The photon structure function in all x region

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- Improvement for the parton distributions of the real photon at all x

  Hadronic initial distributions of the gluon and the sea quark

  -------→ Pomeron cut instead of P-pole

  Point like parts of the parton distributions

  -------→ Containing up to $\mathcal{O}(\alpha_s(Q^2))$ term

  MS-bar(Modified Minimal Subtraction) scheme ----→ Modified MS-bar scheme

1. Introduction

1.1 $e\gamma$ Deep Inelastic Scattering (DIS)

* $e^+e^- \to e^+e^- X$ (Fig.1.1.1)

Equivalent photon approximation (EPA),

$$d\sigma_{ee\to eeX} = d\sigma_{e\gamma\to eX} f_{\gamma/e},$$

$f_{\gamma/e}$: The flux of target photons.

* Cross section for the $e\gamma$ DIS process : $e\gamma \to eX$

(Fig.1.1.2)

$$\frac{d\sigma(e\gamma \to eX)}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1-y)F_2^\gamma + xy^2 F_1^\gamma\right]$$

Single−tag e^+e^- experiments → $y \approx 0 \to F_1^\gamma \approx 0$

*Bjorken scaling variable x (Fig.1.1.3)

$x = \dfrac{Q^2}{2p \cdot q}$: **The fraction of the real photon's momentum carried by the struck parton**

$y = \dfrac{p \cdot q}{p \cdot k}$: **The fraction of the incoming electron energy carried by the exchange photon**

* QCD Factorization Theorem (Fig.1.1.5)

Distribution functions

NS (Non-Singlet) distribution function : $q_{NS}(x,Q^2) = \sum_i^f (e_i^2 - \langle e^2 \rangle)(q_i + \overline{q_i})$

Σ (Singlet) distribution function : $\Sigma(x,Q^2) = \sum_i^f (q_i + \overline{q_i})$

G (Gluon) distribution function : $G(n,Q^2)$

Coefficient function (Fig.1.1.6)

$$\left(1+\frac{\alpha_S(Q^2)}{4\pi}B_q(x)\right),\ \frac{\alpha_S(Q^2)}{4\pi}B_G(x),\ 3f\langle e^4\rangle\frac{\alpha}{4\pi}B_\gamma(x)$$

Photon structure function

$$\frac{1}{x}F_2^\gamma(x,Q^2)=q_{NS}(x,Q^2)+\langle e^2\rangle\Sigma(x,Q^2)$$

$$+\frac{\alpha_S(Q^2)}{4\pi}B_q(x)*[q_{NS}(x,Q^2)+\langle e^2\rangle\Sigma(x,Q^2)\ ]$$

$$+\ \langle e^2\rangle\frac{\alpha_S(Q^2)}{4\pi}B_G(x)*G(x,Q^2)+3f\langle e^4\rangle\frac{\alpha}{4\pi}B_\gamma(x)$$

1.2 PQCD

* QCD Lagrangian

$$L=-\frac{1}{4}F^a_{\mu\upsilon}F^{a\mu\upsilon}-\frac{1}{2\xi}(\partial^\mu A^a_\mu)^2+i(\partial^\mu\chi^a_1)D^{ab}_\mu\chi^b_2+\overline{\varphi^i}(i\gamma^\mu D^{ij}_\mu-m\delta^{ij})\varphi$$

$$F^a_{\mu\upsilon}=\partial^\mu A^a_\upsilon-\partial_\upsilon A^a_\mu+gf^{abc}A^b_\mu A^c_\upsilon$$

$$D_\mu=\partial_\mu-igT^cA^c_\mu$$

* Renormalization

Dimensional regularization

MS-bar (Modified Minimal subtraction) scheme

Next-to Leading Order (NLO)

Effective running coupling

$$\frac{\alpha_s(Q^2)}{4\pi}\cong\frac{1}{\beta_0\ln\frac{Q^2}{\Lambda^2}}-\frac{\beta_1}{\beta_0^3}\frac{\ln\ln\frac{Q^2}{\Lambda^2}}{\left(\ln\frac{Q^2}{\Lambda^2}\right)^2}.$$

$$\beta_0=11-{}^{2f}\!/_3\ ,\ \beta_1=102-{}^{38f}\!/_3$$

1.3 Dokshitzer, Gribov, Lipatov, Altarelli and Parisi (DGLAP) Equation

DGLAP Equation······· $\alpha_S \ln\frac{Q^2}{\Lambda^2}$: the log type factor

Inhomogeneous integro−differential equations

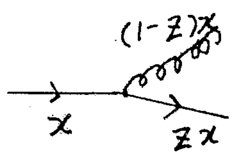
$$\frac{dq_i^\gamma(x,t)}{dt} = P_{q\gamma}(x,\alpha_S(t),\alpha) + \int_x^1 \frac{dz}{z}\left\{P_{qq}\left(\frac{x}{z},\alpha_S(t)\right)q_i^\gamma(z,t) + P_{qG}\left(\frac{x}{z},\alpha_S(t)\right)G^\gamma(z,t)\right\}$$

$$\frac{dG(x,t)}{dt} = P_{G\gamma}(x,\alpha_S(t),\alpha) + \int_x^1 \frac{dz}{z}\left\{P_{Gq}\left(\frac{x}{z},\alpha_S(t)\right)\sum_{j=1}^{2f} q_j^\gamma(z,t) + P_{GG}\left(\frac{x}{z},\alpha_S(t)\right)G^\gamma(z,t)\right\}$$

$$t = \ln\frac{Q^2}{\Lambda^2}$$

* Splitting function $P_{ij}(z,\alpha_s(Q^2))$ (Fig.1.1.7)

Quark with a momentum fraction x



Parton of a momentum fraction $(1-z)x$

Parton of a momentum fraction zx

$$P_{ij}(z,\alpha_s(Q^2)) = \frac{\alpha_S(Q^2)}{2\pi}P_{ij}^{(0)}(z) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 P_{ij}^{(1)}(z) + \cdots\cdots \quad ij = qq, qG, Gq, GG$$

$$P_{q\gamma}(z,\alpha_s(Q^2)) = \frac{\alpha}{2\pi}P_{q\gamma}^{(0)} + \frac{\alpha\alpha_S(Q^2)}{(2\pi)^2}P_{q\gamma}^{(1)}(z) + \cdots\cdots$$

$$P_{G\gamma}(z,\alpha_s(Q^2)) = \frac{\alpha\alpha_S(Q^2)}{(2\pi)^2}P_{G\gamma}^{(1)}(z) + \cdots\cdots$$

2 Photon distributions at small x

[M.Imoto, H.Kan and T.Kikuchi: Prog.Theor.Phys.Vol.102No.4(1999)]

2.1 Hadronic part of photon distributions(Fig.2.1.1)

Solution of DGLAP Equation

Mellin transformation $f(n,Q^2) = \int_0^1 x^{n-1} f(x,Q^2)dx$.

General (hadronic) solutions of the homogeneous equations

$$q_{NS,H}(n,Q^2) = \left\{1 - \frac{2}{\beta_0} R_{NS}(n) \frac{\alpha_s(Q^2)}{2\pi} + \frac{2}{\beta_0} R_{NS}(n) \frac{\alpha_s(Q_0^2)}{2\pi}\right\}$$

$$\times L(Q^2)^{-\frac{2}{\beta_0} P_{NS}^{(0)}(n)} q_{NS}(n, Q_0^2)$$

$$\begin{Bmatrix} \Sigma_H(n,Q^2) \\ G_H(n,Q^2) \end{Bmatrix} = \left[\frac{B(\lambda_-, Q_0^2)\Sigma(n,Q_0^2) - A(\lambda_-, Q_0^2)G(n,Q_0^2)}{A(\lambda_+, Q_0^2)B(\lambda_-, Q_0^2) - B(\lambda_+, Q_0^2)A(\lambda_-, Q_0^2)}\right]$$

$$\times \begin{Bmatrix} A(\lambda_+, Q^2) \\ B(\lambda_+, Q^2) \end{Bmatrix} L(Q^2)^{-\frac{2}{\beta_0}\lambda_+} + [\lambda_+] \leftrightarrow [\lambda_-],$$

$$L(Q^2) = \frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)}, \quad R_{ij} = P_{ij}^{(1)}(n) - (\beta_1 / 2\beta_0) P_{ij}^{(0)}(n) \quad \text{for} \quad ij = (NS,\ qq,\ qG,\ Gq,\ GG)$$

$$\lambda_{+,-} = \frac{1}{2}\sqrt{\left(P_{qq}^{(0)}(n) + P_{GG}^{(0)}(n)\right) \pm \left\{\left(P_{qq}^{(0)}(n) - P_{GG}^{(0)}(n)\right)^2 + 4 P_{qG}^{(0)}(n) P_{Gq}^{(0)}(n)\right\}}.$$

2.2 Hadronic input of photon distributions

Vector meson dominance model (VMD)

Q_0^2 : **Initial value, 1 GeV²**

Regge Theory: Amplitude in large s and small t → $x \approx 0$

Counting rules: Amplitude in large s and large t → $x \approx 1$

$$t = (p-q)^2 = -2p \cdot q - Q^2 = -2p \cdot q(1+x)$$

Regge Theory (Fig.2.2.1)

$$A(s,t) \underset{s\to\infty}{\approx} \sum_{l=0}^{\infty} \frac{\beta(t)(\cos\theta_t)^l}{l-\alpha(t)} \approx \beta(t)(\cos\theta_t)^{\alpha(t)} \approx \beta(t)s^{\alpha(t)}$$ [Regge Pole exchange]

$$A(s,t) \underset{s\to\infty}{\approx} \beta_c(t)\frac{s^{\alpha_c(t)}}{\ln s} \approx \beta_c(t)\frac{x^{-\alpha_c(t)}}{\ln(1/x)}$$ [Regge Cut exchange]

$$s=(p+q)^2=2p\cdot q-Q^2=2p\cdot q(1-x)$$

$q_{NS,H}(n,Q_0^2)$, $\Sigma_H(n,Q_0^2)$, $G_H(n,Q_0^2)$: Hadronic input of photon distributions

Pomeron–cut exchange

$$x\,V^{\pi}(x) = a\ x^{1-\alpha_\rho(0)}(1-x),$$

$$x\,\zeta^{\pi}(x) = c(x^{1-\alpha_C(0)}/\ln(1/x))(1-x)^5$$

$$x\,G^{\pi}(x) = b(x^{1-\alpha_C(0)}/\ln(1/x))(1-x)^3,$$

$$\alpha_{Regge}(t) = \alpha_\rho(t)$$

$$\alpha_c(t) = \{\alpha_P(t)+\alpha_P(t)-1\}$$

$$\alpha_C(0) > \alpha_P(0) = 1.08$$

Effect of the Pomeron–cut exchange

* **Lipatov behavior**: $x^{-\lambda}$.
* $1/\ln(1/x)$ **term: screening effect**
* $x\,G^N(x,Q^2) \propto (x^{1-\alpha_P^B}/[\ln(1/x)]^{1/2})$:

BFKL(Balitzkij,Fadin,Kuraev,Lipatov) Eq.(Fig.2.2.2)

$$\alpha_P^B = 1+\left(12\ln(2)\bar{\alpha}_s/\pi\right)$$ and $\bar{\alpha}_s$ is fixed ,i.e., $\bar{\alpha}_s = 0.2$

Log type factor resummation

$$\alpha_S \ln\left(\frac{Q^2}{s}\right) \approx \alpha_S \ln\left(\frac{x}{1-x}\right)$$: **BFKL equation(**x evolve)

$$\alpha_S \ln\frac{Q^2}{\Lambda^2}$$: **DGLAP equation(**Q^2 evolve)

2.3 Point-like part of photon distributions(Fig.2.3.1)

Particular(point-like) solutions of the inhomogeneous equations

$$q_{NS,PL}(n,Q^2)=\frac{\alpha}{2\pi}\left[\{a_{11}(n)\frac{1-L^{1-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}}{1-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}\right]\left(\frac{\alpha_S(Q^2)}{2\pi}\right)^{-1}$$

$$+\frac{\alpha}{2\pi}\left[a_{12}(n)\frac{1-L^{1-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}}{1-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}+a_{21}(n)\frac{1-L^{-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}}{-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}\right]$$

$$+\frac{\alpha}{2\pi}\left[a_{22}(n)\frac{1-L^{-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}}{-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}+a_{31}(n)\frac{1-L^{-1-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}}{-1-\frac{2}{\beta_0}P_{NS}^{(0)}(n)}\right]\left(\frac{\alpha_s(Q^2)}{2\pi}\right)$$

$$* \quad \left[\left(1-L^{-M-2f(n)/\beta_0}\right)\Big/\left(-M-2f(n)/\beta_0\right)\right]\left(\frac{\alpha_s(Q^2)}{2\pi}\right)^{M}$$

Denominator, $(-M-2f(n)/\beta_0)=0$ at $n=n^M$.

Inverse Mellin transform of the term $1\big/(n-n^M)\ \rightarrow\ (1/x)^{n_M}$

2.4 Results (Fig.2.4.1)

Comparison of our predictions with the OPAL experimental data

K. Ackerstaff et al., OPAL Collab.: Phys. Lett. **B411,** 387(1997).

K. Ackerstaff et al., OPAL Collab.: Z.Phys. **C74,** 33(1997).

3. Photon distributions at large x

[M.Imoto and F.Kawane: submitted to Prog.Theor.Phys.]

3.1 Gluon Distribution Function (Q^2 = 75 GeV²)

(Fig.3.1.1)

- **Our predictions (solid curve)**

  HERA---H1 experimental data [T.Ahmed et al.,H1 Collab.:Nucl.Phys.**B445**,195(1995).]

  GRV predictions.(Valence-type Gluon, $Q_0^2 = 0.3$ **GeV²,** κ = 1.6)

3.2 Unphysical behavior near $x = 1$

$$B_\gamma(x) = 4\left([x^2 + (1-x)^2]\ln\frac{1-x}{x} - 1 + 8x(1-x)\right)$$

Singular part of term, $3f\langle e^4\rangle\frac{\alpha}{4\pi}$ $B_\gamma(x)$ is absorbed into

NS-distribution

$$B_\gamma(x) \Rightarrow B_0(x) = 4\left(x^2\ln(1-x)\right)$$

$$P_{NS\gamma}^{(1)}(x) = 3f(\langle e^4\rangle - \langle e^2\rangle)P_{q\gamma}^{(1)}. \Rightarrow P_{NS\gamma}{}^{(1)}(x) + \delta P_{NA\gamma}{}^{(1)}(x)$$

$$\delta P_{q\gamma}^{(1)}(x) = -(1/2)B_0(x)P_{NS}^0(x)$$

Boundary conditions in the $\overline{MS}$ scheme for the point-like distributions,

$$q_{NS,PL}(x,Q_0^2) = \Sigma_{PL}(x,Q_0^2) = G_{PL}(x,Q_0^2) = 0.$$

3.3 Renormalization—scheme dependence

Renormalization-scheme dependence of **perturbative predictions**

Factorization-schemed dependence

Extra renormalization in addition to the ordinary renormalization

Modified $\overline{MS}$ scheme

4. Results

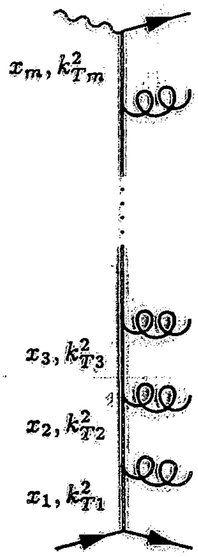
$Q_0^2 = 1$ GeV², $\Lambda = 200$ MeV and $f = 4$

$$F_2^{\gamma}(x,Q^2) = \frac{1}{\pi}\int_0^{\infty} dz\, \mathrm{Im}[e^{i\varphi} x^{-h-ze^{i\varphi}} F_2\,(n = h + ze^{i\varphi}, Q^2)].$$

$$\varphi = \frac{3}{4}\pi,$$

$h = 0.80$	for nonsinglet,
$h = 1.80$	for singlet,
$h = 1.75$	for gluon ,

Dokshitzer, Gribov, Lipatov, Altarelli and Parisi (DGLAP) Equation



DGLAP

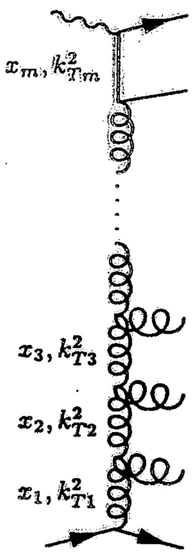
$x_1 > x_2 > x_3 \ldots > x_m = x$

$k_{T1}^2 \ll k_{T2}^2 \ll \ldots \ll k_{Tm}^2 = Q^2$

$$\frac{\mathrm{d}q_i(x,Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z}\left[\sum_j q_j(z,Q^2)P_{ij}^{(0)}\left(\frac{x}{z}\right) + g(z,Q^2)P_{ig}^{(0)}\left(\frac{x}{z}\right)\right]$$

$$\frac{\mathrm{d}g(x,Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z}\left[\sum_j q_j(z,Q^2)P_{gj}^{(0)}\left(\frac{x}{z}\right) + g(z,Q^2)P_{gg}^{(0)}\left(\frac{x}{z}\right)\right]$$

BFKL (Balitzkij, Fadin, Kuraev, Lipatov) Equation



BFKL (only gluon–gluon ladder)

$x_1 \gg x_2 \gg x_3 \ldots \gg x_m = x$

no ordering in k_{Ti}^2

(assume no evolution in Q^2)

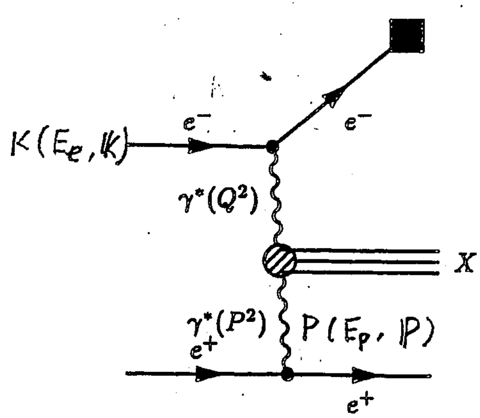
$k_{T1}^2 \simeq Q^2$

$$xg(x,Q^2) = \int_0^{Q^2} \frac{\mathrm{d}k_T^2}{k_T^2} f_g(x,k_T^2)$$

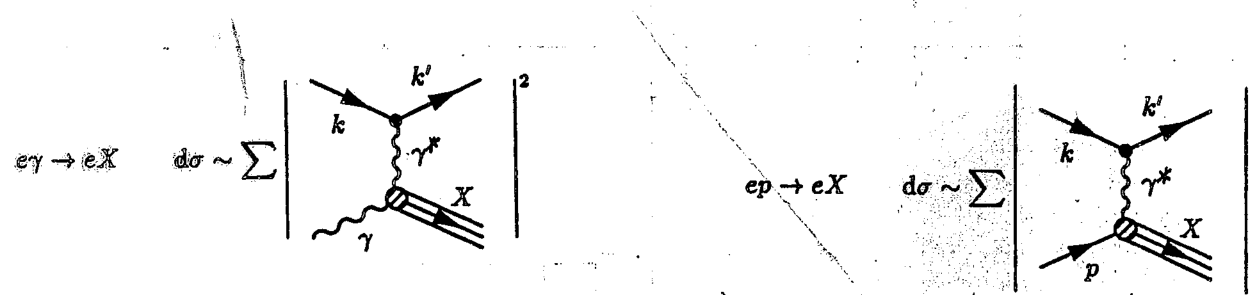
Summing up all ladder diagrams in $\ln(1/x)$ gives the BFKL equation:

$$-x\frac{\partial f_g(x,k_T^2)}{\partial x} = 3\frac{\alpha_S}{\pi}k_T^2 \int_0^\infty \frac{\mathrm{d}k_T'^2}{k_T'^2}\left[\frac{f_g(x,k_T'^2) - f_g(x,k_T^2)}{|k_T'^2 - k_T^2|} + \frac{f_g(x,k_T^2)}{\sqrt{4k_T'^4 + k_T^4}}\right] \equiv$$

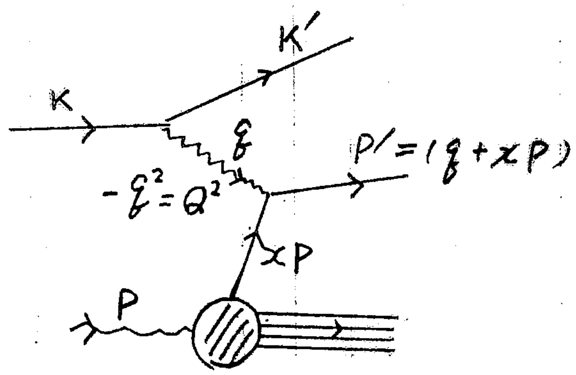
$$\equiv K \otimes f_g$$



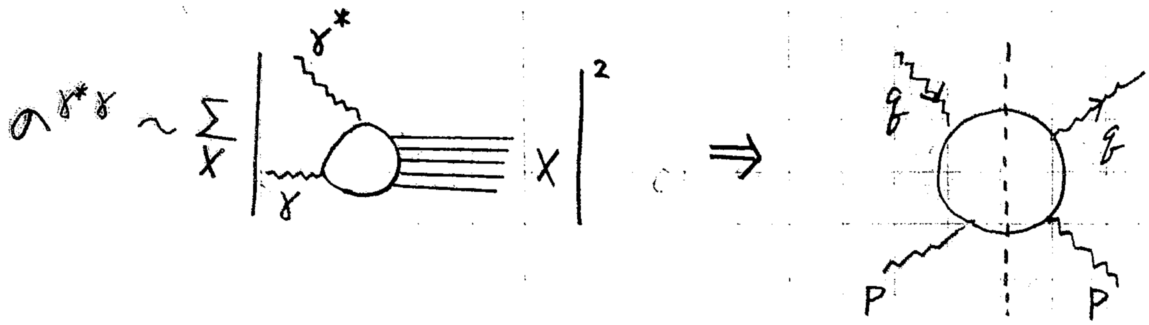
(Fig.1.1.1)



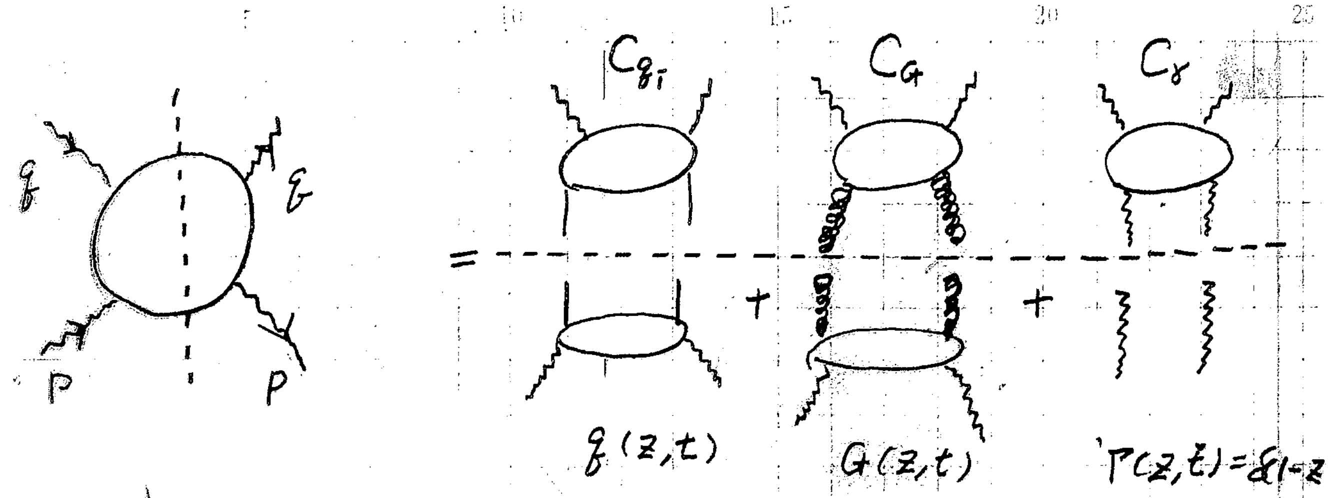
(Fig.1.1.2)



(Fig.1.1.3)



(Fig.1.1.4)



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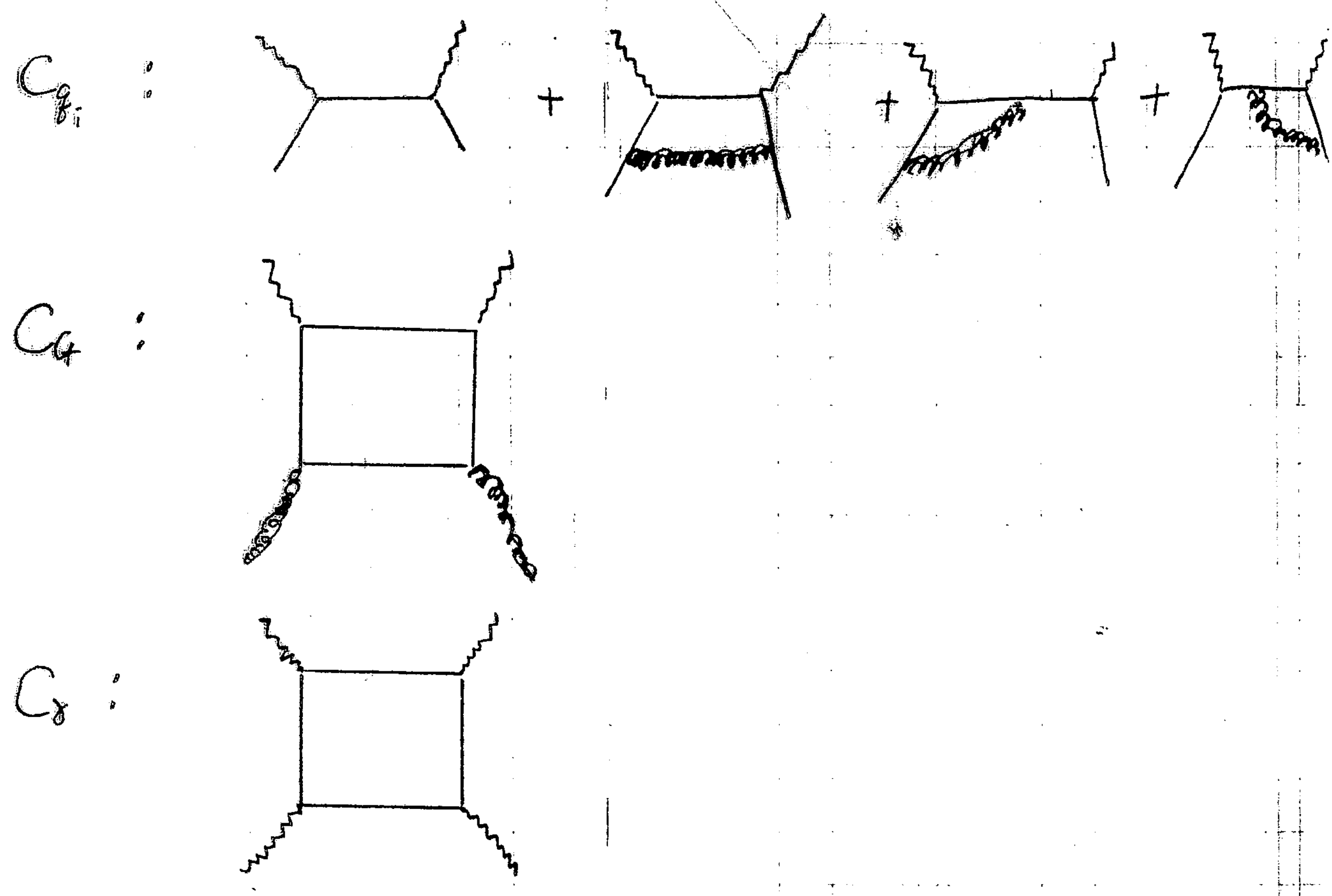
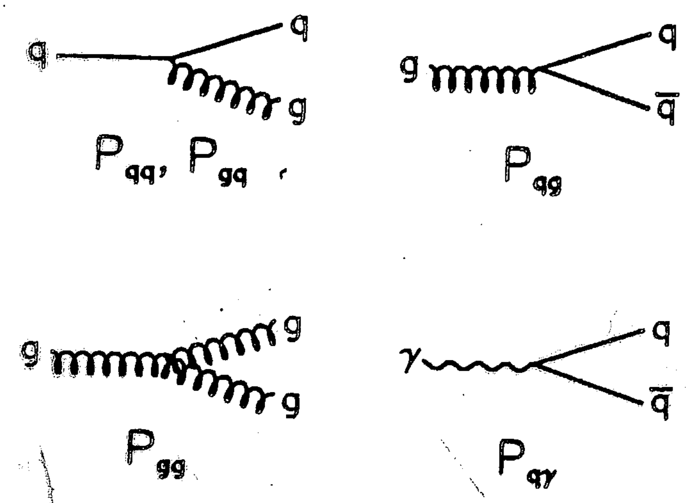
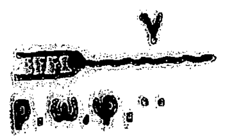
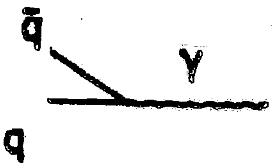


(Fig.1.1.6)



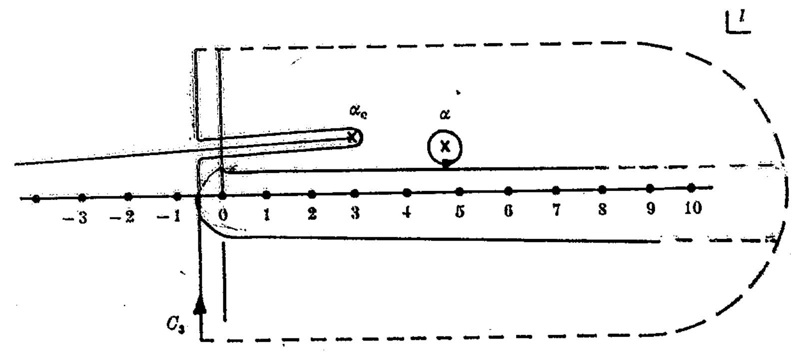
(Fig.1.1.7)





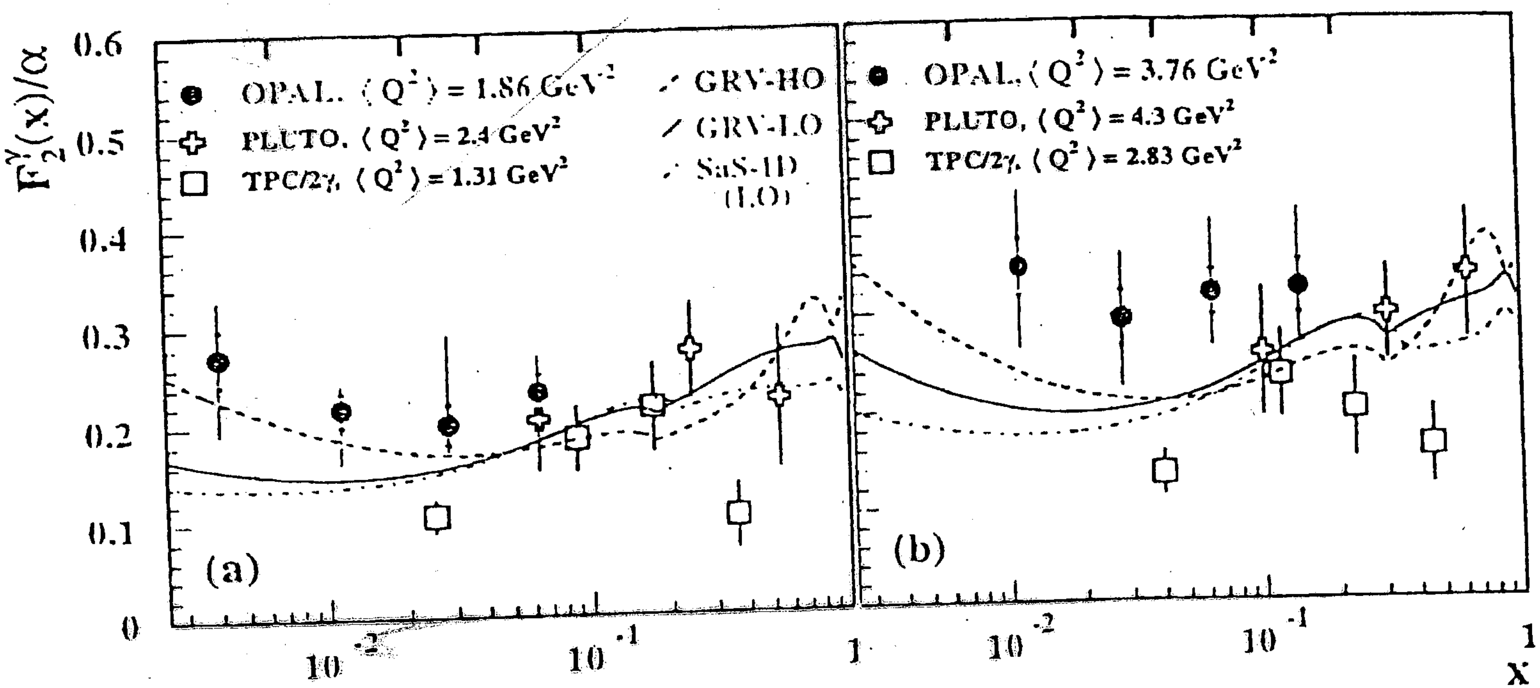
(Fig.2.1.1)

(Fig.2.3.1)



(Fig.2.2.1)

- OPAL (LEP1 data, low x and Q^2)



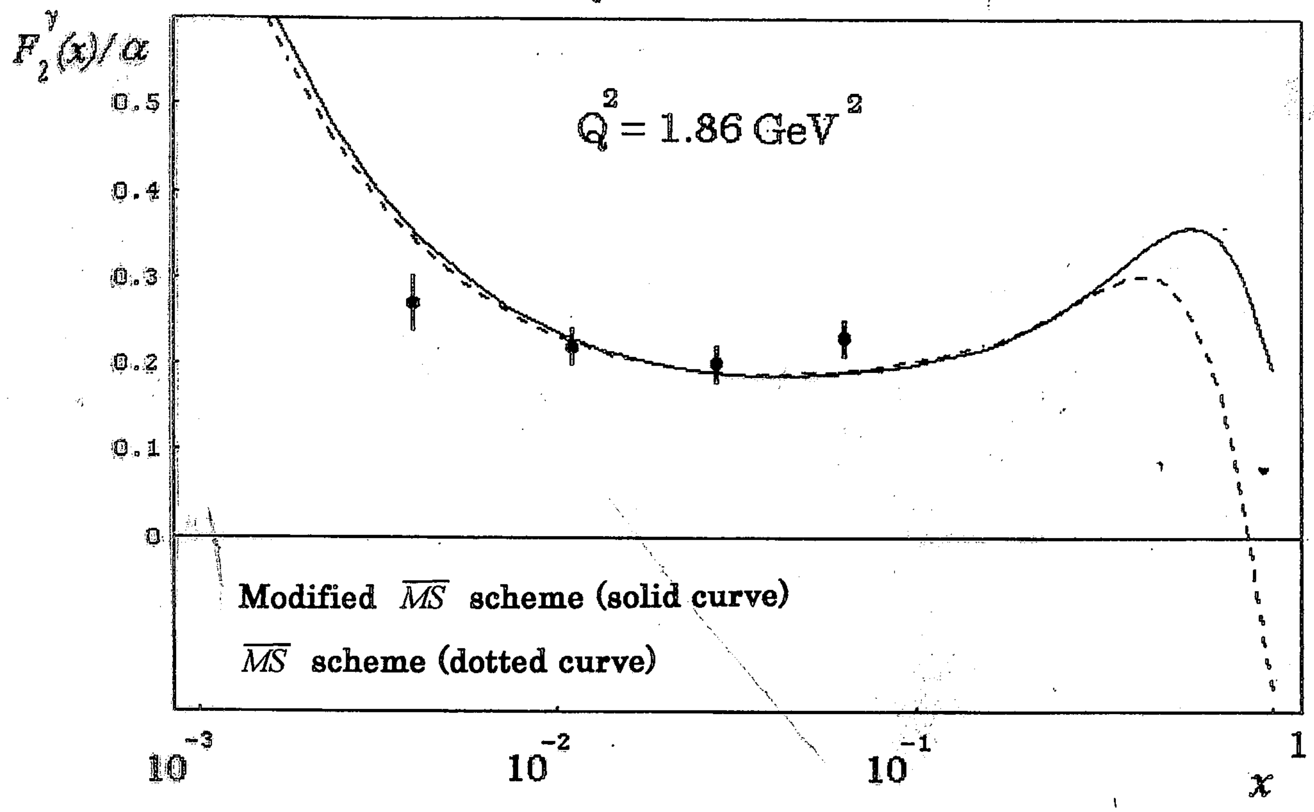


Fig.2.4.1(a)

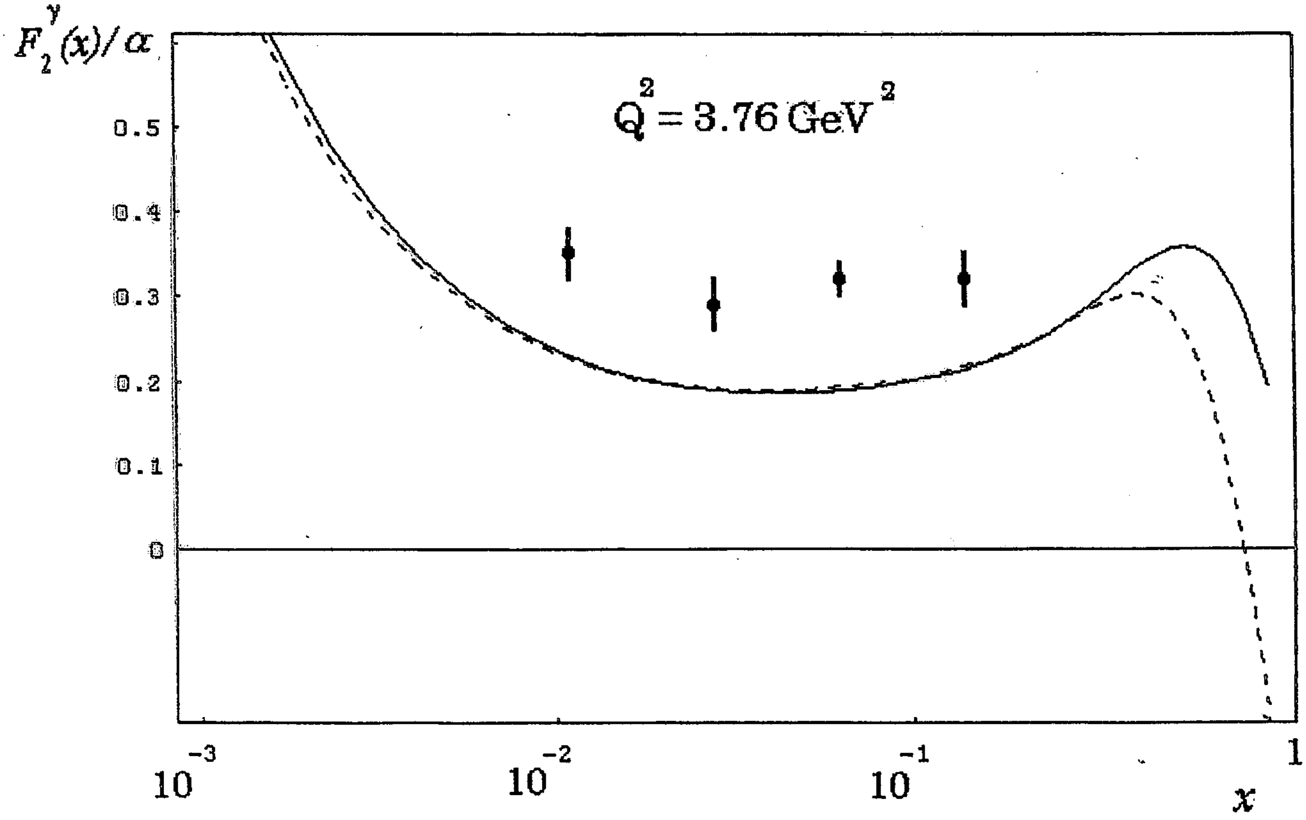
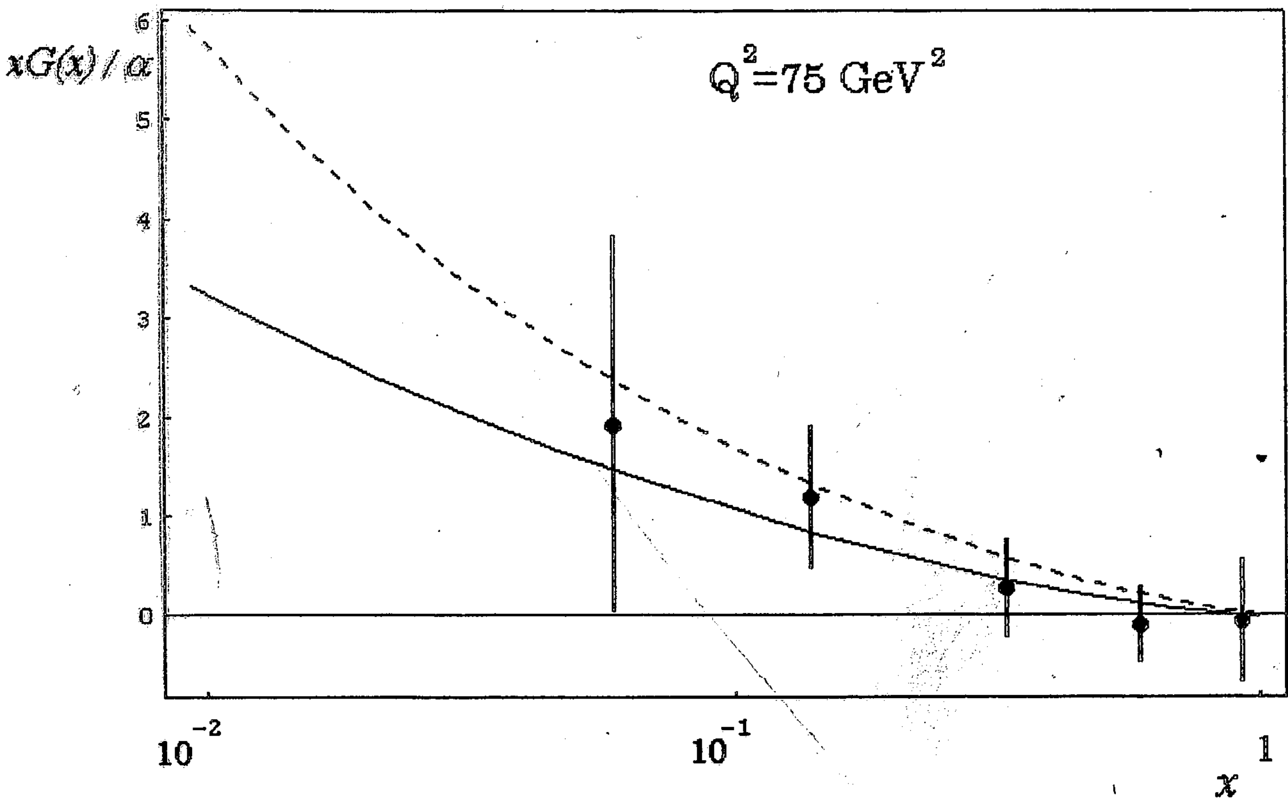
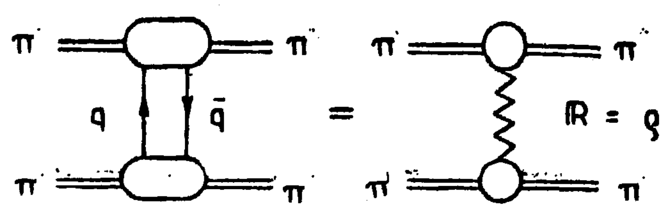


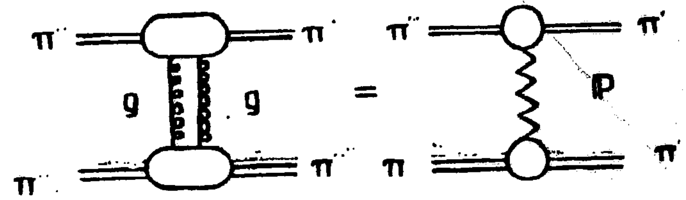
Fig.2.4.1(b)



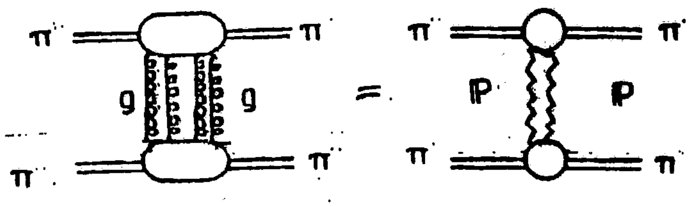
(Fig.3.1.1)



(2a)



(2b)



(2c)

Fig. 5

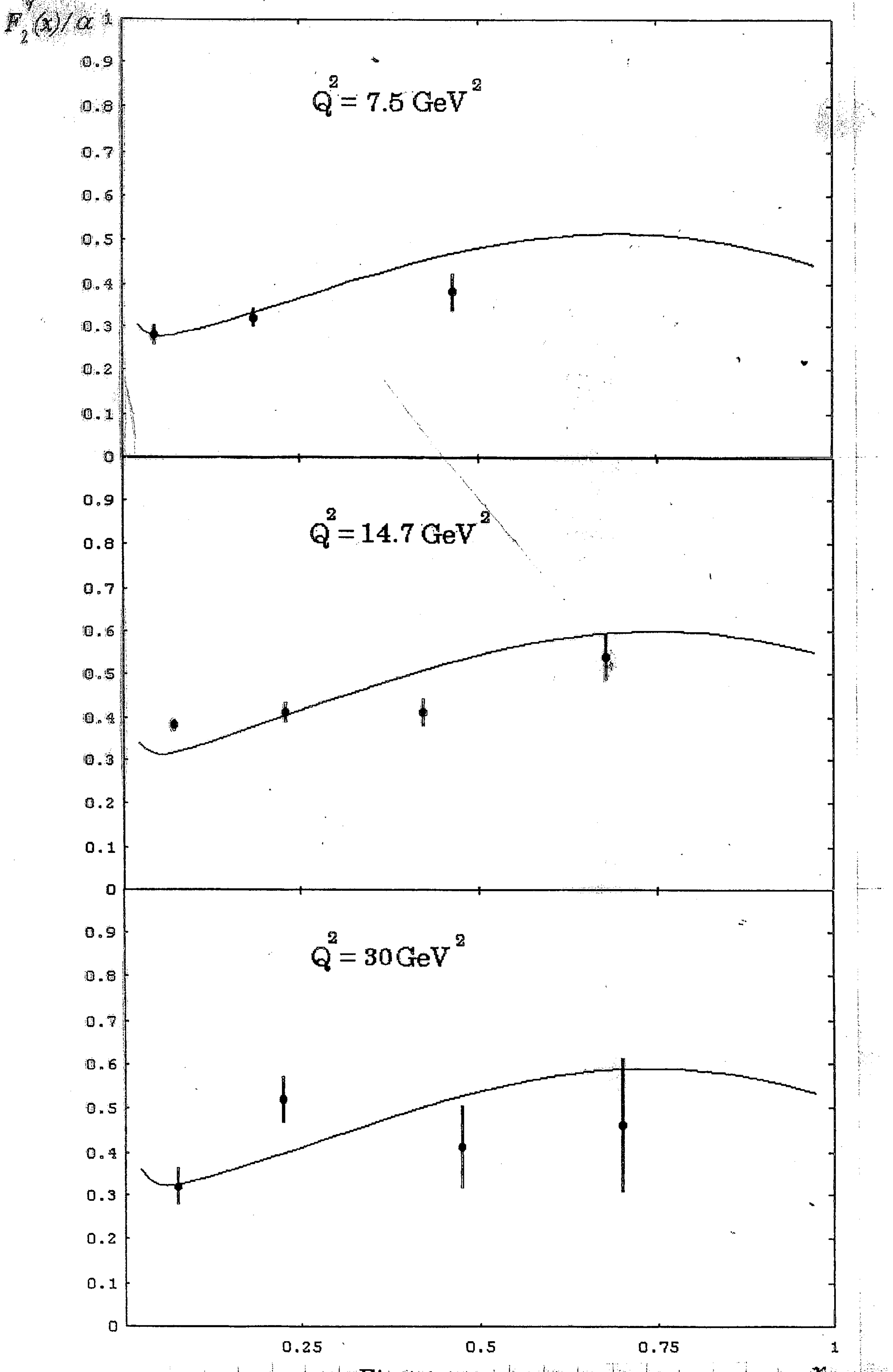


Fig.4.1.1 (b)

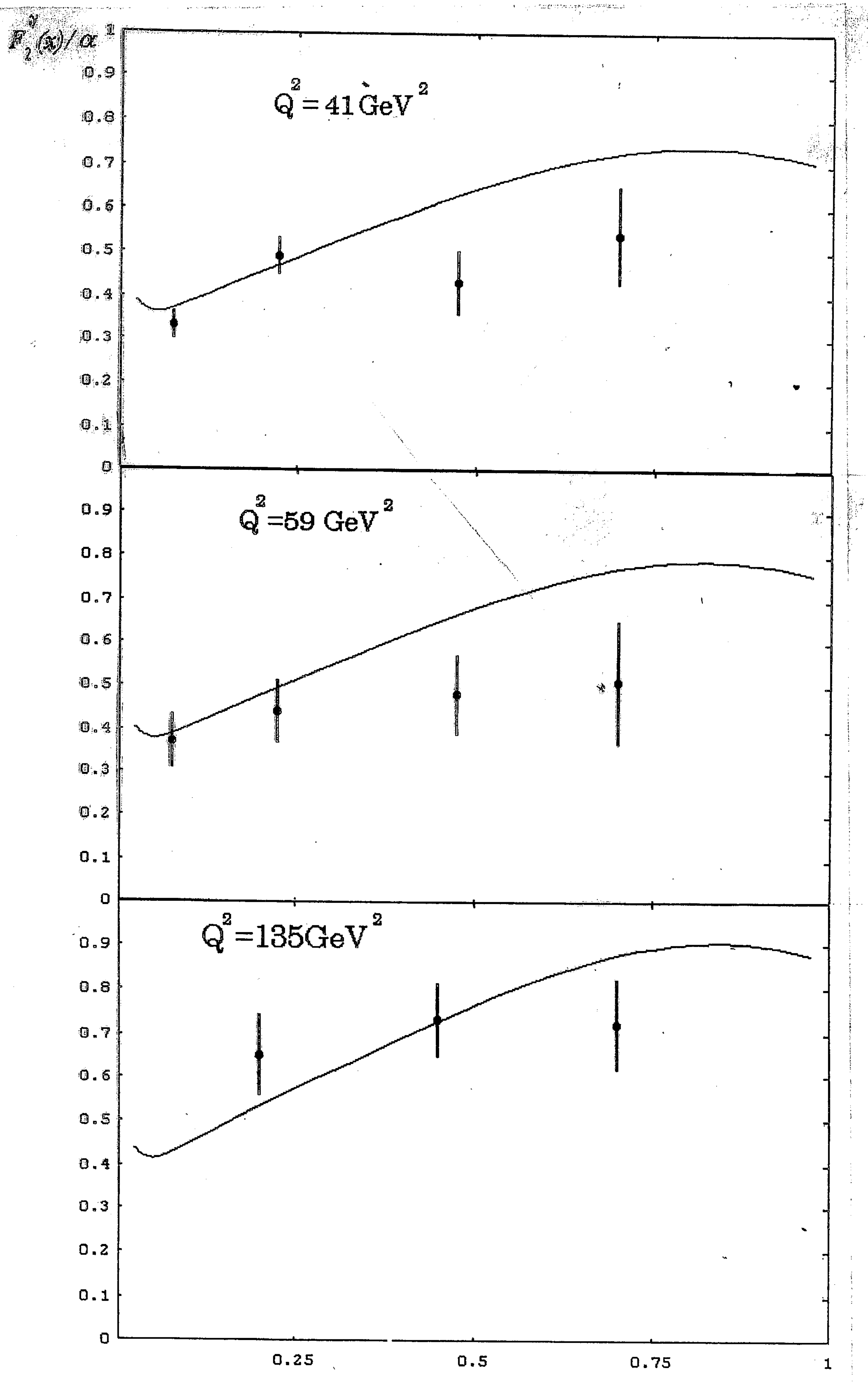

$F_2^\gamma(x)/\alpha$
$Q^2 = 41\,GeV^2$
$Q^2 = 59\,GeV^2$
$Q^2 = 135\,GeV^2$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.25
0.5
0.75
x

Fig.4.1.1 (17)