$$\langle \mathrm{T}^* j_c^{\mp}(x_1) \tfrac{1}{2}\, \delta\, \pounds_I / \delta\, h_{\pm}(x_2) \rangle$$
$$= -3\partial_{\pm} D_F(x_1 - x_2) \cdot \partial_{\pm}^{\,2} D_F(x_1 - x_2)$$

Its violation of conservation law is merely due to the use of T*-product.

Indeed, without T*, D_F is replaced by $D^{(+)}$ in r.h.s., so that conservation law of $j_c^{\mp}$ is perfectly all right.

Thus ***FP-ghost number current anomaly an illusion caused by T*-product.***

(One might say that the existence of FP-ghost number current anomaly is a consequence of Riemann-Roch theorem. This assertion is wrong because this theorem holds only in *global* sense; locally, only one additional point to spacetime manifold can change the conclusion of the theorem.)

Conclusion

① Owing to the use of T*-product, Feynman-diagrammatic or path-integral calculation yields very crazy results in BRS formulation of conformal-gauge two-dimensional quantum gravity.

② Kato-Ogawa's violation of BRS-charge nilpotency for $D \neq 26$ is not an intrinsic result. BRS invariance is not violated for any value of D.

③ FP-ghost number is conserved completely; "FP-ghost number current anomaly" is an illusion caused by T*-product.

Similar misleading discussions are found concerning Virasoro anomaly and gravitational anomaly in a book by Green, Schwarz and Witten ("*Superstring Theory: 1*" pp.141-142 and pp.145-146).

パイオンの崩壊に現れる「T*積の怪」

ベクターボソン(W)理論によるπの崩壊：

$$\pi \rightarrow N + \overline{N} \rightarrow W \rightarrow \mu + \nu$$

において、πはスピン 0、Wはスピン 1 なので、

静止系で**角運動量が非保存**

（標準理論でも$N+\overline{N}$の代わりに$q+\overline{q}$となるだけで、本質的に同じ）

それなのにπ崩壊の振幅は Feynman 図で計算すると、

0 にならない。

「角運動量アノーマリー」か？？？

実は**これは「T*積の怪」の仕業**！！

T*積について

作用積分から直接定義できる共変的摂動論や経路積分法にでてくるのはすべて、**T 積ではなく T*積**。

T 積:　Hamiltonian に基づく正準形式で自然に現れる。

T*積:　これを Lagrangian 形式に改めるときに現れる。

（正準変数q, pを$q, \dot{q}$で表わせば、時間微分の数の数え方が変わるので、時間順序積の意味が変わる。）

T 積では、時間微分を行なってから時間順序積をとるので、一般に共変性を失う。

T*積では、場の量に作用しているように書かれている時間微分もすべて時間順序積をとった**あとから**おこなう。そのため共変性は保たれるが、場の方程式を尊重しない。したがって **Noether current の保存則も尊重しない。**

時間微分を含まない基本場の積に対しても、そのT積とT*積が一致するとは限らない。

この現象は正準共役量を持たない場がある場合に起こる。

このような場合には、T*積をとることと、真空期待値をとることとを別々に定義することは不可能で、「**T*積の真空期待値**」というひとつの概念が経路積分で定義できるだけである。したがってオペレーター形式の理論との対応は成立しなくなる。それゆえ経路積分で計算した結果に基づいて物理的な考察をすることは危険である。

ベクターボソンU_μの Proca 理論

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m^2 U^\mu U_\mu + eU^\mu j_\mu + \text{matter}$$

ただし　$F_{\mu\nu} \equiv \partial_\mu U_\nu - \partial_\nu U_\mu,\qquad \partial^\mu j_\mu = 0$

場の方程式

$$\Box U_\mu - \partial_\mu \partial^\nu U_\nu + m^2 U_\mu = -e j_\mu$$

∂^μを作用させれば、$m^2 \neq 0$ により

$$\partial^\mu U_\mu = 0 \qquad (*)$$

ゆえに　$(\Box + m^2) U_\mu = -e j_\mu$

(*)はU_μのスピン 0 成分がないことを示している**はず**

ラグランジァンは $\dot{U}_0$を含まない。すなわちU_0**は正準共役を持たない場**（正準変数はU_kのみ）。

ゆえに、U_0**を含む T*積には「T*積の怪」が現れる。**

（場の方程式から　$U_0 = m^{-2}(-\partial_k \pi^k - ej_0)$　）

Feynman propagator

$$\langle 0 | \mathrm{T}^* U_\mu(x) U_\nu(y) | 0 \rangle$$

$$= (-\eta_{\mu\nu} - m^{-2}\partial_\mu\partial_\nu)\,\Delta_F(x-y; m^2)$$

∂^μを作用させれば、

$$\langle 0 | \mathrm{T}^* \partial^\mu U_\mu(x) U_\nu(y) | 0 \rangle = i m^{-2} \partial_\nu \delta^4(x-y)$$

となって(*)と整合しない結果が得られる（見かけの角運動量非保存)。

これに対し

$$\langle 0 | \mathrm{T} U_\mu(x) U_\nu(y) | 0 \rangle$$

$$= \langle 0 | \mathrm{T}^* U_\mu(x) U_\nu(y) | 0 \rangle - i m^{-2} \delta_\mu{}^0 \delta_\nu{}^0 \delta^4(x-y)$$

は、ローレンツ共変でないが、角運動量は保存する：

$$\langle 0 | \mathrm{T} \partial^\mu U_\mu(x) U_\nu(y) | 0 \rangle$$

$$= \partial^\mu \langle 0 | \mathrm{T} U_\mu(x) U_\nu(y) | 0 \rangle - i m^{-2} \delta_\nu{}^k \partial_k \delta^4(x-y)$$

$$= i m^{-2} (\partial_\nu - \delta_\nu{}^0 \partial_0 - \delta_\nu{}^k \partial_k) \delta^4(x-y)$$

$$= \quad 0$$

ベクターボゾンA_μのB場形式（ランダウゲージ）

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m^2 A^\mu A_\mu + B\partial^\mu A_\mu$$
$$+ e A^\mu j_\mu + \text{matter}$$

場の方程式

$$\partial_\nu F^{\nu\mu} + m^2 A_\mu - \partial_\mu B = -e j_\mu$$
$$\partial^\mu A_\mu = 0$$

すなわちスピン0部分はない（$m^2 \neq 0$に無関係に）。

∂_μを作用させて得られるのは、B場の方程式

$$\Box B = 0 \qquad (\Rightarrow \text{補助条件})$$

正準変数はA_μの4成分全部で、「T*積の怪」は現れない。

Feynman propagator

$$\langle 0 | \mathrm{T}^* A_\mu(x) A_\nu(y) | 0 \rangle = \langle 0 | \mathrm{T} A_\mu(x) A_\nu(y) | 0 \rangle$$
$$= (-\eta_{\mu\nu} - m^{-2}\partial_\mu\partial_\nu)\Delta_F(x-y; m^2) + m^{-2}\partial_\mu\partial_\nu D_F(x-y)$$

∂^μ を作用させれば0になる。

A_μの漸近場は　$A^{as}{}_\mu = U_\mu - m^{-2}\partial_\mu B$

ベクトルボゾンにProcaを使うかB場形式を使うかで崩壊したりしなかったりする中性中間子のモデル

ψ_N	mass	M_N,	ψ_l	mass	M_l
ϕ	mass	μ ,	W_μ	mass	m

ただし　$2M_N > \mu > 2M_l$

相互作用ラグランジアン

$$e(\overline{\psi}_N\gamma^\mu\psi_N + \overline{\psi}_l\gamma^\mu\psi_l)W_\mu + g\overline{\psi}_N\psi_N\phi$$

W_μにProcaを使うと、スピン0成分により崩壊振幅が0でないが、B場形式を使うと0になる。

ψ_Nのループのカウンター項

$$\lambda\phi\partial^\mu W_\mu$$

をラグランジアンに付け加えると、カレントは保存せず(繰り込み不可能)、W_0が正準共役をもってしまう。
(B場形式の場合は、カウンター項はB場に吸収され、トリヴィアル)

現実のパイオンの崩壊は、標準理論ではどうなっているのか？

ウィークボソンW_μを B 場形式で定式化すると、角運動量保存により、πは W_μを通じて崩壊できない。

標準理論では Higgs 機構が働く。B 場形式では Goldstone 定理が成立し、Higgs 場の虚部χはNGボソンとして存在する（ベクター場に食われて縦波成分になるのではない）。ただし補助条件により非物理的となって、観測されない。しかし virtual には寄与する。

パイオン崩壊は、NG ボソンを通じて起こる：

$$\pi \to q+\bar{q} \to \chi \to \mu + \nu$$

崩壊振幅は Proca で計算したのと一致する。

理由：標準理論はゲージ理論であるため、BRS 不変性があるから。

R_ξゲージでの Feynman propagator

K-I. Aoki et al., PTP Suppl. No. 73(1982)

W_μ

$$\left[\eta_{\mu\nu}-(1-\alpha)\frac{k_\mu k_\nu}{k^2-\alpha m^2+i0}\right]\frac{i}{k^2-m^2+i0}$$

χ

$$\frac{i}{k^2-\alpha m^2+i0}$$

α＝0　のとき Landau ゲージ

α→∞ のとき　Proca に帰着

S 行列のゲージ独立性により、崩壊振幅は少なくとも摂動論的にαに依存しない。

結　論

1. パイオン崩壊で角運動量非保存にしないためには、B 場形式を使わねばならない。このとき **1 粒子中間状態はベクターボゾンでなく、NG ボゾンである。**

2. NG ボゾンは、正ノルムだが非物理的、つまり**閉じ込めが起こっている。Kugo-Ojima-Nishijima** による color confinement の定式化と同じようなことがすでに実現していることになる。

3. パイオン崩壊の計算が正しいためには、ウィークボゾンの理論は、繰り込み可能性の問題を度外視しても、**ゲージ理論でなければならない。**

4. 共変的摂動論や経路積分は最終結果のみ正しいものを与えるが、しばしば途中の式の物理的意味は不明になる。とくに保存則を尊重しないから、**共変的摂動論や経路積分でアノーマリーを論ずることは危険。**

T*-PRODUCT AND FALSE NON-CONSERVATION OF ANGULAR MOMENTUM IN THE PION DECAY

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In the framework of the intermediate vector boson theory, developed in the early 1960's, T*-product induces false non-conservation of angular momentum in the $\pi \to \mu + \nu$ decay. It is pointed out that if correctly formulated, the pion cannot decay. It is clarified how this trouble is resolved in the electroweak theory. In this consideration, some unexpected facts are found.

According to the intermediate vector boson theory, a charged pion (π) decays into a muon (μ) and a neutrino (ν) through a vector boson (W) intermediate state:

$$\pi \to N + \overline{N} \quad (\text{or } q + \overline{q}) \to W \to \mu + \nu \, .$$

Since the spins of π and W is 0 and 1, respectively, this process does *not* conserve the angular momentum at the rest frame of π. Nevertheless, if one calculates the decay amplitude of this process by means of the covariant perturbation theory, one obtains a *non-vanishing* result. This is not anomaly. The reason for this false non-conservation is the use of T*-product in perturbation theory, which does not generally respect Noether theorem.

As is well known (to old physicists but not necessarily to younger people!), one encounters T*-product when T-product in the Hamiltonian formalism is rewritten into the Lagrangian formalism.[1] While the T-product quantities are not necessarily covariant, the T*-product ones are always covariant. The price paid for this is the fact that the T*-product quantities generally violate field equations and, therefore, Noether theorem. The amount of the violation of field equations can be seen explicitly in terms of the path-integral formula by using the invariance of the functional measure under functional translation. The violating term can be derived also from the canonical operator formalism, *provided that every fundamental field has its own canonical conjugate.* When this condition is not satisfied, however, the path-integral result, which reproduces the perturbative one, does not necessarily coincide with the result of operator formalism. Then the concept of T*-product cannot be defined separately from taking vacuum expectation value, and therefore it may become inadequate to discuss physical contents on the basis of path-integral formalism or covariant perturbation theory. Detailed accounts and explicit examples of the pathological phenomena caused by T*-product are given in our recent papers.[2,3,4]

Traditionally, a massive vector field is described by the Proca formalism, because it does not require the introduction of indefinite metric. Its Lagrangian density is given by

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m^2U^{\mu}U_{\mu} + eU^{\mu}j_{\mu} + \text{matter} \qquad (1)$$

with $F_{\mu\nu} \equiv \partial_{\mu}U_{\nu} - \partial_{\nu}U_{\mu}$ and $\partial^{\mu}j_{\mu} = 0$. From (1), the equation for the Proca field U_{μ} is

$$\Box U_\mu - \partial_\mu \partial^\nu U_\nu + m^2 U_\mu = -ej_\mu \,. \tag{2}$$

Since $m^2 \neq 0$, (2) yields

$$\partial^\mu U_\mu = 0 \,. \tag{3}$$

This equation *should* imply the absence of spin-0 part.

Since (1) does not involve $\partial_0 U_0$, U_0 does not have its canonical conjugate [It is expressed as $U_0 = m^{-2}(-\partial_k \pi^k - ej_0)$ from (2), where π^k denotes the canonical conjugate of U_k.]. Hence, as noted above, we need extreme care about T*-product.

The Feynman propagator is given by

$$\langle 0 | \mathrm{T}^* U_\mu(x) U_\nu(y) | 0 \rangle = (-\eta_{\mu\nu} - m^{-2}\partial_\mu \partial_\nu)\Delta_F(x-y; m^2) \,. \tag{4}$$

Hence the commutativity between T* and ∂_0 implies

$$\langle 0 | \mathrm{T}^* \partial^\mu U_\mu(x) U_\nu(y) | 0 \rangle = im^{-2}\partial_\nu \delta^4(x-y) \tag{5}$$

in contradiction with (3). This is the origin of the false non-conservation of angular momentum. On the other hand, if T-product is considered, we have

$$\langle 0 | \mathrm{T} U_\mu(x) U_\nu(y) | 0 \rangle = \langle 0 | \mathrm{T}^* U_\mu(x) U_\nu(y) | 0 \rangle - im^{-2}\delta_\mu{}^0 \delta_\nu{}^0 \delta^4(x-y) \,. \tag{6}$$

From (6), using the non-commutativity between T and ∂_0, we can show that

$$\langle 0 | \mathrm{T} \partial^\mu U_\mu(x) U_\nu(y) | 0 \rangle = 0 \tag{7}$$

in conformity with (3).

Three decades ago, the present author proposed a more satisfactory formalism for the vector field, called the B-field formalism,[5,6] which has a smooth massless limit. Its Lagrangian density is given by

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m^2A^{\mu}A_{\mu} + B\partial^{\mu}A_{\mu} + eA^{\mu}j_{\mu} + \text{matter}. \quad (8)$$

We have $\partial^{\mu}A_{\mu} = 0$ directly from (8). Since (8) involves $\partial_0 A_0$ explicitly, A_0 has its canonical conjugate. Canonical quantization requires the introduction of indefinite metric, but the physical appearance of negative-norm states are excluded by a subsidiary condition, as in QED. The Feynman propagator is given by

$$\langle 0|\mathrm{T}^* A_{\mu}(x)A_{\nu}(y)|0\rangle = \langle 0|\mathrm{T}\, A_{\mu}(x)A_{\nu}(y)|0\rangle$$

$$=(-\eta_{\mu\nu} - m^{-2}\partial_{\mu}\partial_{\nu})\Delta_F(x-y;m^2) + m^{-2}\partial_{\mu}\partial_{\nu}D_F(x-y). \quad (9)$$

It is consistent with $\partial^{\mu}A_{\mu} = 0$. Hence there arises no false non-conservation of angular momentum. Note that the asymptotic field of A_{μ} is a linear combination of a free Proca field and $\partial_{\mu}B$.

Now, we can construct a model of a (neutral) scalar meson ϕ such that the meson *either* decays *or* does not decay according as one employs the Proca formalism *or* the B-field formalism for the intermediate vector boson *W*. Let the masses of ψ_N, ψ_l and ϕ be M_N, M_l and μ, respectively, where $2M_N > \mu > 2M_l$. The interaction Lagrangian density is given by

$$\mathcal{L}_{\text{int}} = e(\overline{\psi}_N\gamma^{\mu}\psi_N + \overline{\psi}_l\gamma^{\mu}\psi_l)W_{\mu} + g\overline{\psi}_N\psi_N\phi. \quad (10)$$

Then, the decay amplitude of ϕ is non-vanishing [a] if the free Lagrangian density of *W* is chosen to be the Proca one, while it vanishes if the B-field formalism is employed. Thus the choice of formalism changes the physical contents of the theory. Which choice is right? *The correct one is, of course, the*

B-field formalism, because the Proca formalism contradicts the angular momentum conservation law. Thus we should conclude that the intermediate vector boson theory cannot explain the pion decay.

Then the following question naturally arises: *Why can the actual charged pion decay without violating the angular momentum conservation law?* The correct theory is, of course, the electroweak theory, which is a non-abelian gauge theory. Its B-field quantization is carried out under the BRS invariance.[7,8] The vector bosons W acquire their mass by the Higgs mechanism, which is realized by a complex scalar field doublet having four degrees of freedom; one of them is nothing but the Higgs field, while the remaining three are the Nambu-Goldstone (NG) bosons, which we denote by χ. (Note that the widespread catchphrase "The NG boson is *eaten* by the gauge field to become its longitudinal component" is quite misleading. In the covariant formalism, the NG boson *does survive* but becomes *unphysical* by the subsidiary condition.[9,8]) The NG bosons χ are massless and *spinless*. Our crucial observation is that *the pion decays through the NG boson*:

$$\pi \rightarrow q+\bar{q} \rightarrow \chi \rightarrow \mu + \nu .$$

This process is, of course, consistent with the angular-momentum conservation law. Note that the asymptotic field of the gauge field is a linear combination of a free Proca field, $\partial_\mu B^{\text{asymp}}$ *and* $\partial_\mu \chi^{\text{asymp}}$.

Then the final question is this: Does the decay amplitude calculated by the above process coincide with the conventional one calculated on the basis of the Proca formalism? The answer is "*Yes*". This is the specialty of the gauge theory: The gauge-fixing plus FP-ghost term is BRS-exact, and any physical amplitude does not depend on it. To see this more explicitly, we quote the Feynman propagators of W and χ (in momentum space) obtained in the R_ξ-gauge:[10]

$$\left[\eta_{\mu\nu} - (1-\alpha)\frac{k_\mu k_\nu}{k^2 - \alpha m^2 + i0}\right]\frac{i}{k^2 - m^2 + i0} \qquad \text{for } W, \tag{11}$$

$$\frac{i}{k^2 - \alpha m^2 + i0} \qquad \text{for } \chi. \tag{12}$$

Here α is a parameter involved in the gauge fixing; for $\alpha = 0$ these propagators reproduce those of the B-field formalism, while for $\alpha \to \infty$ they tend to the Proca ones. The physical amplitudes are independent of α because of the gauge independence of the physical S-matrix.

Our conclusions are as follows.

1. In order to avoid the false non-conservation of angular momentum, it is necessary to employ the *B-field formalism* for the vector boson.
2. The pion decays not through the vector boson but *through the NG boson.*
3. The NG boson of the electroweak theory is an elementary particle, which decays into observable particles. Hence it provides an *example of confinement.* This fact encourages the color confinement schemes [b] proposed by Kugo and Ojima[7] and by Nishijima.[11]
4. Historically, the intermediate vector boson theory was replaced by a gauge theory because of renormalizability requirement. But the present consideration indicates that this replacement was necessary even if renormalizability was not required.

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Footnotes

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[a] If a counter term $\lambda\phi\partial^\mu W_\mu$ is introduced in order to remove the loop divergence, the current is no longer conserved. Then the theory becomes unrenormalizable and the positivity of norm is no longer guaranteed.

[b] In these schemes quarks and gluons are *unphysical* particles in contrast with the treatment in the Wilson-type scheme of confinement. Note that the latter is not *color* confinement because it deals with the quark-antiquark system only.

Illusory are the conventional anomalies in the conformal-gauge two-dimensional quantum gravity *

Noboru Nakanishi †

The exact solution in terms of Wightman functions is given to the BRS-formulated conformal-gauge two-dimensional quantum gravity coupled with D massless scalar fields. The solution is seen to be free of various anomalies. Its anomalous feature appears only in the B-field equation. The nilpotency violation of the BRS charge for $D \neq 26$, found by Kato and Ogawa, is shown to have been caused by their elimination of B-field. Covariant perturbation theory and path-integral formalism are shown to yield various misleading results (e.g., FP-ghost number current anomaly), owing to the fact that these approaches are based on T*-product, which does not generally respect field equations and, therefore, Noether theorem.

— 1 —

I discuss the BRS-formulated conformal-gauge two-dimensional quantum gravity coupled with D ($\neq 26$ in general) massless scalar fields.[1-3] The purpose of my talk is to point out that the conventional belief on various anomalies must be reconsidered. To avoid possible misunderstanding, I stress here that I do not intend to criticize non-field-theoretical approaches to string theory such as conformal field theory, path-integral approach without ghosts, etc. Rather, I wish to remark that those approaches are not equivalent to the genuine field-theoretical formalism based on the BRS invariance.

As is well known, the conformal-gauge two-dimensional quantum gravity reduces to a free field theory if the B-field (=BRS daughter of

FP-antighost) is eliminated (or path-integrated out) at the starting point. This fact is usually discarded because it contradicts the expectation from string theory. Indeed, Fujikawa's path-integral analysis [4] and Kato and Ogawa's operator-theoretical analysis,[5] in which also the B-field is eliminated at a certain step, are (wishfully) believed to give the right answer. It has never been discussed seriously the reason why the reduction to the free-field theory should be regarded as inadmissible. This point must be clarified *logically*. The truth can be found by constructing explicitly the exact solution to the model without eliminating the B-field.

— 2 —

Although what I discuss explicitly is a very particular model, the problem itself is the one at very deep level. Hence I first discuss the basic aspects of two fundamental approaches —— operator formalism approach and path integral approach —— to quantum field theory in the general framework. To be precise, I restrict my consideration to the path integral approach which reproduces all the Green's functions obtainable by Feynman diagrammatic method.

There are two essential differences between these two approaches: The operator formalism consists of two steps —— operator algebra and its representation in terms of state vectors. On the other hand, the path integral approach directly gives the solution without separating operator level from representation level. The important consequence of this difference will be considered later.

The other difference is that the path integral based on the Lagrangian density can describe only the quantities which are expressed in terms of *T*-product*, while operator formalism can describe any kind of operator products. I emphasize that T*-product is different from T-product; T*-product appears when T-product in the Hamiltonian formalism is rewritten into the Lagrangian formalism. It is quite unfortunate that many authors do not seriously take care of the distinction between these two notions. Because of non-commutativity between time differentiation and time-difference θ-function, T-products are non-covariant in general. This defect is remedied by introducing T*-product, which is defined by the prescription that all differentiations should be made after taking T-product of fundamental field operators. The cost paid for securing covariance is, however, that *T*-product generally violates field equations.* Hence the Noether theorem no longer holds inside T*-product. *This violation of current conservation is often misidentified with current anomaly.* A concrete example will be given later.

It is, of course, possible to evaluate the amount of the field-equation violation in path integral. By using the fact that the path integral measure is invariant under any functional translation of a field, one obtains

$$\langle \mathrm{T}^*(\delta / \delta\varphi) S \cdot F \rangle + i \langle \mathrm{T}^*(\delta / \delta\varphi) F \rangle = 0, \qquad (1)$$

where φ, S and F are a generic field $\varphi(x)$, the action integral and an arbitrary function $F(y, z, \ldots)$ of fields, respectively. The second term of (1) expresses the deviation from the field equation $(\delta / \delta\varphi) S = 0$.

It is quite instructive to derive (1) by means of operator formalism. From the equivalence between the field equation and the Heisenberg equation, one has

$$(\delta / \delta\varphi) S = -\partial_0 \pi + i[H, \pi]. \qquad (2)$$

Substituting (2) into

$$\langle \mathrm{T}(\delta / \delta\varphi) S \cdot F \rangle = 0 \qquad (3)$$

and pulling out ∂_0 to the outside of T-product, one encounters an equal-time commutator between π and F, which yields the second term of (1). Thus as long as the canonical conjugate, π, of φ exists, the T*-product effect can be reproduced by operator formalism. The B-field of the model considered, however, does *not* have its canonical conjugate. Hence the path integral approach *may* give results different from those of the operator formalism approach.

— 3 —

Since it is not widely known how to construct the solution in the operator formalism approach,[6] I explain its outline, stressing its essential points. Given a quantum field-theoretical action S, one has a system of field equations $(\delta/\delta\varphi_A)\,S = 0$ and equal-time commutation (including anti-commutation) relations by the standard procedure. Rewriting the field equations into the form

$$[(\delta/\delta\varphi_A(x))S,\ \varphi_B(y)] = 0, \qquad (4)$$

one can set up a *q-number Cauchy problem* for unequal-time commutators $[\varphi_A(x),\ \varphi_B(y)]$. Solving it, one obtains an infinite-dimensional (nonlinear) Lie algebra of field operators.

Then one proceeds to constructing its matrix representation by introducing state vectors. This is realized by constructing a complete set of Wightman functions (=vacuum expectation values of simple products) under the vacuum-characterizing conditions (=energy positivity and generalized normal product). The crucial mathematical point to be noted here is that singular products of field operators may not necessarily faithfully represented (owing to the use of generalized normal product). It is therefore possible, in principle, that one (or more) of the original field equations, if it is *nonlinear*, is not completely satisfied at the representation level. If this situation happens, I call it "field-equation anomaly". Its explicit examples are found in various two-dimensional quantum-gravity models.[7] It is important to note that *field-equation anomaly cannot be described by path integral.*

Now, I discuss the conformal-gauge two-dimensional quantum gravity in the BRS formalism.[1] The gravitational field $g_{\mu\nu}$ is gauge-fixed to $\rho\eta_{\mu\nu}$, where $\rho(x)>0$. The matter fields are D massless scalar fields ϕ_M ($M=0, 1, \ldots, D-1$). Parametrizing $g_{\mu\nu}$ by a traceless symmetric tensor $h^{\mu\nu}$, one has *an action independent of* ρ. As is well known, FP-ghost c^λ is a vector, while B-field $\tilde{b}_{\mu\nu}$ and FP-antighost $\bar{c}_{\mu\nu}$ are traceless symmetric tensors. But it is possible to rewrite $h^{\mu\nu}$, $\tilde{b}_{\mu\nu}$ and $\bar{c}_{\mu\nu}$ into vector-like quantities h_λ, $\tilde{b}^\lambda$ and $\bar{c}^\lambda$ because the two-dimensional Lorentz group is abelian.

With $x^\pm = (x^0 \pm x^1)/\sqrt{2}$, the Lagrangian density is given by

$$\mathcal{L} = [-\tfrac{1}{2}\tilde{b}^+ h_+ - i\bar{c}^+\partial_- c^+ + (+ \Leftrightarrow -)] + \partial_+\phi_M \cdot \partial_-\phi^M + \mathcal{L}_{\text{int}}, \tag{5}$$

$$\mathcal{L}_{\text{int}} = \tfrac{1}{2}h_+[-2i\bar{c}^+\partial_+ c^+ - i(\partial_+\bar{c}^+ \cdot c^+ + \partial_-\bar{c}^+ \cdot c^-) + \partial_+\phi_M \cdot \partial_+\phi^M] + (+ \Leftrightarrow -) + O(h^2), \tag{6}$$

where $O(h^2)$ denotes higher-order terms of h_λ. Field equations are

$$h_\pm = 0, \tag{7}$$

$$\tilde{b}^\pm = 2\,\delta\mathcal{L}_{\text{int}}/\delta h_\pm, \tag{8}$$

$$\partial_\mp c^\pm = 0, \qquad \partial_\mp \bar{c}^\pm = 0, \tag{9}$$

$$\partial_+\partial_-\phi_M \overset{=}{=} 0. \tag{10}$$

Here $O(h^2)$ does not contribute to the r.h.s. of (8) because $h_\pm = 0$ from (7), and also the terms involving $\partial_\mp \bar{c}^\pm$ in (6) do not contribute to it because of (9). Furthermore, differentiating (8), one finds

$$\partial_\mp \tilde{b}^\pm = 0; \tag{11}$$

therefore any of $\tilde{b}^\pm$, $c^\pm$, $\bar{c}^\pm$, $\partial_\pm\phi_M$ is a function of $x^\pm$ *alone*. It should be emphasized that *this remarkable simplicity is valid only in the operator formalism, but not in the path integral formalism* because of the use of T*-product in the latter as pointed out above.

According to the prescription stated above, it is possible to construct the exact solution in terms of Wightman functions explicitly. The nonvanishing truncated Wightman functions are the n-point functions consisting of *either*

ϕ_M, ϕ_N and $n=2$ B-fields *or* $c^{\pm}$, $\bar{c}^{\pm}$ and $n=2$ B-fields. Thus if B-field is not considered at all, the solution is a free-field one. I emphasize that the exact solution is completely consistent with BRS invariance and with FP-ghost number conservation.

A subtlety arises, however, for the B-field equation (8): It is "slightly" [i.e., *modulo* (11)] violated at the level of representation; that is, it suffers from field-equation anomaly. Indeed, one obtains

$$\langle \tilde{b}^{\pm}(x_1)[\tilde{b}^{\pm}(x_2) - 2\,\delta\,\mathcal{L}_{\rm int}/\,\delta\, h_{\pm}(x_2)]\rangle \;=\; -2(D-26)\left[\partial_{\pm}{}^2 D^{(+)}(x_1-x_2)\right]^2 \quad (12)$$

in contradiction to (8) for $D\neq 26$.

— 5 —

When the same model is calculated by the path integral or Feynman diagrammatic method, the results become very crazy [3] owing to the fact that T*-product does not respect field equations.

First, in addition to the natural expressions for the free propagators, $\langle {\rm T}^* c^{\pm}(x_1)\bar{c}^{\pm}(x_2)\rangle$ and $\langle {\rm T}^* \phi_M(x_1)\phi_N(x_2)\rangle$, there is a nonvanishing 2-point function

$$\langle {\rm T}^* \tilde{b}^{\pm}(x_1) h_{\pm}(x_2)\rangle \;=\; -2i\delta^2(x_1-x_2), \qquad (13)$$

which arises from the second term of (1). Since it does not respect the field equation (7), it induces many pathological effects, which are absent in the operator formalism.

Owing to (13), Feynman diagrammatic calculation based on (6) implies the existence of one-loop diagrams for the n-point functions consisting of B-fields only; for example, one finds

$$\langle {\rm T}^* \tilde{b}^{+}(x_1)\tilde{b}^{+}(x_2)\rangle \;=\; 2(D-26)\left[\partial_{+}{}^2 D_F(x_1-x_2)\right]^2, \qquad (14)$$

$$\langle {\rm T}^* \tilde{b}^{+}(x_1)\tilde{b}^{-}(x_2)\rangle \;=\; -\tfrac{1}{2}(D-2)\left[\delta^2(x_1-x_2)\right]^2. \qquad (15)$$

Since $\tilde{b}^{\pm}$ is the BRS transform of $\bar{c}^{\pm}$, (14) shows that BRS anomaly appears for $D\neq 26$.

Second, as was already remarked, T*-product does not respect the fact that any of $\widetilde{b}^{\pm}$, $c^{\pm}$, $\bar{c}^{\pm}$, $\partial_{\pm}\phi_M$ is a function of $x^{\pm}$ *alone*. Therefore, the Green's functions consisting of *both* +-components *and* −-components can remain nonvanishing in contrast to Wightman functions. For example, one has (15) and

$$\langle \mathrm{T}^{*} c^{+}(x_1)\widetilde{b}^{+}(x_2)\bar{c}^{-}(x_3)\rangle = -2\delta^2(x_1-x_2)\partial_{-}D_F(x_2-x_3). \qquad (16)$$

Thus *the well-known decoupling of right-mover and left-mover is not realized in the path integral approach*. Furthermore, as is seen from (15), one often encounters *false* divergences, which require the introduction of false counter terms.

Third, in contrast to the operator formalism, an infinite number of ~~higher-order terms~~ $O(h^2)$ ~~in~~ (6) ~~*do contribute*~~ to the ~~higher-point functions~~ containing more-than-one B-fields. Thus it becomes impossible to write down the solution completely.

— 6 —

The BRS Noether current is given by

$$j_b^{\mp} = j_b'^{\mp} + (\widetilde{b}^{\pm} - 2\,\delta\mathcal{L}_{int}/\delta h_{\pm})c^{\pm} \qquad (17)$$

with

$$j_b'^{\mp} \equiv -\widetilde{b}^{\pm}c^{\pm} + i\bar{c}^{\pm}c^{\pm}\,\partial_{\pm}c^{\pm}, \qquad (18)$$

and the FP-ghost number Noether current is given by

$$j_c^{\mp} = -i\bar{c}^{\pm}c^{\pm}. \qquad (19)$$

They are, of course, conserved. Indeed, both BRS invariance and FP-ghost number conservation exactly hold in the exact solution of the operator solution.

However, if one defines the BRS charge by using the BRS Noether current (17), which does not involve B-field, one encounters anomaly for $D \neq 26$. This is nothing but the famous result found by Kato and Ogawa [5] (The finiteness effect of string length can be taken into account without bringing any essential change into the conclusion.[2]). On the other hand, if the BRS charge is defined by using $j_b'^{\mp}$ given by (18), which, owing to (8), is completely *equal to the Noether current* $j_b^{\mp}$ *at the operator level,* then one can show that *there is no anomaly even for* $D \neq 26$. *Thus the violation of the nilpotency of the BRS charge, claimed by Kato and Ogawa, is not an intrinsic result* but a consequence of unconsciously taking in field-equation anomaly.

As for the FP-ghost number current anomaly, I claim that it is nothing more than *an illusion caused by T*-product.* Remember that the well-known "anomaly" is implied by the Feynman diagrammatic result:[8]

$$\langle \mathrm{T}^* j_c^{\mp}(x_1)\, \delta \mathcal{L}_{int}/\delta h_{\pm}(x_2)\rangle = -6\partial_{\pm} D_F(x_1 - x_2) \cdot \partial_{\pm}{}^2 D_F(x_1 - x_2). \quad (20)$$

The non-conservation of j_c^{λ} is, however, merely due to the use of T*-product. Indeed, without T*, D_F is replaced by $D^{(+)}$ in the r.h.s. of (20), and therefore, owing to $\partial_{\mp}\partial_{\pm}D^{(+)}=0$, the conservation law $\partial_+ j_c^+ + \partial_- j_c^- = 0$ is seen to be perfectly all right.

One might say that the non-existence of the FP-ghost number current anomaly contradicts the Riemann-Roch theorem. This assertion is wrong because this theorem is valid only for a *compact* manifold and a two-dimensional manifold having *Lorentzian metric* is compactified into a torus, whose *Euler characteristic* χ *is zero.* This fact is consistent with the absence of anomaly.

— 7 —

My conclusions are as follows.

(1) *Owing to the use of T*-product, the Feynman diagrammatic or path integral approach yields very crazy results in the BRS-formulated conformal-gauge two-dimensional quantum gravity.*

(2) *The violation of the nilpotency of the BRS charge for* $D \neq 26$, *claimed by Kato and Ogawa, is not an intrinsic result. One can construct*

another BRS charge nilpotent for any D. What is intrinsic to the model is the existence of the field-equation anomaly for the B-field equation.

(3) *The FP-ghost number current is conserved perfectly; the "anomaly" is nothing more than an illusion caused by T*-product.*

Similar misleading discussions are found concerning Virasoro anomaly and gravitational anomaly in the literature.[9] Moreover, I have recently noted that T*-product induces false non-conservation of angular momentum in the charged pion decay; the pion decays *not through the vector boson W but through the Nambu-Goldstone boson*.[10]

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FOOTNOTES

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