Generalized BRST Transformations

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PLAN OF THE TALK

- Introduction to BRST
- Generalized BRST
- Applications of generalized BRST
- Conclusions

INTRODUCTION TO BRST

- **Problem in gauge invariant theory**

In Feynman method of PI the vacuum to vacuum transition amplitude is,

$$W = \int \mathcal{D}A_\mu \; e^{iS_0} \tag{1}$$

where

$$S_0 = -\frac{1}{4}\int d^4x \; F^\alpha_{\mu\nu}F^{\alpha\mu\nu} \tag{2}$$

The measure, $\mathcal{D}A_\mu$ and the action, S_0 are invariant under Gauge transformations

$$A^\omega_\mu = A_\mu + D_\mu\omega \tag{3}$$

This implies that functionally integrating over both A^ω_μ and A_μ will over count the integral repeatedly and W becomes infinite.

The solution to this problem is to choose a particular gauge to break the gauge symmetry,

$$S = S_0 - \frac{1}{2\lambda}\int d^4x\ F^2[A] \tag{4}$$

But now the theory depends on arbitrary function, $F[A]$ which is not desirable,

Faddeev-Popov tackle this problem by modifying the measure $\int d\omega \rightarrow \int dF$. They sorted out this problem by giving an effective action,

$$S^{FP}_{eff} = S_0 + Sgf + S_g \tag{5}$$

where

$$S_g = -\int d^4x\ \bar{c}Mc \tag{6}$$

With

$$M^{\alpha\beta} = \frac{\delta F^{\alpha}}{\delta A^{\gamma}} D^{\beta\gamma}_{\mu} \tag{7}$$

Now since the gauge symmetry is broken it leads to the serious problem in trying to prove the renormalizability of the theory. Gauge invariance restricts the forms of the terms in the Lagrangian that are available as counter terms to absorb UV divergent.

Remarkablely the Path Integral till does have symmetry related to gauge invariance . This symmetry was discovered by Becchi, Rouet and Stora (and independently by Tyutin) and known as **BRST** symmetry.

For pure Yang-Mills theory BRST Transformations are given as,

$$\begin{aligned} \delta A_\mu^\alpha &= D_\mu^{\alpha\beta} c^\beta \delta\Lambda \\ \delta c^\alpha &= -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \delta\Lambda \\ \delta \bar{c}^\alpha &= \frac{\partial \cdot A}{\lambda} \delta\Lambda \end{aligned} \quad (8)$$

For covariant gauge fixing, $F[A] = \partial^\mu A_\mu$. These transformations are symmetry of S_{eff}^{FP}

- **BRST Charge**

Using *Noether theorem* one can construct the conserved current corresponding to BRST transformations ,

$$J_\mu = \sum_i \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi_i} \frac{\delta_{BRST} \phi_i}{\delta \Lambda} \tag{9}$$

And for these particular BRST,

$$J_\mu = -F^\alpha_{\mu\nu} D^\nu c^\alpha - \frac{g}{2} \partial_\mu \bar{c}^\alpha f^{\alpha\beta\gamma} c^\beta c^\gamma \tag{10}$$

Then the BRST charge,

$$Q_{BRST} = \int d^3x \ J_0 \tag{11}$$

Q_{BRST} is the generator of BRST transformations ,

$$[Q, \phi]_\mp = is\phi \tag{12}$$

And it can be shown explicitly that,

$$Q^2_{BRST} = 0 \tag{13}$$

The condition for a *physical state* in BRST formulation is

$$Q|\alpha >= 0 \tag{14}$$

This is obtained by demanding the fundamental physical requirement that matrix element between two physical states should be independent of choice of gauge.

The Eigenstates of H can be divided in 3 subspaces,

$$\begin{aligned} \mathcal{H}_1 &= \{\psi_1; Q\psi_1 \neq 0\} \\ \mathcal{H}_2 &= \{\psi_2; \psi_2 = Q\psi_1\} \\ \mathcal{H}_3 &= \{\psi_3; Q\psi_3 = 0; \psi_3 \neq Q\psi_1\} \end{aligned} \tag{15}$$

The physical state condition implies only $\mathcal{H}_3$ subspace is the physical subspace, as $< \alpha'|\alpha >=< \psi_3'|\psi_3 >$

Properties and Applications

Properties

- **Symmetry of the S_{eff}**
- **Nilpotent**
- **Source terms for BRST variation composite operators can be added to the S_{eff}**

Applications

- These transformations are extremely useful in deriving Identities between the Green,s functions , known as Slavnov-Taylor identities,

$$\mathcal{G}\Gamma[< A >, < c >, < \bar{c} >] = 0 \qquad (15)$$

Where $\mathcal{G}^2 = 0$. Renormalization of gauge theories are greatly simplified because of this.

- A renormalizable Lagrangian that obeys BRST invariance must take the form

$$S = S_0 + s\Psi \tag{16}$$

where the 2nd term generates gauge fixing and ghost terms. However for the complete proof of renormalization one has to show that UV divergent terms are BRST invariant.

Finite field dependent BRST transformation [FFBRS]

If we observe the infinitesimal BRST transformation and its properties carefully, we notice that the properties of infinitesimal BRST do not depend on whether,

- **Λ is finite or infinitesimal.**
- **Λ is field dependent or not.**

as long as Λ is anti-commuting and explicit space-time independent.

Therefore, we are at liberty to choose Λ finite and field dependent. In fact, we integrate the infinitesimal BRS to construct **finite field dependent BRST[FFBRS]** as

$$\begin{aligned} A_\mu^{\alpha\prime} &= A_\mu^\alpha + D_\mu^{\alpha\beta} c^\beta \Theta[A, c, \bar{c}] \\ c^{\alpha\prime} &= c^\alpha - \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \Theta[A, c, \bar{c}] \\ \bar{c}^{\alpha\prime} &= \bar{c}^\alpha - \frac{1}{\alpha} \partial^\mu A_\mu^\alpha \Theta[A, c, \bar{c}] \end{aligned} \qquad (11)$$

Where $\Theta[A, c, \bar{c}]$ is finite, field dependent, anti-commuting and x_μ independent quantity.

What do we mean by finite anti-commuting quantity ?.

Let us consider an example ,

$$\Theta[A, c, \bar{c}] = \int \bar{c}^\alpha(y)(\partial \cdot A^\alpha)(y) d^4y \qquad (12)$$

Now if we calculate the Greens' function of $\Theta[A, c, \bar{c}]$ between vacuum and a state with a ghost and a gauge field it has finite value (as opposed to infinitesimal).

Also one can show easily that $\Theta^2[A, c, \bar{c}] = 0$.

FFBRST are non-local transformations but they are exact symmetry of the theory, more over these can be used to relate generating functional, W corresponding to different effective theories. For example, W corresponding to **FP effective action in linear gauge with gauge parameter** α, can be related to W corresponding to,

- **Most general BRST/anti-BRST symmetric action in linear gauges.**

- **FP effective action in quadratic gauges.**

- **FP effective action with another distinct gauge parameter** α'.

- **FP effective action in axial gauge.**

Note, Jacobian of these **FFBRST** are no longer 1. The extra terms in the latter action arise from the non-trivial Jacobian.

Construction of FFBRST

We introduce some parameter $\kappa[0 \leq \kappa \leq 1]$. And make all the fields κ dependent. We make infinitesimal BRST field dependent and write as

$$
\begin{aligned}
\frac{d}{d\kappa} A_\mu^\alpha(x,\kappa) &= D_\mu^{\alpha\beta} c^\beta(x,\kappa)\Theta'(A,c,\bar{c}) \\
\frac{d}{d\kappa} c^\alpha(x,\kappa) &= -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta(x,\kappa) c^\gamma(x,\kappa)\Theta'(A,c,\bar{c}) \\
\frac{d}{d\kappa} \bar{c}^\alpha(x,\kappa) &= \frac{\partial \cdot A^\alpha}{\alpha}(x,\kappa)\Theta'(A,c,\bar{c})
\end{aligned}
\tag{13}
$$

Let denote generically

$$\phi(x,\kappa) \equiv \{A_\mu^\alpha(x,\kappa), c^\alpha(x,\kappa), \bar{c}^\alpha(x,\kappa)\}$$

With $\phi(x,\kappa=0) \equiv \phi(x)$; $\phi(x,\kappa=1) \equiv \phi'(x)$.

Above Eqs. then can be written compactly as;

$$\frac{d}{d\kappa}\phi(x,\kappa) = \delta_{BRS}[\phi(x,\kappa)]\Theta'[\phi(x,\kappa)] \qquad (14)$$

We shall show that as these are integrated w.r.t κ they preserve the form and yield FF-BRST of generic form,

$$\phi(x,\kappa) = \phi(x,0) + \delta_{BRS}[\phi(x,0)]\Theta(\phi(x,0)) \qquad (15)$$

Now setting $\kappa = 1$ we obtain,

$$\phi'(x) = \phi(x) + \delta_{BRS}[\phi(x)]\Theta(\phi(x)) \qquad (16)$$

We consider:

$$\begin{aligned} \frac{d\Theta'(\kappa)}{d\kappa} &= \frac{d}{d\kappa}\phi[x,\kappa]\frac{\delta\Theta'}{\delta\phi} \\ &= \frac{\delta\Theta'}{\delta\phi}\delta_{BRS}\phi[x,\kappa]\Theta'(\kappa) \\ &\equiv f[\phi(x,\kappa)]\Theta'(\kappa) \end{aligned} \qquad (17)$$

This is solved to yeild ,

$$\Theta'[\phi(\kappa)] = \Theta'[\phi(0)]\,\mathrm{exp}[\int d\kappa f[\phi(x,\kappa)] \quad (18)$$

This implies $\Theta'(\phi[\kappa])$ always contains $\Theta'[\phi(0)]$ as a factor.

Now if we expand $f(\phi[\kappa])$,

$$f(\phi[\kappa]) = f(\phi[0]) + \kappa \left[\frac{\delta f}{\delta \phi_i}\frac{\delta \phi_i}{\delta \kappa}\right]_{\kappa=0} + \cdots \quad (19)$$

Except 1st term all terms in the RHS of the above Eq. contain $\Theta'[\phi(0)]$

Therefore Eq. 17 can be written as, [because $[\Theta'(\phi(0))]^2 = 0]$

$$\frac{d\Theta'(\kappa)}{d\kappa} = f[\phi(x,0)]\Theta'(\kappa) \quad (20)$$

And corresponding to Eq. 18 we have,

$$\Theta'[\phi(\kappa)] = \Theta'[\phi(0)]\ \mathrm{exp}[\kappa f[\phi(0)]] \quad (21)$$

With similar arguments it can be shown that $\delta_{BRS}[\phi(x,\kappa)]$ can be replaced by $\delta_{BRS}[\phi(x,0)]$ in Eq. 14

$$\frac{d}{d\kappa}\phi(x,\kappa) = \delta_{BRS}[\phi(x,0)]\Theta'[\phi(x,0)]\ \exp[\kappa f[\phi(0)]] \quad (22)$$

This can be integrated easily from $\kappa = 0$ to 1 to obtain

$$\phi(x,1) = \phi(x,0) + \delta_{BRS}[\phi(x,0)]\Theta[\phi(0)] \quad (23)$$

Where $\Theta[\phi(x,0)] = \int_0^1 d\kappa \Theta'[\phi(\kappa)]$

$$= \Theta'[\phi(x,0)]\frac{\exp f - 1}{f}.$$

The Jacobian

We express the vacuum to vacuum amplitude in two different gauges

$$<0|0>= W = \int \mathcal{D}\phi \exp[iS^{L}_{eff}[\phi]] \\ = \int \mathcal{D}\phi' \exp[iS^{A\prime}_{eff}[\phi']] \quad (24)$$

We seek a field transformation $\phi \to \phi'$ such that W with S^{L}_{eff} when expressed in terms of ϕ', becomes converted by this field transformations into W with S^{A}_{eff}, an effective action in axial gauge.

Now under $\phi \to \phi'$

$$W \to W = \int \mathcal{D}\phi' J[\phi'] \ \exp[iS^{L}_{eff}(\phi')]$$

For many cases, the Jacobian for the non-local transformation can effectively be replaced by $\exp\{iS_1(\phi')\}$ with $S_1(\phi')$ a local action. In such cases, the transformation takes you from the effective action in Lorentz gauges to that some other family of gauges. The Jacobian explains the difference between the two effective actions.

The mathematical condition for the effective replacement $J \to \exp[iS_1]$ is formulated in terms of the Jacobian for the infinitesimal transformation as

$$0 = \int \mathcal{D}\phi \left[\frac{1}{J}\frac{dJ}{d\kappa} - i\frac{dS_1(\phi(x,\kappa),\kappa)}{d\kappa}\right] \exp\left[i(S_{eff}^L + S_1)\right] \tag{25}$$

- For $W^{FP}_{LG} \to W^{FP}_{QG}$

$$\Theta'(\phi) = i\beta \int d^4y \, d^{\alpha\beta\gamma} \overline{c}^{\alpha}(y,\kappa) A^{\beta}_{\mu}(y,\kappa) A^{\gamma\mu}(y,\kappa) \tag{27}$$

- For $W^{FP}_{\lambda} \to W^{FP}_{\lambda'}$

$$\Theta'(\phi) = i\gamma \int d^4y \, \overline{c}^{\alpha}(y,\kappa) \partial \cdot A^{\alpha}_{\mu}(y,\kappa) \tag{28}$$

- For $W^{FP}_{LG} \to W^{FP}_{AG}$

$$\Theta'(\phi) = i\beta \int d^4y \overline{c}^{\alpha}(y,\kappa) [\partial \cdot A - \eta \cdot A]^{\alpha}(y,\kappa) \tag{29}$$

Steps for construction of FFBRST

1. **Write down S_{eff} for two different effective theory**

2. **Postulate an infinitesimal field dependent BRST**

3. **Make an ansatz for $S_1[\phi(\kappa)]$**

4. **Evaluate the Jacobian for an IFBRST**

5. **Impose the condition of the replacement $J \rightarrow \exp(iS_1)$**

Examples

- For $W^{FP} \rightarrow W^{BRS-anti-BRS}$

$$\Theta'(\phi) = i\beta \int d^4y \; f^{\alpha\beta\gamma}\bar{c}^{\alpha}(y,\kappa)\bar{c}^{\beta}(y,\kappa)c^{\gamma}(y,\kappa) \tag{26}$$

APPLICATIONS OF FINITE BRST

- **Dynamical generation of Mass?**

Recently Fujikawa et al. have shown that the following theories have identical physical meaning at the quantum level,

(i) Classical massive gauge theory, given by,

$$\mathcal{L}_M = \mathcal{L}_{YM} - \frac{M^2}{2} A_\mu A^\mu \qquad (1)$$

and (ii) Gauge invariant theory whose gauge symmetry is broken by a gauge fixing term,

$$\mathcal{L}_g = \mathcal{L}_{YM} - \frac{1}{2}[\partial \cdot A]^2 \qquad (2)$$

Both of these theories represent an effective BRST invariant theory at the quantum level.

We have shown that these two theories are related by finite BRST transformations. Further we have constructed the finite BRST transformations and applied on gauged fixed theory (Lorentz gauge) and obtained the quantum theory with a mass term.

We start with the generating functional for the Yang-Mills theory in the Lorentz gauge,

$$Z = \int \mathcal{D}A_\mu \mathcal{D}c \mathcal{D}\bar{c} \exp(iS^L_{eff}) \tag{3}$$

where

$$S^L_{eff} = S_0 - \frac{1}{2}\int d^4x(\partial^\mu A_\mu)^2 - \int d^4x \bar{c} W c \tag{4}$$

with $W = \partial^\mu D_\mu$ is the Faddeev-Popov determinant. We now apply FFBRST with

$$\Theta' = i\int d^4y \bar{c}^a(y)\left[\partial^\mu A^a_\mu - m\frac{\omega^\mu}{|\omega|}A^a_\mu\right](y) \tag{5}$$

where ω^μ is an arbitrary 4-vector, to the expression for the generating functional to obtain,

$$Z = \int \mathcal{D}A'_\mu \mathcal{D}c' \mathcal{D}\bar{c}' \exp i(S^L_{eff} + S_1) \quad (6)$$

The additional piece in the action comes from the non-trivial Jacobian

of the FFBRST and can be written as,

$$S_1 = \int d^4x \left[-\frac{1}{2|\omega|^2} m^2 (\omega^\mu A_\mu)^2 + \frac{1}{2} (\partial^\mu A_\mu)^2 - \bar{c}(W' - W)c \right] (7)$$

with $W' = m\frac{\omega^\mu}{|\omega|} D_\mu$. Hence we obtain the generating functional for a new effective action given by,

$$S_{eff} = S_0 - \int d^4x \left[\frac{1}{2} A_\mu M^{\mu\nu} A_\nu + \bar{c} m \frac{\omega^\mu}{|\omega|} D_\mu c \right] \quad (8)$$

where $M^{\mu\nu}$ is a generalized mass matrix,

$$M^{\mu\nu} = m^2 \frac{\omega^\mu \omega^\nu}{|\omega|^2} \quad (9)$$

This effective action (8) corresponds to the Yang-Mills lagrangian with a generalized mass term. It shows the connection between the Lorentz gauge and a generalized 'mass' gauge $\frac{1}{2}A_\mu M^{\mu\nu}A_\nu$ in the context of Yang-Mills theory. To exactly reproduce the familiar mass term, we restrict the arbitrary vector ω^μ to be of infinitesimal form satisfying the

symmetric multiplication rule,

$$\frac{\omega^\mu \omega_\nu}{\|\omega\|^2} = \frac{g^\mu_\nu}{4} \tag{10}$$

In that case the gauge fixing term is $\frac{1}{8}m^2 A_\mu A^\nu$ which coincides with the standard mass term, after a proper normalization of m.

- **Prescription for the poles in non-covariant gauges.**

  Practical calculations in Non-abelian gauge theory requires a choice of gauge. Lorentz gauge is used widely principally because of simplicity in Feynman rules, Lorentz covariance. They do however need FP ghosts and these complicates Feynman diagram calculations, OPE etc.

  Hence another class of gauge have found favor in calculations, non-covariant gauge. Consider the axial gauges, $\eta \cdot A = 0$, Where η is arbitrary 4 vector. The main advantage of axial gauges is that these are formally ghost free and hence this simplifies Feynman diagram calculations.

  However it has disadvantages : (1) Lack of manifest covariance. More importantly, (2) the gauge field propagator has spurious poles at $\eta \cdot k = 0$

Therefore for the loop integrations involving gauge fields one needs some prescription for the poles. Various prescriptions have been proposed,

For examples;

Principle Value Prescription : [**PVP**]

Mandelstam - Liebrandt Prescriptions [**MLP**]

However these prescriptions are ad-hoc and leads to variety of problems, like, **PVP** violets WT identity to one loop order for $\eta^2 = 0$

MLP in the light cone gauge $\eta^2 = 0$ leads to Lorentz non-invariant integral and/or non-local counter terms.

Our FFBRST transformation can relate the action in different gauges and find solution for the prescription problem. In this regard

FFBRST has been used to study the following problems,

- The correct treatment of $\frac{1}{(\eta\cdot k)^p}$ type singularities in the axial gauge propagator has been derived using FFBRST [Ref: hep-th/9909123].

- FFBRST has been used to the problems of Coulomb gauge. [Ref. hep-th 0105042].

Problems with Coulomb gauge:

The time like propagator $D_{00}(k) = \frac{1}{|\vec{k}|^2}$ is not damped as as $k_0 \to \infty$ unlike Feynman gauge propagator [which goes like k_0^{-2}]. As a result k_0 integration may not be convergent in loop calculations. Thus it requires special treatment to get rid of this extra divergent over and above the UV divergent. Cheng and Tsai have provided a treatment in the form of extra regularization put by hand to get the actual result in loop calculations. They pointed out this extra contribution is due *anomalous Coulomb interaction*

This problem is solved by using FFBRST which connect the Lorentz gauge theory with the theory in Coulomb gauge. The outline of the approach is as follows,

$\frac{1}{k^2}$ singularities in the Landau type gauges is correctly dealt with effective replacement $k^2 \to k^2 + i\epsilon$. This amounts, in practice, to an addition of terms $-\frac{i\epsilon}{2}A_\mu A_\mu + i\epsilon\bar{c}c$ to the S_{eff}^L. This modification in action takes care of the singularity problem in Lorentz gauge.

We then start with

$$\tilde{S}_{eff}^L = S_{eff}^L - \frac{i\epsilon}{2}A_\mu A^\mu + i\epsilon\bar{c}c \qquad (31)$$

and perform the FFBRST which takes the Lorentz gauge theory to Coulomb gauge theory.[We have to chose a particular parameter in FFBRST for this purpose, $\Theta'[\phi(\kappa)] = i\int d^4y\bar{c}^\gamma(y,\kappa)\partial_0 A_0^\gamma(y,\kappa)]$

New terms will be generated from the (1) **Jacobian of FFBRST** (2) added extra terms in the $\tilde{S}^L_{eff}$.

The new terms arise from Jacobian will take the theory from Lorentz gauge to Coulomb gauge on the other hand the new terms arise in (2) will modify the propagator in a nontrivial way.

We have calculated the free propagator explicitly in this frame work.

$$G^{0C}_{00}(k) = \frac{1}{(|k|^2 - \lambda k_0^2 - i\epsilon)} + \frac{(4\lambda - \lambda^2)k_0^2}{(k^2 + i\epsilon)(|k|^2 - \lambda k_0^2 - i\epsilon)} + 0(\epsilon) \tag{32}$$

We note the following facts from our calculations:

(1) The propagator has now good high energy behavior $[k_0^{-2}]$ for $\lambda \neq 0$ and $\epsilon \neq 0$. No ad hoc regularization is needed.

(2) We can reproduce the result of Cheng & Tsai in path integral frame work. The anomalous Coulomb contribution is calculated.

(3) We also find that the singular Coulomb gauge has to be treated as the gauge parameter $\lambda \to 0$ limit.

FFBRST and Field-Anti-field formalism

Batalin & Vilkovisky addressed the question, Given a set of classical fields $\phi^i(x)$ and a classical action $S[\phi^i]$, what is the full quantum action $W(\phi)$ such that the functional integral

$$Z[J] = \int \mathcal{D}\phi e^{\frac{i}{\hbar}W(\phi)+J(\phi)} \tag{1}$$

determines all physical quantities.

To answer this question they introduce anti-fields, ϕ^*_A, corresponding to each field, ϕ^A, with opposite statistic. The sum of ghost numbers to a field and its anti-field is equal to -1.

Then they define the quantum action, $W[\phi, \phi^*]$ which satisfy some equation,

$$\Delta e^{\frac{i}{\hbar}W} = 0$$
$$\Delta \equiv \frac{\partial_r}{\partial \phi^A} \frac{\partial_r}{\partial \phi^*_a} (-1)^{\epsilon_A + 1}$$

Called **Master Equation**.

This can be written as

$$(W,\ W) = 2i\hbar\Delta W \tag{2}$$

Where the anti-bracket is defined as,

$$(X,\ Y) = \frac{\partial_r X}{\partial\phi^A}\frac{\partial_l Y}{\partial\phi^*_A} - \frac{\partial_r X}{\partial\phi^*_A}\frac{\partial_l Y}{\partial\phi^A} \tag{3}$$

This is very powerful equationr. It reflects the gauge symmetry if you consider the master equation in 0th order of antifields. In 1st order of anti-fields it reflects the nilpotency of the BRST transformations.

There are different solutions to this master equation, but these solutions are related through **anti-canonical transformations** . We have observed that, The solutions to the master equations is related through the FFBRST [Checked for particular pair of solutions]. Therefore, in this sense, we can interpret FFBRST as anti-canonical transformations in field - anti-field formulation.

We have started with generating functional corresponding to a specific solution of master equation and then applied some FF-BRST and landed with generating functional corresponding to another solution of the Master equation.

The work is not completed yet.

Conclusion

We have generalized the BRST transformations in two different ways:

- **Local BRST**

  The BRST parameter is space-time dependent: It is NOT symmetry of the theory. But leads to the non-trivial WT identities. Usual Slavanov-Taylor identities are special case of these identities. Local BRST is related to partial OSp(3,1—2) symmetry in some superspace formulation. [Ref: SDJ & BPM , Z. Phys. C74, 172 (1997)]

  Other approach in Local BRST is to extend the theory by introducing auxiliary fields such that local BRST could be exact symmetry of the extended theory.

- **FFBRST**

S.D. Joglekar & myself first introduced this concept in 1995. Here the BRST parameter is finite, field dependent, but no explicit x_μ dependence.

We have already seen many applications of these transformations, and we hope theses transformations will find many other applications in future.

- Other kinds of generalization of BRST transformations are also considered in literature recently. These are not symmetry of the action but leaves W, generating functional invariant.

References

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