[hep-ph/0011387]

Ambiguities of theoretical parameters and CP/T violation in neutrino factory

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1. Introduction

Background

i) CP violation; Massive neutrinos.

ii) Possible large mixing angles of the lepton sector.

iii) Long baseline neutrino oscillation experiments in the future: Muon storage rings as neutrino beam sources.

iv) Possibility of sizable CP/T-violation effect.

i) CP violation; Massive neutrinos

Hadron sector

K^0-$\overline{K}^0$ system (1964) ⟶ B^0-$\overline{B}^0$ system (2001)

Belle (KEK)
BaBar (SLAC)

Lepton sector

Nonexistent if $m_\nu = 0$

No hint.

Description: Imaginary Kobayashi-Maskawa matrix ⟶ Unitarity triangle

Prediction: Origin? Magnitude?

ii) Possible large mixing angles

$$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13} \\ & 1 & \\ -s_{13} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

$(c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij})$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Magnitude of CP violation

$$J \equiv c_{13}^2 s_{13} c_{23} s_{23} c_{12} s_{12} \sin\delta$$

$\sin 2\theta_{23} \sim 1$ **Atmospheric neutrino anomaly**
Zenith angle dependence of neutrino flux

$\sin 2\theta_{12} \sim 1$ **Solar neutrino deficit**
(Except SMA Solution)

but

$\sin 2\theta_{13} << 1$ **Chooz reactor experiments**

iii) Long baseline experiments & μ storage rings

Long baseline experiments

K2K	KEK~Kamioka; 250km
MINOS	FNAL~Soudan; 730km
ICANOE/OPERA	CERN-Gran Sasso; 732km

CP/T violation search (in the future)

T violation: $P(\nu_\alpha \to \nu_\beta) \neq P(\nu_\beta \to \nu_\alpha)$

CP violation: $P(\nu_\alpha \to \nu_\beta) \neq P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$

(beware of matter effect contamination)

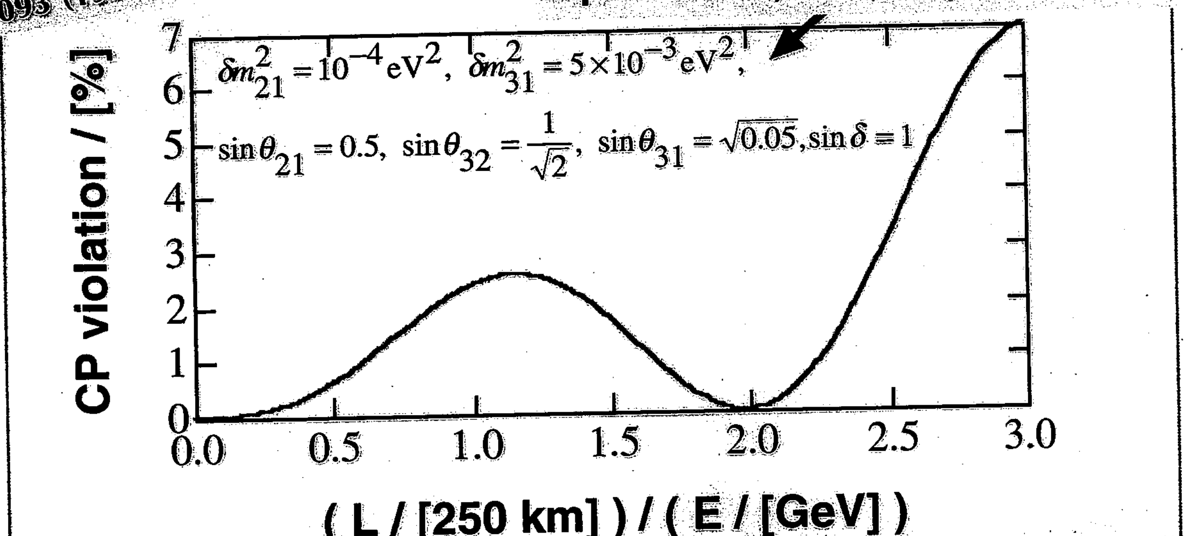
- **Conventional beam:** $\pi^+ \to \mu^+ \nu_\mu, \ \pi^- \to \mu^- \overline{\nu}_\mu$
- **μ storage rings:** $\mu^+ \to e^+ \nu_e \overline{\nu}_\mu, \ \mu^- \to e^- \overline{\nu}_e \nu_\mu$
  - ν_e available
  - high intensity
  - well-known spectra
  - no ν_μ-ν_e contamination

$\nu_\mu \to \mu^-$ "wrong sign"

iv) Possibility of sizable CP-violation effect

Jiro Arafune, Masafumi Koike and Joe Sato, Phys. Rev. D 56, 3093 (1997).

experimentally allowed parameter set



- CP-violation effect is enhanced for small E_ν.
- Sizable CP-violation effect even for small δm^2's.
- Genuine CP violation and matter effect can be distinguished.

In our previous works,

Precise values of parameters (except δ) are assumed to be known.

What we have in our hands is only the allowed regions of the parameters.

Oscillation probabilities are considered.

The quantity to be observed is the neutrino event number.

We now try to overcome these shortcomings.

2. Indication of CP Violation

TWO WAYS for CP-Violation Search

[1] Compare $N(\delta)$ with $N(\{0,\pi\})$

Parameter Fitting

Sensitive to the Real Part of the Lagrangian

$$N(\nu;\delta)-N(\nu;\{0,\pi\})$$

$$\stackrel{(E \gg \delta m^2_{31}L)}{\sim} E^3 \times \frac{1}{E^2}\frac{\delta m^2_{21}}{\delta m^2_{31}} j(\cos\delta \mp 1)\left[1-\frac{1}{3}\left(\frac{aL}{4E}\right)^2\right]$$

[2] Compare $N(\nu;\delta)$ with $N(\bar{\nu},\delta)$

Difference between ν and $\bar{\nu}$

Sensitive to the Imaginary Part of the Lagrangian

$$N(\nu)-N(\bar{\nu})$$

$$\stackrel{(E \gg \delta m^2_{31}L)}{\sim} E^3 \times \frac{1}{E^3}\left[\frac{aL}{4E}\times(\cdots)-\frac{\delta m^2_{21}}{\delta m^2_{31}} j\sin\delta\right]$$

We propose to consider

$$\Delta N(\delta) \equiv N(\nu;\delta)-N(\bar{\nu};\delta)$$

as a more direct indication of CP violation effect.

Taking into account ambiguities of parameters,

$\{\delta m_{21}^2, \delta m_{31}^2; \theta_{12}, \theta_{23}, \theta_{13}; \rho\}$

* What is the optimum setup of experiments?

* How many neutrino events, or

- How many muon decay
- How large exposure

is required to observe CP/T-violation effect?

Assumptions$_{\text{(Ideal...)}}$

1. Three generations, no ν_s. [LSND is not taken into account]
2. Both e and μ can be detected with its charge.
3. E_ν can be perfectly reconstructed.
4. Same count number expected for ν and $\bar{\nu}$ for no-oscillation case.
5. No muon polarization.
6. No ambiguities due to detection efficiency, no backgrounds condiered, etc.

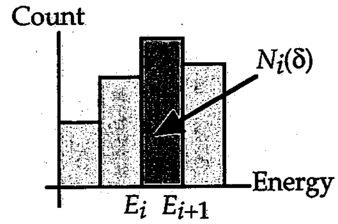
3. CP Violation

A parameter set

$\{\theta_j\} \equiv \{\delta m^2_{21}, \delta m^2_{31}; \theta_{12}, \theta_{23}, \theta_{13}; \rho\}$

↓

Count of i-th energy bin $E_i < E < E_{i+1}$



$$\Delta N_i(\{\theta_j\}, \delta)$$
$$\equiv N_i(\nu_\alpha \to \nu_\beta; \{\theta_j\}, \delta) - N_i(\bar{\nu}_\alpha \to \bar{\nu}_\beta; \{\theta_j\}, \delta)$$

↓

Experiment
$\Delta N_i(\{\theta_j\}, \delta)$'s

⟺

Expected for No-CP
$\Delta N_i(\{\theta_j\}, \delta = 0)$'s
$\Delta N_i(\{\theta_j\}, \delta = \pi)$'s

unknown...

$\{\theta_j\} \to \{\tilde{\theta}_j\}$

both

Can we distinguish $\delta \neq 0$ from $\delta = 0$, $\delta = \pi$?

Nature — **Hypothesis**

Goodness-of-fit analysis:

Nature — Hypothesis

$$\chi^2(\{\tilde{\theta}_j\},\delta_0) \equiv \sum_i \frac{\left[\Delta N_i(\{\theta_j\};\delta) - \Delta N_i(\{\tilde{\theta}_j\};\delta_0)\right]^2}{N_i(\nu;\delta)+N_i(\bar{\nu};\delta)}$$

$\delta_0 \in \{0,\pi\}$

~~CP~~ at 90% (99%) C.L.

$$\min_{\{\tilde{\theta}_i,\delta_0\}}\left[\chi^2(\{\tilde{\theta}_i\},\delta_0)\right] > \chi^2_{90\%(99\%)}$$

$$N_i(\{\theta_j\},\delta) = (\mathrm{Const})\cdot N_\mu M_{\mathrm{detector}} \frac{E_\mu^2}{L^2} R_i(\{\theta_j\},\delta)$$

$$\left(R_i(\{\theta_j\},\delta) \equiv \int_{E_{i-1}}^{E_i} E_\nu f_{\nu_\alpha}(E_\nu) P(\nu_\alpha \to \nu_\beta;\{\theta_j\},\delta)\frac{dE_\nu}{E_\mu} \right)$$

$$\min_{\{\tilde{\theta}_j,\delta_0\}}\left[N_\mu M_{\mathrm{detector}}\right] > \frac{1}{(\mathrm{Const})}\frac{L^2}{E_\mu^2}\frac{\chi^2_{90\%(99\%)}}{\sum_i \frac{[\Delta R_i(\delta)-\Delta\tilde{R}_i(\delta_0)]^2}{R_i(\nu;\delta)+R(\bar{\nu};\delta)}}$$

(Exposure)

→ **Graphs...**

$\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ are considered

Parameters — LMA MSW + Atm + Chooz

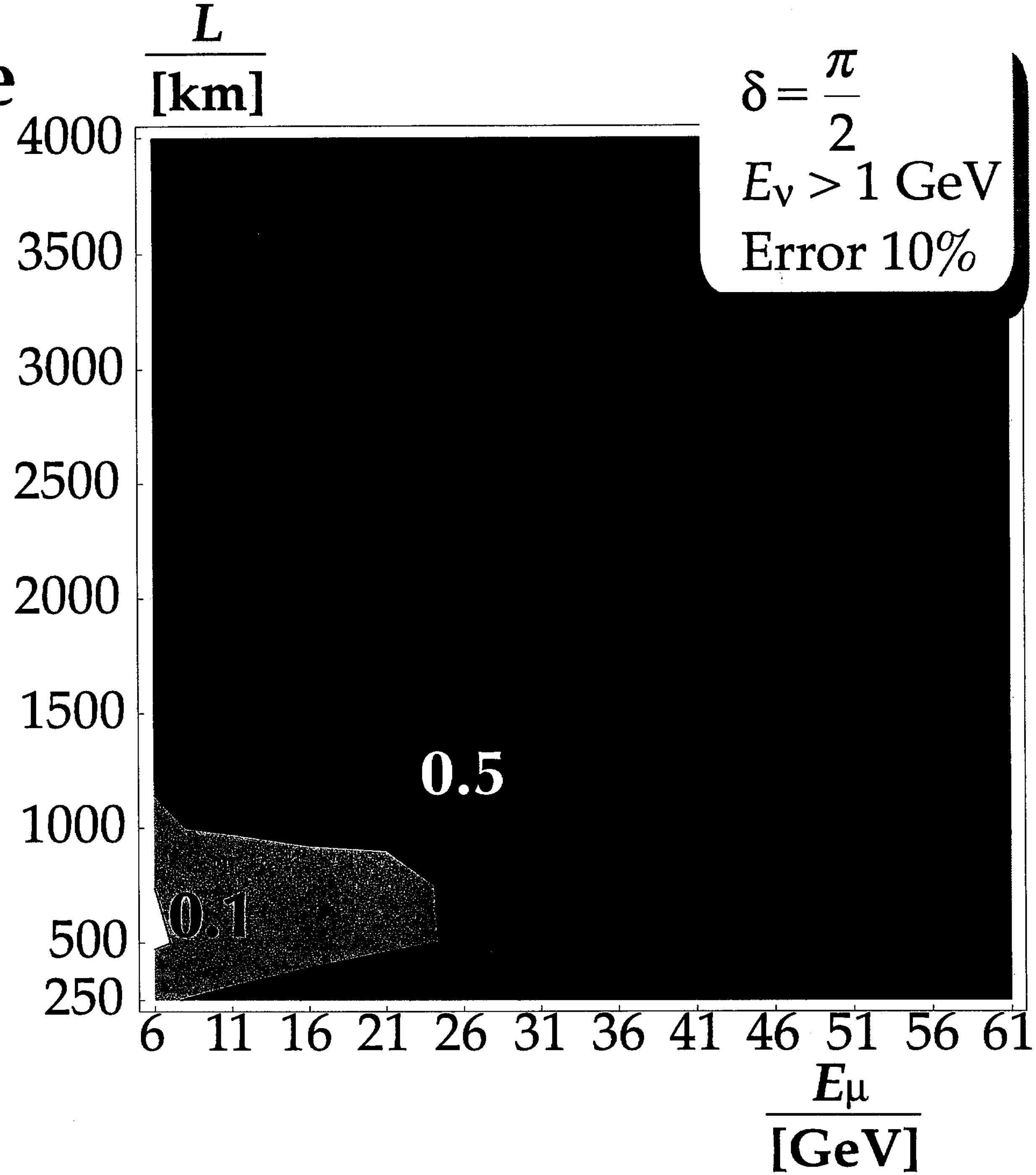
$$\delta m_{31}^2 = 3.0\times 10^{-3}\mathrm{eV}^2, \delta m_{21}^2 = 1.0\times 10^{-4}\mathrm{eV}^2$$

$$\sin\theta_{12} = 0.5,\ \sin\theta_{23} = \frac{1}{\sqrt{2}},\ \sin\theta_{13} = 0.1$$

Errors = 10%

Required Exposure (CP violation)

$\frac{N_\mu}{[10^{21}]}$ $\frac{M_{\text{detector}}}{[100\text{ kt}]}$



$\sin\tilde{\theta}_{13} \approx 0.01$

$\sin\tilde{\theta}_{23} \approx \frac{1}{\sqrt{2}}$

$\sin\tilde{\theta}_{12} \approx 0.5$

$\tilde{\rho}(L) \approx \rho_{\text{PREM}}(L)$

$\delta\tilde{m}^2_{31} \approx 3.0 \times 10^{-3}\,\text{eV}^2$

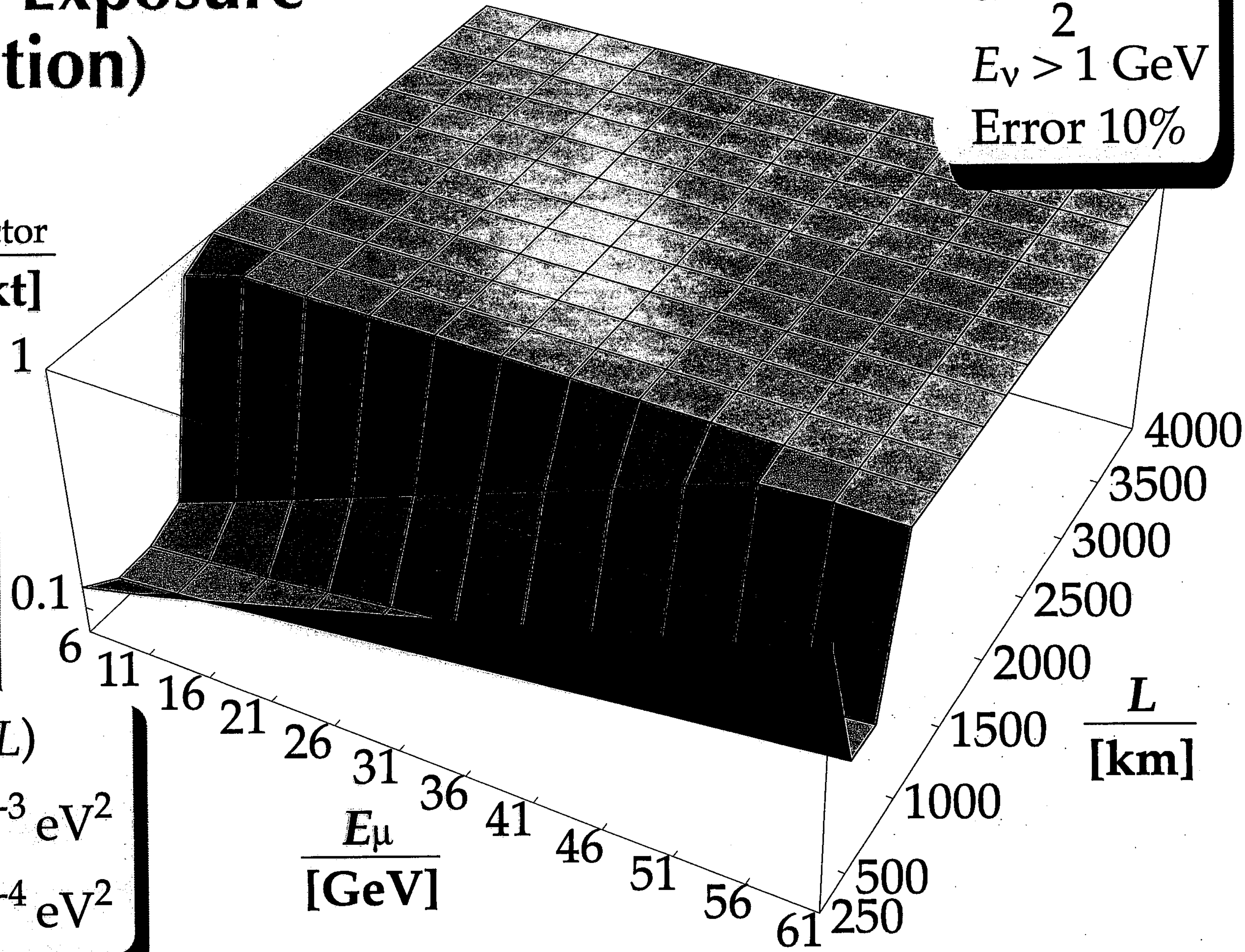
$\delta\tilde{m}^2_{21} \approx 1.0 \times 10^{-4}\,\text{eV}^2$

Required Exposure (CP violation)

$\delta = \frac{\pi}{2}$

$E_\nu > 1$ GeV

Error 10%



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$\tilde{\rho}(L) \approx \rho_{PREM}(L)$

$\delta\tilde{m}^2_{31} \approx 3.0 \times 10^{-3}\ \mathrm{eV}^2$

$\delta\tilde{m}^2_{21} \approx 1.0 \times 10^{-4}\ \mathrm{eV}^2$

4. T Violation

Different energy spectra for ν_e's and ν_μ's.

$$N_i(\nu_\mu;\delta) - \frac{\tilde{N}_i(\nu_\mu;\delta_0)}{\tilde{N}_i(\nu_e;\delta_0)} N_i(\nu_e;\delta)$$

(Re)normalize.

Vanishes when $\delta = \delta_0$ and $\{\tilde{\theta}_j\} = \{\theta_j\}$.

Goodness-of-fit analysis:

$$\chi^2(\{\tilde{\theta}_j\},\delta_0) \equiv$$

$$\sum_i \frac{\left[\tilde{N}_i(\nu_e;\delta_0)N_i(\nu_\mu;\delta) - \tilde{N}_i(\nu_\mu;\delta_0)N_i(\nu_e;\delta)\right]^2}{\tilde{N}_i(\nu_e;\delta_0)^2 N_i(\nu_\mu;\delta) + \tilde{N}_i(\nu_\mu;\delta_0)^2 N_i(\nu_e;\delta)}$$

~~CP~~ at 90% (99%) C.L.

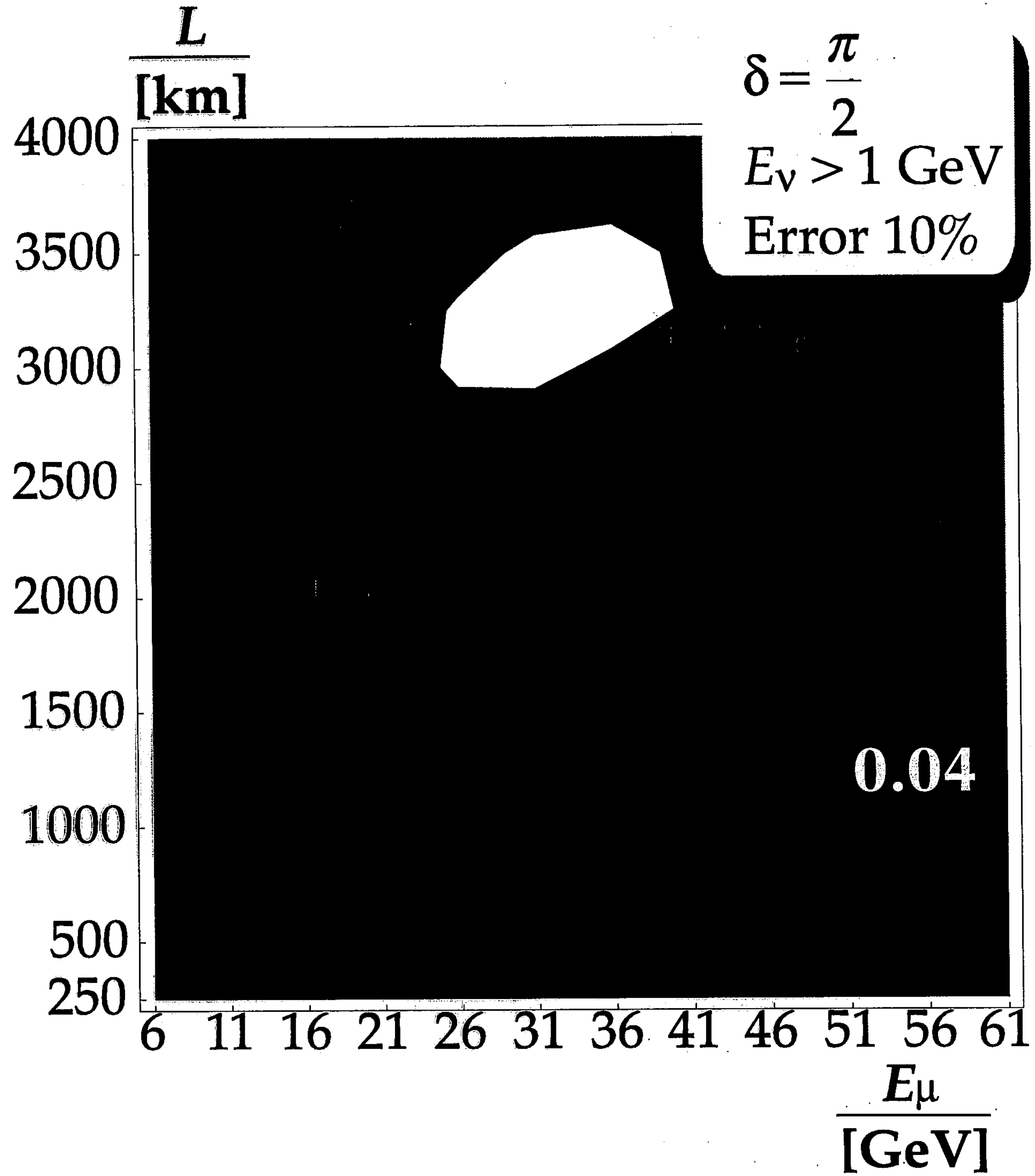
$$\min_{\{\tilde{\theta}_i,\delta_0\}}\left[\chi^2(\{\tilde{\theta}_i\},\delta_0)\right] > \chi^2_{90\%(99\%)}$$

→ **Graphs...**

Required Exposure (T violation)

$\frac{N_\mu}{[10^{21}]}$ $\frac{M_{detector}}{[100\ kt]}$

$$\sin\tilde{\theta}_{13} \approx 0.01$$
$$\sin\tilde{\theta}_{23} \approx \frac{1}{\sqrt{2}}$$
$$\sin\tilde{\theta}_{12} \approx 0.5$$
$$\tilde{\rho}(L) \approx \rho_{PREM}(L)$$
$$\delta\tilde{m}^2_{31} \approx 3.0\times10^{-3}\,eV^2$$
$$\delta\tilde{m}^2_{21} \approx 1.0\times10^{-4}\,eV^2$$



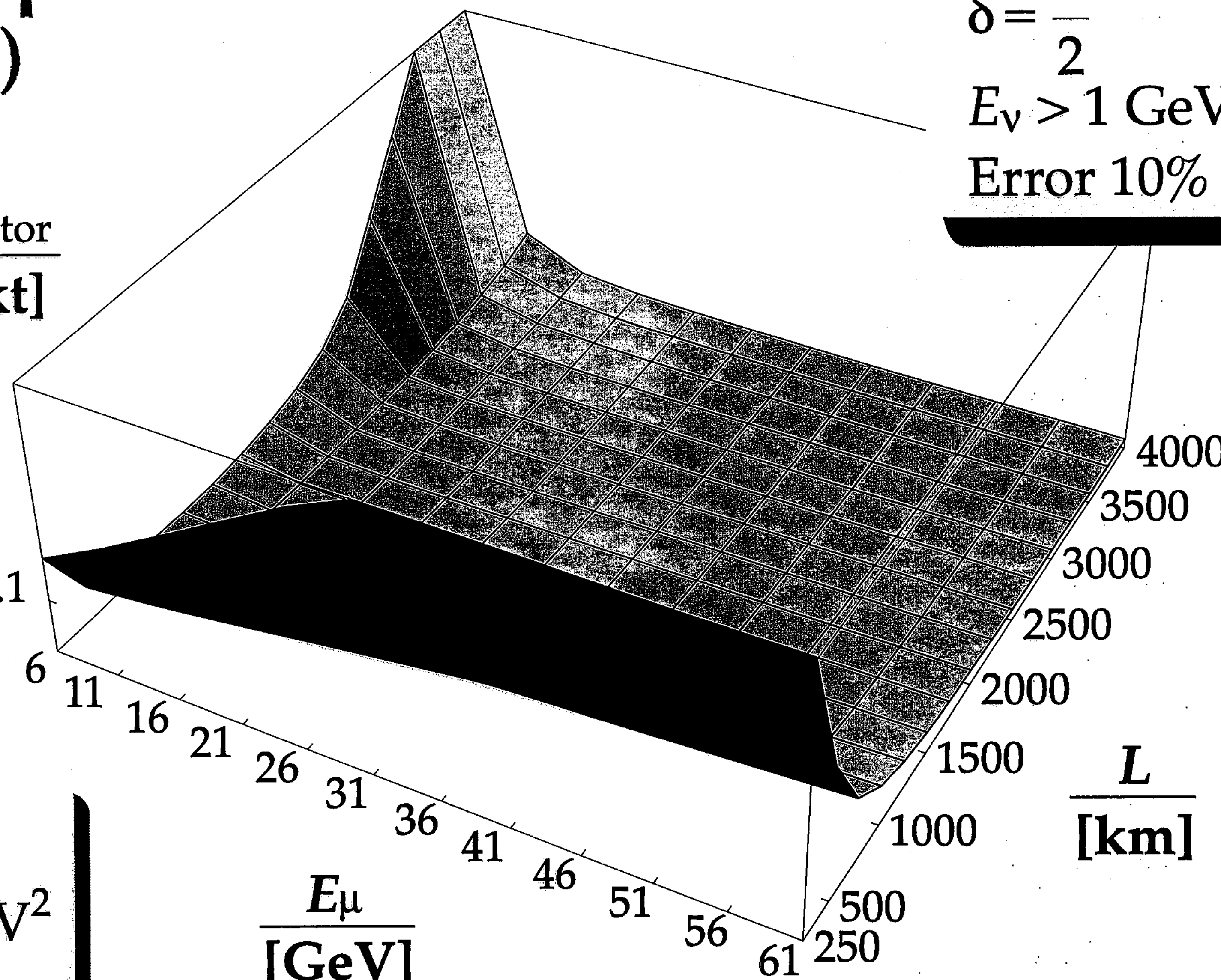
Required Exposure (T violation)

$$\delta = \frac{\pi}{2}$$

$E_\nu > 1$ GeV

Error 10%

$\frac{N_\mu}{[10^{21}]} \quad \frac{M_{\rm detector}}{[100\ {\rm kt}]}$



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$\tilde\rho(L) \approx \rho_{\rm PREM}(L)$

$\delta\tilde{m}^2_{31} \approx 3.0 \times 10^{-3}\ {\rm eV}^2$

$\delta\tilde{m}^2_{21} \approx 1.0 \times 10^{-4}\ {\rm eV}^2$

5. Discussion

CP violation

Ambiguity of ρ ⟶ Matter effect? True CP violation?

The sensitivity is lost in large L.

T violation

Consistent to intuitive condition

$$\delta m_{21}^2 < \frac{E_\nu}{L} < \delta m_{31}^2$$

Considering σ and $f_\nu(E_\nu)$,

$$\frac{E_\nu}{L} \sim \frac{\delta m_{31}^2}{2}$$

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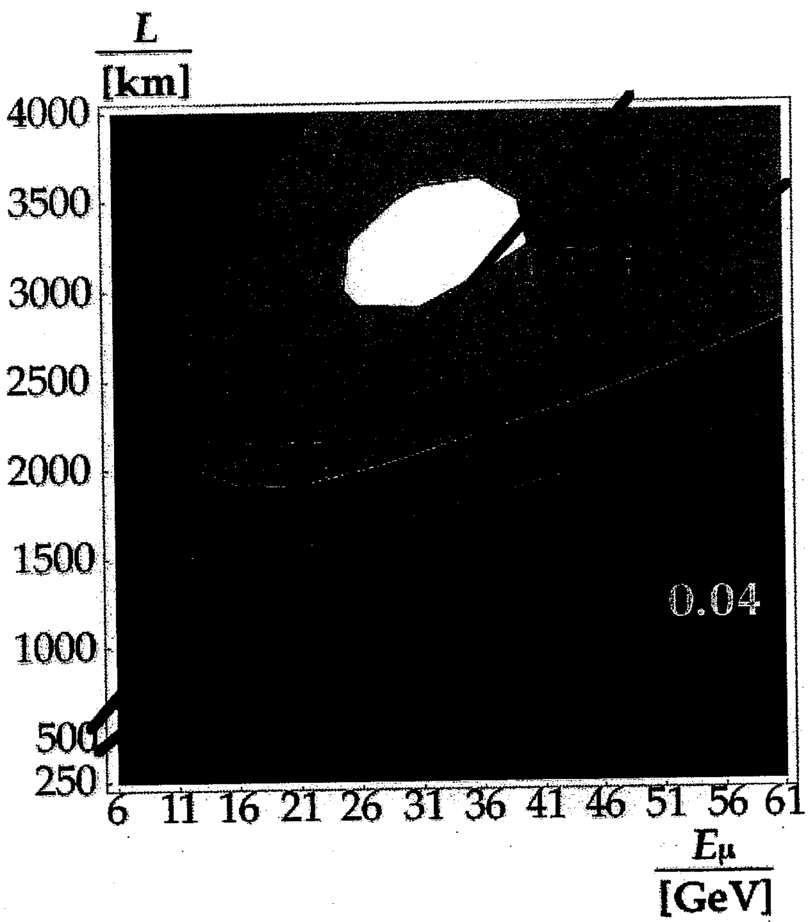
PRD**61**,073012
(Erratum *ibid*.**62**,079903)



Note: Many other sources of ambiguities that are not considered here.

6. Conclusion

CP violation search

- Make up a statistics that is sensitive to IMAGINARY part of the Lagrangian.

CP violation

- Ambiguity of ρ eliminates the sensitivity to CP-violation effects.
- $L <$ (~ 1000)km is required.

T violation

- $L \sim 3000$km, $E_\mu \sim 30$GeV is the most effective for T-violation search.

Outlook — Left to improve

- Errors, backgrounds...
- Consider the energy of charged leptons.
- Search for better statistical quantity.

Definition

3×3 CKM matrix applied to the first and third columns yields

$$(V^\dagger V)_{bd} = V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ . \qquad (11.19)$$

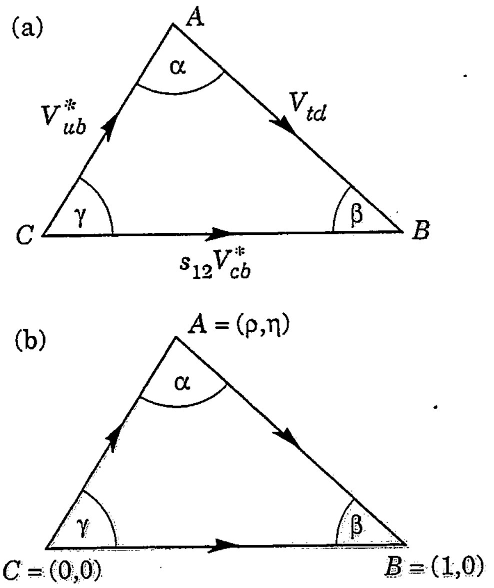
The unitarity triangle is just a geometrical presentation of this equation in the complex plane [43]. We can always choose to orient the triangle so that $V_{cd}\ V_{cb}{}^*$ lies along the horizontal; in the parametrization we have chosen, V_{cb} is real, and V_{cd} is real to a very good approximation in any case. Setting cosines of small angles to unity, Eq. (11.19) becomes

$$V_{ub}{}^* + V_{td} = s_{12}\ V_{cb}{}^* \ , \qquad (11.20)$$

which is shown as the unitarity triangle in Fig. 11.1(a). Rescaling the triangle by a factor $[1/|s_{12}\ V_{cb}|]$ so that the base is of unit length, the coordinates of the vertices become

$$A(\mathrm{Re}(V_{ub})/|s_{12}\ V_{cb}|\ ,\ -\mathrm{Im}(V_{ub})/|s_{12}\,V_{cb}|)\ ,\ B(1,0)\ ,\ C(0,0)\ . \qquad (11.21)$$

In the Wolfenstein parametrization [4], the coordinates of the vertex A of the unitarity triangle are simply (ρ, η), as shown in Fig. 11.1(b). The angle $\gamma = \delta_{13}$.



The angles of the triangle are

$$\phi_1 = \pi - \arg\left(\frac{-V_{tb}^* V_{td}}{-V_{cb}^* V_{cd}}\right) = \beta \ ,$$

$$\phi_2 = \arg\left(\frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}}\right) = \alpha \ ,$$

$$\phi_3 = \arg\left(\frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}}\right) = \gamma \ .$$

Figure 11.1: (a) Representation in the complex plane of the triangle formed by the CKM matrix elements $V_{ub}{}^*$, V_{td}, and $s_{12}\ V_{cb}{}^*$. (b) Rescaled triangle with vertices $A(\rho, \eta)$, $B(1,0)$, and $C(0,0)$.

Constraints

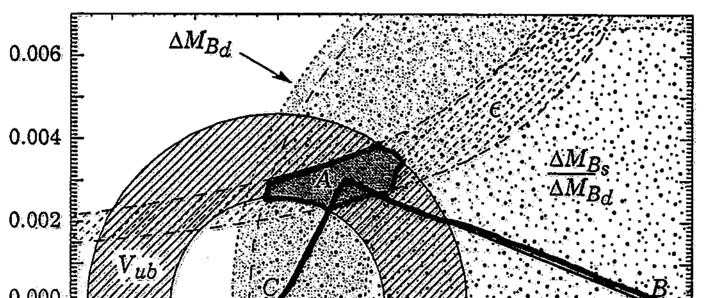
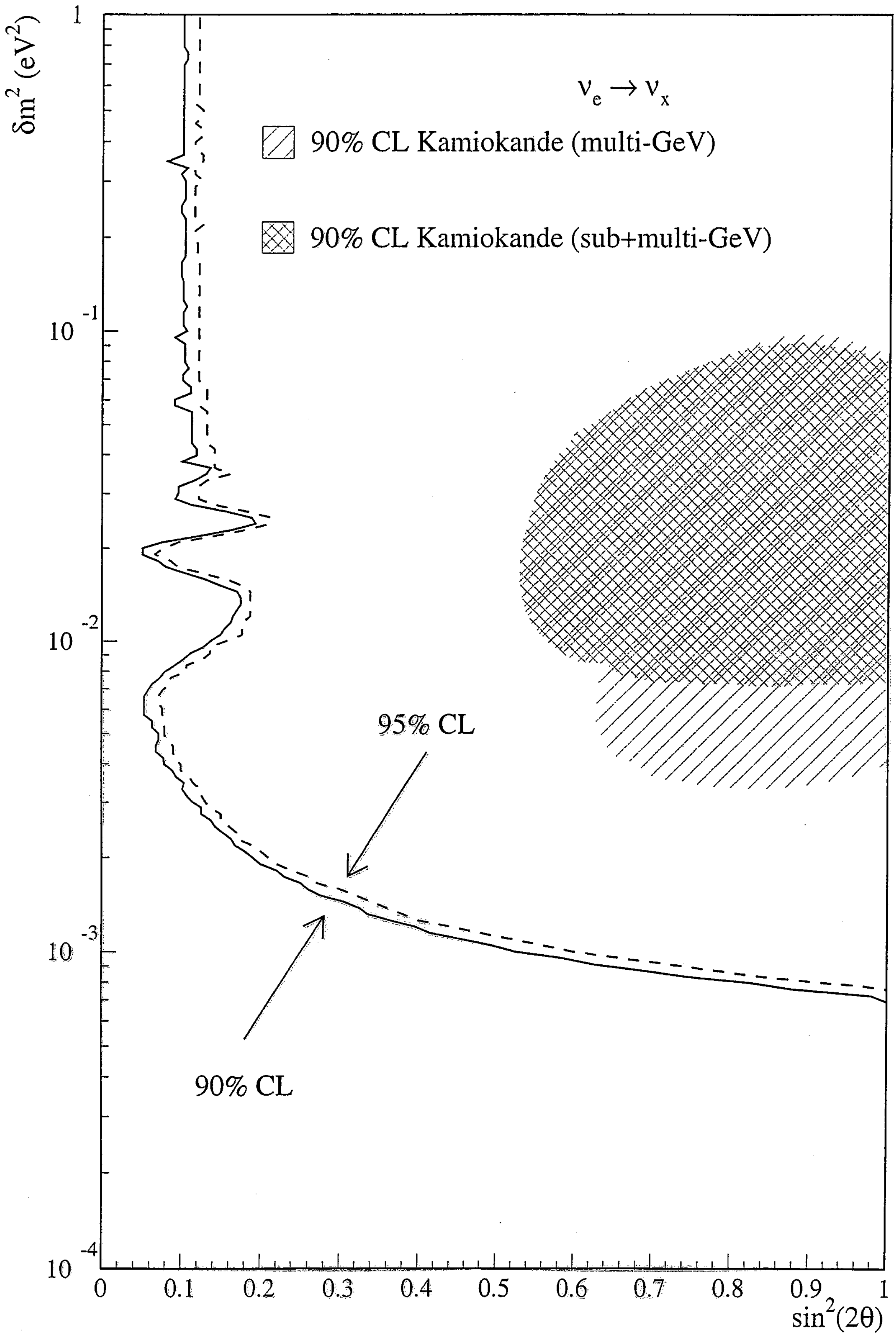


Figure 11.2: Constraints on the position of the vertex, A, of the unitarity triangle following from $|V_{ub}|$, B mixing, and the CP-violating parameter ϵ. A possible unitarity triangle is shown with A in the preferred region.

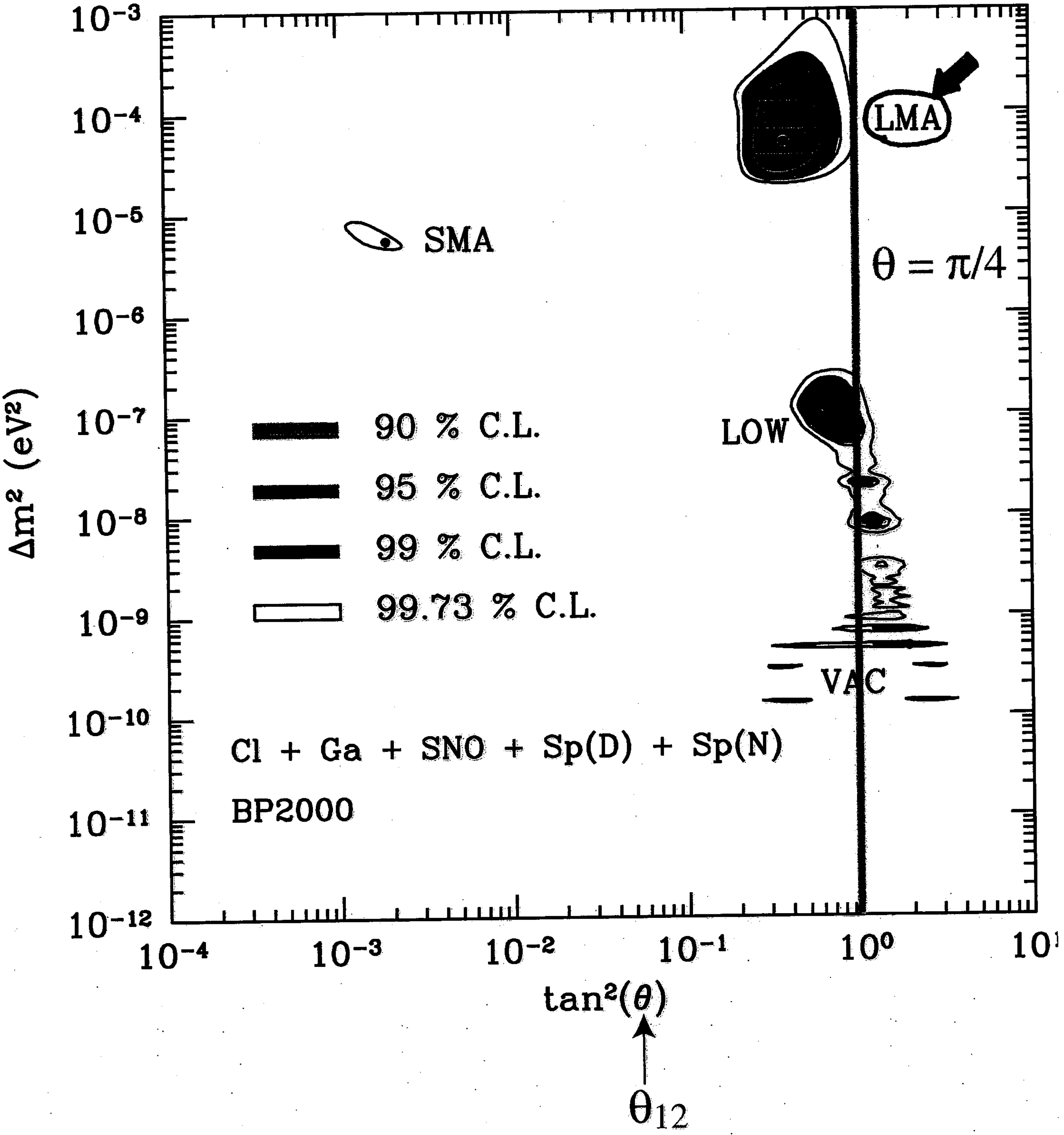
PDG(2000)

Exclusion plot of Chooz Experiments

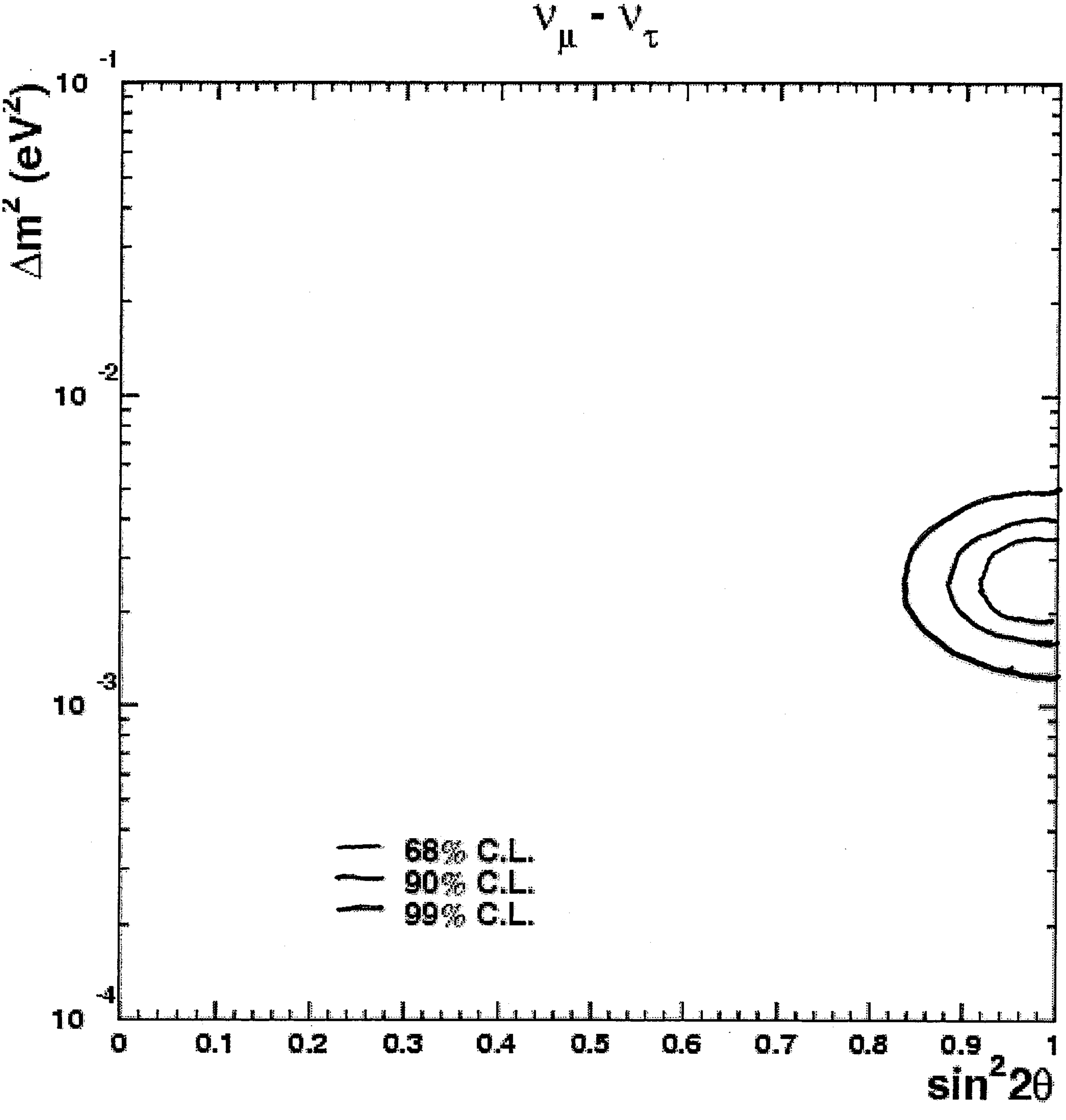


Phys.Lett. B466 (1999) 415

Krastev&Smirnov, hep-ph/0108177



Allowed region of atmospheric neutrino observations at Super-Kamiokande



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Oscillation Probability

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_{i=1}^{3} U_{\beta i}\, e^{i p_i L} (U^\dagger)_{i\alpha} \right|^2$$

$$\left(\begin{array}{l} p_i = \sqrt{E^2 - m_i^2} \simeq E - \dfrac{m_i^2}{2E} \\ p_i - p_j \simeq \dfrac{\delta m_{ij}^2}{2E} \ ; \ \delta m_{ij}^2 \equiv m_i^2 - m_j^2 \end{array} \right)$$

$$= \sum_{ij} U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^* \, e^{-i \frac{\delta m_{ij}^2 L}{2E}}$$